Differential Evolution for Optimal Electrical Distribution Network Reconfiguration

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Abstract- *Electrical distribution network reconfiguration is a complex optimization process aimed at finding a radial operating structure that minimizes the system power loss whilst satisfying operating constraints. This paper presents the application of Differential Evolution to solve the reconfiguration problem. The method is tested on two sets of test systems, one composed of two commonly used systems in academia and the other of a real-world system. Simulation results are presented for the proposed method, which is also compared to three Genetic Algorithms approaches.*

I. INTRODUCTION

Under normal operating conditions, distribution feeders may be frequently reconfigured by opening and closing switches to reduce line losses, improve feeder voltage profile and increase network reliability whilst meeting all load requirements and maintaining a radial network. These requirements result in a very complicated non-linear integer optimization problem. Enumerative schemes have been used to find the exact optimal solution of such a problem, requiring prohibitively long computational time. This is so because the number of switch options is usually very large in practical distribution systems. This problem is not easily solvable by standard optimization methods and yet its accurate solution can result in vast savings for electricity utilities.

Several methods have been reported to solve the network reconfiguration problem: enumerative (exhaustive), heuristics and random search methods. Enumerative schemes have been considered in [1], the search algorithm starts looking at objective function values at every point in the search space, one at a time and thus is computationally intensive. Therefore an enumerative search is infeasible in practice. Heuristic search method is guided by a simplified optimization procedure. These methods include Discrete Branch and Bound [2, 3] and Switch Exchange Type Heuristic [4-8].

In recent years, Evolutionary Algorithms (EA) have been applied to find the optimal solutions of many engineering applications. Under this class is grouped three well-known algorithms: Genetic Algorithms, Evolutionary Programming and Evolutionary Strategies. Moreover, there are also hybrid methods based on features of the above paradigms which can be classified into the broad class of EA methods. Population set based algorithms are direct search methods where a population of points are used and evolved to improve the solutions towards the optimum. Population set based algorithms are different from traditional methods in 4 ways: (1) they work with coding of parameters set not the parameters themselves; (2) they search for a population of points not a single point; (3) they use payoff (objective function) information, not derivatives or other auxiliary knowledge; (4) they use probabilistic transition rules, not deterministic rules.

Genetic Algorithms (GA) [9] and Differential Evolution (DE) [10] are population set based algorithms where the search procedure uses random choice as a tool to guide a highly exploitative search through a coding of parameter space. DE is a simple method that is based on stochastic searches, in which function parameters are encoded as floating-point variables. DE has been shown to be a promising candidate for solving real-valued optimization problems [9]. For the distribution network reconfiguration problem, the performance and applicability of GA compared to enumerative search and heuristic search have been recently presented in [11]. Furthermore, certain improvements were obtained with a fuzzy controlled GA [12]. In this paper DE is applied to solve the distribution network reconfiguration problem and its performance is compared to previous approaches using GA.

II. MATHEMATICAL MODEL OF DISTRIBUTION NETWORK RECONFIGURATION

Finding a radial operating structure that minimizes the system power loss while satisfying operating constraints is the objective of the distribution network reconfiguration problem. Thus network reconfiguration for loss minimization can be formulated as follows:

$$\underline{\text{Minimize}}\quad f_c = \sum_{i=1}^{n} Loss_i \qquad i \in NL$$

$$\therefore f_c = \sum_{i=1}^{n} \left|I_i\right|^2 k_i R_i \qquad i \in NL \tag{1}$$

where NL is the set of branches;
$Loss_i$ is loss within branch i;
I_i is the current in branch i;
R_i is the resistance of branch i;

k_i represents the topological status of the branches, k_i=1 if the branch i is closed, and k_i=0 if the branch i is open;

subject to the following constraints:

1. Radial network constraint
Distribution network should be composed of radial structure considering operational point of view.
$$\varphi(k)=0 \tag{2}$$
2. Power source limit constraint
The total loads of a certain partial network cannot exceed the capacity limit of the corresponding power source.
3. Node voltage constraint
Voltage magnitude at each node must lie with their permissible ranges to maintain power quality.
$$V_{i\,min} \le V_i \le V_{i\,max} \quad i \in N \tag{3}$$
where N is the set of nodes.
Subscripts 'min' and 'max' represent the lower and upper bounds of the constraint.
4. Branch current thermal stability constraints.
Current magnitude of each branch (feeder, laterals and switches) must lie with their permissible ranges.
$$k_i|I_i| \le I_{i\,max} \qquad i \in NL \tag{4}$$
5. Kirchhoff's current laws
$$g_i(I,k)=0 \tag{5}$$
6. Kirchhoff's voltage laws
$$g_v(V,k)=0 \tag{6}$$

To solve the distribution network reconfiguration problem, a network-topology-based three-phase distribution power flow algorithm developed by Teng [12] is used to determine the bus current injections and bus voltages. Power loss of the radial configuration is calculated as in [1].

III. Differential Evolution

Differential Evolution (DE) is a population set based algorithm designed for minimizing a function $f(x), \quad x \in \Omega \quad \subset R^n$ (Ω is assumed to be defined by specifying upper and lower limits of the domain of each variable). The DE algorithm is simple and extremely robust in locating the global minimum. DE is quite similar to GA with the following differences. Unlike simple GA, which uses binary coding for representing variables, DE uses real coding of floating point numbers. DE as in GA attempts to guide an initial set $S=\{x_1, x_2, \ldots\ldots, x_{NP}\}$ of points in Ω to the vicinity of the global minimum through repeated cycles of selection, reproduction (mutation and crossover) and acceptance. However, DE attempts to replace all points in S by new points at each generation while GA replaces m_1 points of S by the new m_1 points (children) per generation.

A. Description of DE

In each generation, *NP* competitions are held to determine the members of S for the next generation. Each competition is held to replace x_i (i = 1, 2,…, *NP*) in S. Point x_i is considered as the target point and a trial point y_i is found from two points (parents), x_i and another point $\hat{x}_i$ determined by the mutation operation. The mutation operator in DE is different from other evolutionary algorithms since it is neither based on the alteration of genes nor based on a defined probability distribution function. In DE, the mutation operator three distinct points $x_{p(1)}$, $x_{p(2)}$, $x_{p(3)}$ are randomly selected from the current set S excluding the current target x_i. The mutated point $\hat{x}_i$ is obtained by adding the weighted difference of any two points to the target point as follows:

$$\hat{x}_i = x_{p(1)} + F \times (x_{p(2)} - x_{p(3)}), \tag{6}$$

where $F \le 1$ is a scaling factor which affects the differential variation between the two points. The trial point y_i is found from its parents x_i and $\hat{x}_i$ using the following crossover rule:

$$y_i^j = \begin{cases} \hat{x}_i^j & if \quad R^j \le C_R \quad or \quad j = I_i \\ x_i^j & if \quad R^j > C_R \quad and \quad j \ne I_i \end{cases}, \tag{7}$$

where I_i is a randomly chosen integer in the set I, i.e., $I_i \in I=\{1,2,\ldots,n\}$; the superscript j represents the j-th component of respective vectors; $R^j \in (0,1)$, drawn randomly for each *j*. The crossover parameter C_R ensures that the components of target vector x_i and mutated vector $\hat{x}_i$ are combined to obtain the trial vector. It is noted that for C_R = 1 the trial vector y_i is the same as the mutated vector $\hat{x}_i$ and in this case only the mutation operation is used in reproduction. Finally, in the acceptance phase, the function value at the trial point, $f(y_i)$, is compared to $f(x_i)$, the value at the target point. If $f(y_i)< f(x_i)$, then y_i replaces x_i in S, otherwise, S retains the original x_i. This process is repeated for each member in S. The stopping condition for the algorithm can be defined as:

$$f_{max} - f_{min} \le \varepsilon, \tag{8}$$

where ε is a small number, f_{max} and f_{min} are the maximum and minimum function values, respectively.

B. The Differential Evolution Algorithm

The steps involved in the Differential Evolution algorithm are given below:

Step 1: Determine the initial set $S = \{x_1, x_2, \ldots, x_{NP}\}$ where the points x_i (i = 1, 2,…, NP) are sampled randomly. Set generation counter k = 0.

Step 2: Determine best and worst points in S with respective function values f_{max} and f_{min}. Check for convergence using equation (8).

Step 3: Generate points to replace points in S. For each $x_i \in S$, determine y_i by mutation and crossover operations as follows:

- Mutation: Randomly select three points from S except x_i, the running target and find the second parent $\hat{x}_i$ by the mutation rule in equation (6).
- Crossover: Calculate the trial vector y_i corresponding to the target x_i from x_i and $\hat{x}_i$ using the crossover rule in equation (7).

Step 4: Replace points in S. Select each trial vector y_i for the (k+1)-th generation using the acceptance criterion: replace $x_i \in S$ with y_i if $f(y_i) < f(x_i)$ otherwise retain x_i. Set k:= k+1 and go to Step 2.

C. DE applied to Network Reconfiguration

Each solution (point) is encoded as a list of indices of the normally open sectionalizing switches in the distribution network [11]. The status of each of these switches is decided according to graph theory subject to the radiality constraint of distribution network.

The evaluation function is formed by combining the objective function and the penalty function, i.e.

$$\text{Min } f = L \tag{7}$$

where

$$L = \Sigma_i |I_i|^2 k_i R_i + \beta_1 max\{0, (|I_i| - I_{i\,max})^2\} + \beta_2 max\{0, (V_{i\,min} - V_i)^2\} + \beta_3 max\{0, (V_i - V_{i\,max})^2\} \tag{9}$$

where $\beta_i (i = 1,2,3)$ is a large constant.

IV. SIMULATION RESULTS

The Differential Evolution Algorithm was applied to three test systems: 16-bus [7], 33-bus [1] and a real distribution system of the Central Electricity Board (CEB) of Mauritius (Bramsthan network) [11]. The following DE parameters were used in the simulations:

- Number of population members, NP = 20
- Maximum number of iterations (generations), *itermax* = 100
- DE step size from interval [0, 1], F = 0.7
- Crossover probability constant from interval [0, 1], C_R = 0.9

It is noted that DE is much more sensitive to the choice of the step size F than it is to the choice of crossover probability C_R. C_R is more like a fine tuning element. High values of C_R like C_R = 1.0 give faster convergence if convergence occurs.

A. 16-bus system

The 16-bus system (Fig. 1) has 3 open switches at 4, 11 and 13 and initial power loss of 0.5114 MW. DE found the global optimal configuration with open switches at 6, 9 and 11 and corresponding power loss of 0.4661 MW.

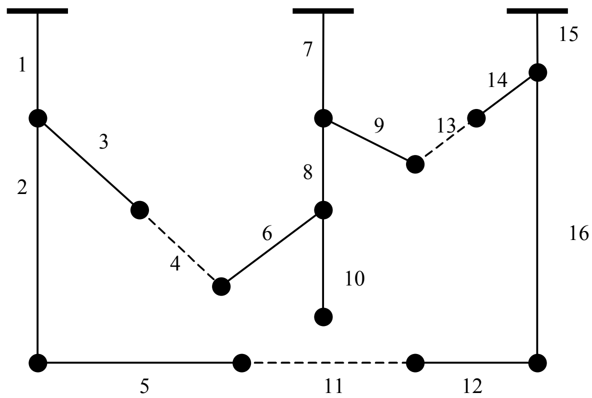

Figure 1. 16-bus system

From Fig. 2, it is observed that convergence of DE is achieved at the 8th generation for the 16-bus system.

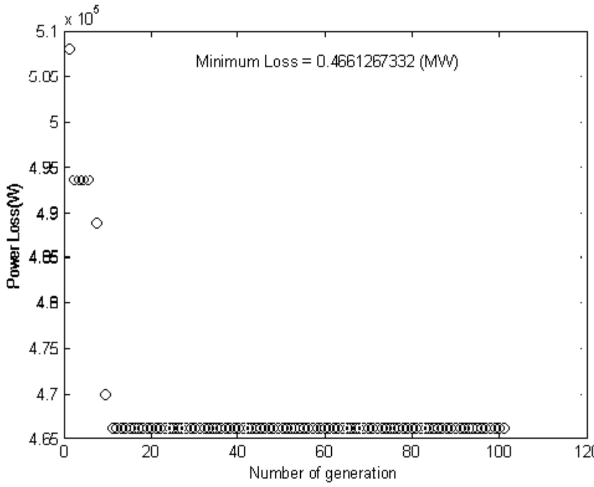

Figure 2: Convergence characteristic of DE for 16-bus distribution network

B. 33-bus system

The 33-bus system (Fig. 3) has 5 open switches at 33, 34, 35, 36 and 37 and initial power loss of 0.202674 MW. DE found the global optimal configuration with open switches at 7, 9, 14, 32 and 37 and power loss of 0.139532 MW.

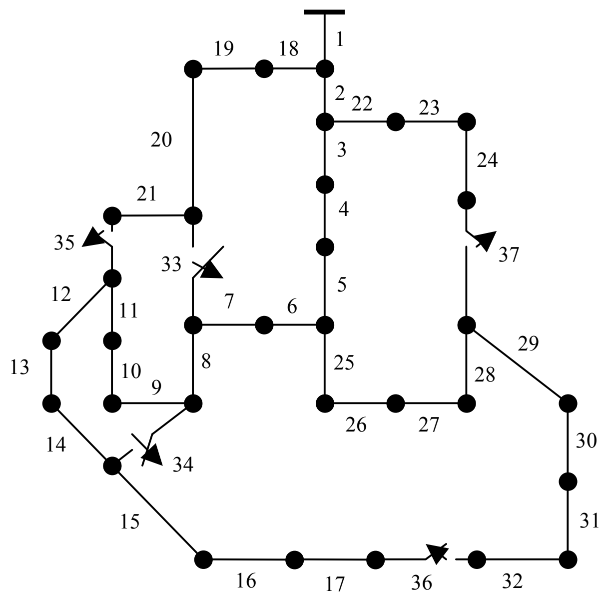

Figure 3. 33-bus system

In the case of the 33-bus system, DE converges to the global optimum at the 21st generation as shown in Fig. 4.

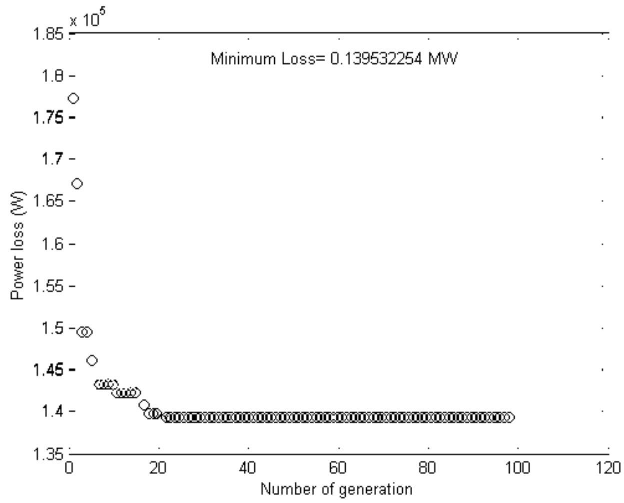

Figure 4: Convergence characteristic of DE for 33-bus distribution network with C_R = 0.9 and F = 0.7

C. Bramsthan system

The Bramsthan system [11] is part of the real distribution network of the Central Electricity Board found in the eastern part of Mauritius (Indian Ocean). The Bramsthan distribution network (Fig. 5) has 4 open switches at 33, 34, 35 and 36 and an initial power loss of 1.2333 MW. DE found the global optimal configuration with open switches 33, 34, 21 and 36 and the corresponding loss is 1.1759 MW.

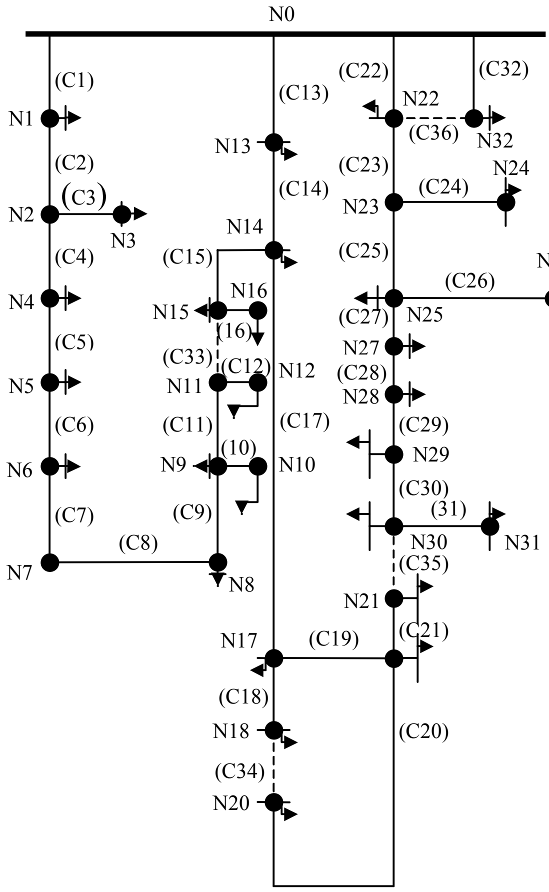

Figure 5. Bramsthan distribution network

Fig. 6 shows the convergence of DE to the global optimum at the 11th generation for the Bramsthan system.

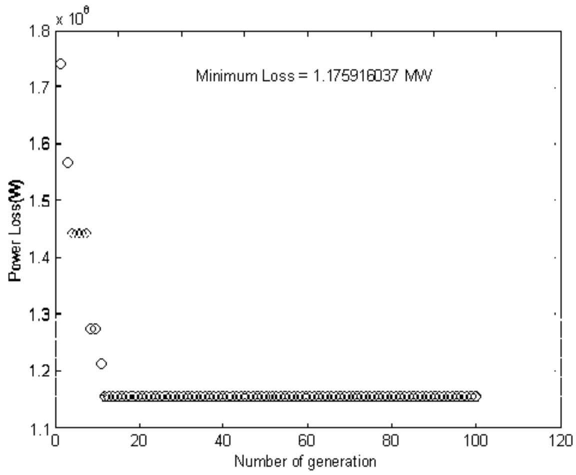

Figure 6: Convergence characteristic of DE for Bramsthan distribution network

Table 1: Comparison of Differential Evolution with other GA-based approaches on 20 runs

System	GA with fixed parameters [13]		GA with fixed crossover and adaptive mutation [11]		Fuzzy Controlled GA [13]		Differential Evolution Algorithm	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
16 bus	15.1	5.11	7.3	3.7	7.1	1.76	6.2	1.32
33 bus	43.4	11.78	29.3	7.63	23.0	5.24	19.6	4.79
Bramsthan	18.3	6.33	11.6	5.92	10.4	4.17	9.5	3.65

To confirm the performance of DE on the test systems, the simulations were repeated 20 times each with different initial populations. DE obtained the global optimum for each system in each of the runs. Table 1 shows the results of the simulations for DE (mean and standard deviation of the number of generations required to converge to the optimum for 20 runs) as compared to 3 other GA-based approaches. The first GA is a fixed parameter GA with crossover probability, p_c = 0.6 and mutation probability, p_m = 0.4 [13], the second GA is p_c = 0.6 and adaptive mutation probability [11] and the third GA is one with fuzzy controlled crossover and mutation probabilities [13]. It can be deduced that DE finds the global optimum much faster than the GA-based approaches in terms of the number of generations. Moreover, the algorithm is more consistent since it has a smaller standard deviation in the number of generations required to find the global optimum.

V. CONCLUSIONS

This paper has presented the numerical results obtained using Differential Evolution for solving the distribution network reconfiguration problem. The algorithm has been applied to 16-bus, 33-bus and a real distribution system in Mauritius. It has been found that Differential Evolution always reaches the global optimum and has proved to have faster convergence than a genetic algorithm using fixed crossover and mutation probabilities, another genetic algorithm with fixed crossover and adaptive mutation, and a fuzzy controlled genetic algorithm. As such, Differential Evolution is a simple and fast algorithm for solving the problem at hand. Therefore, Differential Evolution is a potential candidate for solving real-world distribution systems problems where there are a large number of normally open sectionalizing switches, to achieve minimum real power losses in a few number of generations.

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