An intuitional fuzzy graph drawing by use GA

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Summary − Research on fuzzy graph drawing has desired with progress in research on fuzzy graph. In this paper, for realizing understandable and intuitional fuzzy graph, we show the drawing method that the degrees of relation between nodes correspond to length of fuzzy graph edge. To draw this type of fuzzy graph, the drawing rule for the fuzzy graph are induced, and realizing this drawing rule, genes of the inversion in genetic algorithm (GA) are evaluated directly. The fuzzy graph we propose here has high understandability and good intuitional characteristic.

The fuzzy graph drawing method we shown in this paper is useful not only for a fuzzy graph drawing method but also for improving convergence characteristic in inversion of GA.

I. Introduction

As it is well known, the quantitative analysis of the social phenomenon and dealing with others is called sociometry, and the graph for analysis is called socioguram.

Increasingly advances in study of fuzzy graph bring to apply a fuzzy graph as analysis tool in sociometry. It seems that the fuzzy graph strongly influences an analysis results in sociometry, if the fuzzy graph with comfortable degree of understandability could be drawn , it sometimes bring us the adequate analysis results more than it is expected.

In general, to draw a graph automatically using a computer is called the automatic graph drawing.

Many studies of the automatic graph drawing for a conventional graph have been reported [1]-[4] and argument about a framework for the automatic graph drawing have been deepened although.

A framework for the automatic graph drawing is divided into two processes; the drawing conventions and the drawing rules [5]. In the drawing convention process, the coordinate system and the drawing purpose are clarified. In the drawing rule process, to avoid overlapping among nodes and edges, and minimizing edge crossings are done.

The main theme for the graph drawing problems is to give an algorithm for realizing drawing rules; Kamada's method [6], improving Kamada's methods [7], linear time algorithm, drawing algorithm for a general directed graph etc. are well known. Some of the drawing rules are indicated that these are NP-complete; as for the drawing rules, an expression in the formulation and the algorithm is hard. In this case, genetic algorithm is one of useful way. On the other hand, although the partition tree and the graph which has the fuzzy relationship on edges are well known, drawing method of a fuzzy graph which can express clearly the fuzzy relationship between nodes has been needed.

From this point, this paper propose the fuzzy graph drawing method which combines the relationship between nodes and the length of the edge and show the crossover in GA to realize the fuzzy graph drawing method.

Also we evaluate the fuzzy graph drawn by the method we propose.

2. Applying the drawing rule

The framework of the graph drawing method is composed by two steps; the drawing conventions and the drawing rule, and the main point is to actualize the drawing rule.

2.1 Evaluating a graph

A drawing rule will significantly influences an evaluating a graph. Because in the fuzzy graph the drawing result tends to be complicated, understandable and intuitional fuzzy graph is especially expected. To improve an intuitional characteristic of a fuzzy graph, it is an important to make the graph as simple as possible under given condition. Also, in drawing a graph, The beauty of drawing result so called aesthetic graph is one of the important factors. On the other hand, it is undesirable to limit the displaying region of a graph more than necessary by simplifying.

The characteristic of fuzzy graph having readability and intuitional characteristic and ability of enough displaying information is called understandability for convenience in this paper.

And, if beauty of symmetry and balance of the fuzzy graph are estimated, the word aesthetics is used in generally.

Furthermore, we call the difference between maximum and minimum fuzzy relation of nodes the fuzzy display range in this paper.

2.2 The drawing rule of the fuzzy graph

So-called nice fuzzy graph having the high understandability, the good aesthetics and the wide displaying range can derive humans' intuition.

In this paper, we show the drawing rule for the fuzzy graph that can derive the humans' intuition. Table 1 is the drawing rules for the fuzzy graph we propose in this paper.

Table 1 Drawing rule

Drawing rule	type
(a) To avoid overlapping among nodes and edge	semantic
(b) To combine different type of edge length	semantic
(c) A specified set of nodes are drawn near to each other	semantic
(d) Nodes are drawn with a specified size	semantic
(e) A specified set of nodes is placed near the center	structural
(f) Nodes of high degree relation are placed near the center	structural
(g) Keep adequate distance between isolated and non isolated node	structural
(h) Minimum edge crossing	structural
(i) The density of the nodes is uniform.	structural

We arrange the drawing rules for a conventional graph from the view of the fuzzy graph, and add (b) to the drawing rules of the conventional graph.

In this paper, we adopt the drawing rules (a), (b) and (h) as a drawing rule for the fuzzy graph we propose. In table 1, the semantic rules give a greater effect on understandability more than structural rules in general. In generally, some drawing rules are needed to draw a graph automatically, but these drawing rules often come into conflict with each other. In this case, to keep understandability of a fuzzy graph, it seems reasonable that semantic rules take the precedence over the structural rules. The semantic rule is changed into the structural rule depending on analysis theme. The converse is also the possible.

3. Characteristics of the fuzzy graph

A conventional graph shows the relationship between nodes, and a fuzzy graph show the fuzzy relation between nodes. Because fuzzy relation can show diverse conditions between nodes, the construction of fuzzy graph tends to be complex, and it can sometimes be difficult to draw. However, if we decide the desirable nodes arrangement and combination of nodes and edges, fuzzy graph can derive human's intuition and display excellent characteristics.

4. Integration of the relationship between nodes

Fuzzy graph can show relationship between nodes in detail, whereas, drawing result of the graph tend to lose the characteristic of the deriving humans' intuition and aesthetics. Also, to divide vague information with respect to recognition and behavior of a human in detail do not sometimes accord with a human's sense. In this case, it is a useful means that to draw the fuzzy graph of which degree of relation between nodes is integrated into some group. In this paper, we draw the fuzzy graph as simply as possible by use genetic algorithm. This allows the fuzzy graph which derives a human's intuition. We define the fuzzy graph F_G as following.

$$F_G = (V, F)$$

V is a finite set of the nodes, and F means fuzzy matrix

$$V = \{V_i \mid 1 \le i \le n\}$$

$$F = (f_{ij}) \qquad 0 \le f_{ij} \le 1$$

Where, f_{ij} means the relationship between node V_i and V_j.

We integrate f_{ij} into the following.

$$0 \le f_{ij} < l_1, \ \ l_1 \le f_{ij} < l_2, \ \ \ldots l_m \le f_{ij} \le 1$$

The fuzzy graph which the integrated degree of relation correspond to length of edge is drawn. The value l_1, l_2, $\cdots$ is

Table 2　Degree of freedom to combine three type of edges length in EUC,　MAN,　NEI

V_2 - V_3	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
V_3 - V_1	1	1	1	2	2	2	3	3	3	1	1	1	2	2	2	3	3	3	1	1	1	2	2	2	3	3	3
V_1 - V_2	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
EUC	*	○	*	○	*	*	*	*	*	○	*	*	*	*	○	*	○	*	*	*	*	*	○	*	*	*	*
MAN	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*	○	*
NEI	○	○	*	○	○	○	*	○	○	○	○	○	○	○	○	○	○	○	*	○	○	○	○	○	○	○	○

EUC: Euclid distance　MAN: Manhattan distance　NEI: Neighbor distance　○ : possible　＊ : impassible

decided by an appropriate method depending on an analysis purpose.

5. Indicating the fuzzy edges

By integrating degree of relation of edges into the some part and by corresponding to the integrated degree of relation and the fuzzy edges, we improve an intuitional characteristic of the fuzzy graph. For drawing the intuitional fuzzy graph, high degree of freedom in defining a length of edge is needed. We compare the degree of freedom for displaying edges in Euclid distance, Manhattan distance and neighbor distance. In neighbor distance, 8, 16 and 32 neighbor of the node in a lattice plane correspond to distance 1, 2 and 3 each.

Table 2 show the number of combinations made by three length edge and three nodes. One of three nodes can take lattice frames, 24 of 27 frame for neighbor distance, 13 of 27 frames for Manhattan distance and 6 of 27 frames for Euclid distance. From these results, Euclid distance has high degree of freedom for arranging the nodes, and neighbor distance has high degree of freedom for displaying edge length we use the neighbor distance following.

6. Drawing the fuzzy graph

As sociogram, we automatically draw the fuzzy graph of which degree of relation between edges correspond to fuzzy edge length. GA is used for realizing the drawing rules.

We use the subject following to explain the drawing method of the fuzzy graph we proposed here.

The questions following are given to elementary school pupil [8], [9].

Question: Write down the names of 10 people who you would like to have in your group under the following circumstance from the most to the least.

(a) Study group

(b) Lunch group

(c) Exercise group

From Sociometry research, fuzzy matrix is given by quantizing degree of relation between pupils. Fig. 1 is the fuzzy matrix which is given by Sociometry, and show relationship between pupils.

	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
01	1.0	.00	1.0	.00	.00	.48	.00	.27	.18	.86	.00	.00	.47	.44	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
02	.00	1.0	.00	.31	.00	.80	.00	.00	.33	.69	.00	.09	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
03	1.0	.00	1.0	.00	.00	.00	.00	.00	.00	.92	.00	.00	.78	.00	.73	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
04	.00	.31	.00	1.0	.00	.73	.00	.16	.24	.00	.00	.00	.00	.71	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
05	.00	.00	.00	.00	1.0	.00	.31	.00	.00	.00	.00	.00	.00	.38	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
06	.48	.80	.00	.73	.00	1.0	.00	.00	.00	.30	.00	.00	.00	.52	.35	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
07	.00	.00	.00	.00	.31	.00	1.0	.11	.29	.21	.00	.59	.00	.41	.00	.00	.12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
08	.27	.00	.00	.16	.00	.00	.11	1.0	.41	.22	.00	.49	.00	.00	.10	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
09	.18	.33	.00	.24	.00	.00	.29	.41	1.0	.26	.00	.24	.00	.20	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
10	.86	.69	.92	.00	.00	.30	.21	.22	.26	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
11	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
12	.00	.09	.00	.00	.00	.00	.59	.49	.24	.00	.00	1.0	.00	.00	.16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
13	.47	.00	.78	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
14	.44	.00	.00	.71	.38	.52	.41	.00	.20	.00	.00	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
15	.00	.00	.73	.00	.00	.35	.00	.10	.00	.00	.00	.16	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
17	.00	.00	.00	.00	.00	.00	.12	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.0	.00	.35	.45	.00	.44	.00	.00	.16	.43	.00
18	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.0	.00	.83	.12	.00	.09	.00	.27	.47	.00
19	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.35	.00	1.0	.00	.00	.00	.00	.00	.00	.21	.00
20	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.45	.83	.00	1.0	.00	.00	.44	.00	.00	.00	.00
21	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.12	.00	.00	1.0	.00	.45	.97	.14	.26	.23
22	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.44	.00	.00	.00	.00	1.0	.00	.00	.00	.00	.00
23	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.09	.00	.44	.45	.00	1.0	.31	.00	.59	.00
24	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.97	.00	.31	1.0	.00	.00	.00
25	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.16	.27	.00	.00	.14	.00	.00	.00	1.0	.00	.00
26	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.43	.47	.21	.00	.26	.00	.59	.00	.00	1.0	.00
27	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.23	.00	.00	.00	.00	.00	1.0

Fig. 1　Fuzzy matrix

The numbers in the most left column and the top of row mean pupils in the class. The numbers until 15 means school boys and the number from 16 to 27 mean school girls. In the question a) - c), if both pupils write down name each other, it is decided that they have some fuzzy relationship. In Fig. 1, value 1 means the most relationship. Table 4, shown later, show the analysis results obtained from Fig. 1.

6.2 Rearranging the nodes by GA

1) Inversion

Since in the inversion, genes' swapping is performed by use only one chromosome, it does not essentially produce the lethal hole. However, the inversion does not have the sufficient convergence characteristic within itself. We use inversion as a crossover and evaluate directly genes correspond to the drawing rules. To evaluate genes directly has effect to improve characteristic of inversion. Also, to accelerate convergence time, sequential inversion is performed.

Fig. 5 shows a normal inversion. p means the parent individual and is the chromosome before crossover. o means the child individual and is the chromosome after crossover.

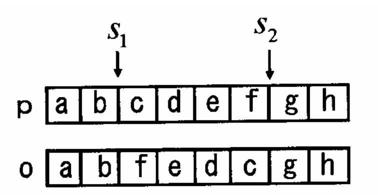


Fig. 2 Inversion

In inversion in Fig. 2, two pairs of genes, (c, f), (d, e), between s1 and s2 are yield, and then genes in each pair are exchange and yield the child individual o.

2) Genes arrangement

In this paper, nodes are arranged in a lattice plain. The nodes are arranged in the lattice frames according to random number.

Fig. 3 shows the nodes which are randomly disposed in the lattice frames.

*	*	3	11	*	*	19	20
26	14	1	10	*	*	18	9
17	*	24	16	*	*	*	*
*	15	27	*	*	*	*	*
*	*	*	8	5	13	*	*
12	*	21	*	*	*	*	*
*	*	*	*	*	2	22	*
7	*	4	*	25	23	*	6

Fig.3 Node arrangement of a fuzzy graph

The symbols * in the figure indicate that no nodes were disposed. And, upper - left frame in lattice plain is a start point of the frame and it means locus, also the most left frame in the next row follows the most right one in just upper row. Numbers in the lattice frames mean nodes and correspond to gene.

Fig. 4 shows the part of chromosome for nodes allocation shown in Fig. 3.

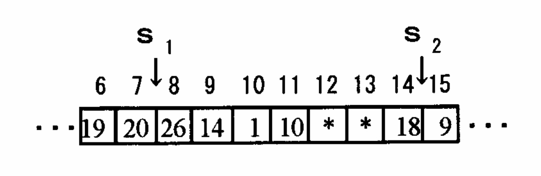


Fig. 4 Chromosome

The numbers above the frames 6, 7, 8, … show loci, and the numbers in frames 19, 20, 26 … mean genes.

3) Evaluation of drawing rule

As drawing rules for intuitional fuzzy graph, the drawing rules (a), (b), (h) in Table 1 are realized together.

(1) Evaluation of the fuzzy graph drawing

Initially, realizing drawing rule (b) is explained.

Table 3 shows the degree of integrated relation between nodes and fuzzy edges length.

Table 3 Corresponding degree of relation with edge length

Degree of fuzzy relation	Edge length
$1 \geq f_{ij} \geq 0.7$	1 (:red)
$0.7 > f_{ij} \geq 0.5$	2 (or 1:green)
$0.5 > f_{ij} \geq 0.3$	3 (or 2:blue)
$0.3 > f_{ij} > 0$	no edge
$f_{ij} = 0$	no edge

In Table 3, by using three type of fuzzy edge length, the relationships between nodes are divided into five. The drawing rule (h) in Table 1 is realized based on Table 3.

The node V_1 (1) and V_3 (3) in Fig.3 correspond to the pupil 01 and 03 in Fig.1. The degree of relation between 01 and 03 is 1. Then, the edge length between V_1 and V_3 must be numeric 1 as indicated Table.1. Because in Fig.3, $V_1$and V_3 keep neighbor distance 1 each other, gene 1 and 3 in Fig. 4 are given numeric 1 as an evaluated value (gene 3 is not displayed in Fig.4). Gene 20 and gene 18 are the same as gene 1 and gene 3 in Fig.4.

Fig. 5 is the part of evaluation table for initial generation in GA. Numeric number 0, 1, 0, … in the lowest row in Fig. 5 show the evaluating results, and the loci and gene correspond to them in

Fig. 4. GA refers the evaluating result in Fig. 5.

6	7	8	9	10	11	12	13	14	15
19	20	26	14	1	10	*	*	18	9
0	1	0	0	2	1	0	0	1	0

Fig. 5 Evaluation of genes

(2) Evaluation of superimposition between node and edge

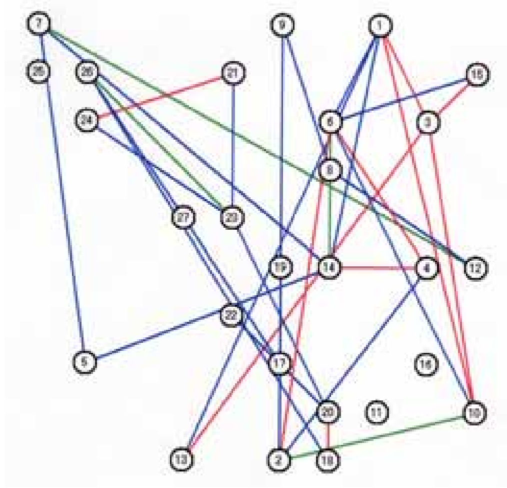
Inaccurate judgment of connection between node and edge will be caused by superimposition of node and edge; we don't judge that GA is converged on solution till all superimposition is dissolved. Fuzzy relation between 14 and 05 in Fig. 1 is 0.38 and distance between node V_{14} and V_5 in Fig.3 keep distance 3. The correspondence between integrated degree of relation and length of edge satisfy the relation between node and edge in Table.3. However, since V_{24} is put at the frame between node V_{14} and V_5, fuzzy edge and node V_{24} is superimposed. In this case, The evaluation number is not given to gene 14 and 5.

(3) Evaluation for number of nodes' crossover

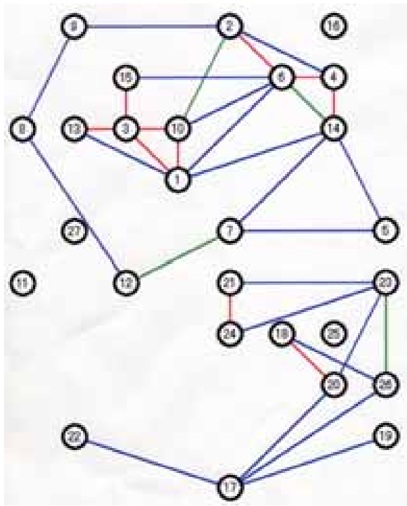
In i trials, number of crossover at j-th ($j \leq i$) trial is less or equal to that at (i-1)-th trial is required as condition for convergence.

7. The drawing results

Fig. 6 (a) and (b) is an example of a fuzzy graph we proposed, it has the fuzzy edges colored. Fig. 6 (a) is the initial generation in GA and Fig. 6 (b) is the result at 850 generation. We selected the graph having the least crossover and better nodes arrangements among 8 solutions in 20 times trials. The total time for 8 solution needs 1428 sec. and the mean convergence generation is 1524.6. And mean deviation of 20 times trials is 1.9% for mean and 5.9 for the worst. Fig.6 (b) based on the fuzzy graph of which nodes are combined by three kinds of length edges. In this type of fuzzy graph, as shown in Table 2, physically impossible combination of nodes and fuzzy edges are required. In this case, edge length 1 approximate edge length 2, and length 2 approximate length 3. These approximations decrease the understandability of a fuzzy graph. In order to avoid this, we combine length and color in edges. Sometimes we combine length and line width instead of color for convenience. Thus if length and color are combined, understandability of the fuzzy graph will be improved. Table 4 is the summary of the analysis results in Fig.4 and Fig. 6(b). Analysis results in Fig. 1 and Fig. 6 (b) are very close.



(a) Initial generation



(b) 850 generation

Fig. 6 The graph of which edges has length and color together

Because the integration has focused on strong relation between nodes, both analysis results about large degree of relationship in Fig. 1 and Fig. 6 (b) are very close. An important thing, Analysis time for Fig. 6 (b) drastically becomes shorter than that for Fig.1. Although the integration must be done according to purpose of analysis, in Fig. 6 (b), analysis results about the week relation give different impression from that in Fig. 1, because Fig.6 (b) focalizes on the strong relationship between nodes. Since pupil 25 and 27 has maximum degree of relation 0.27 and 0.23 each in Fig.4, it is sometimes hard to say that 25 and 27 are isolated.

Table 4 The analysis results using Fig.4 and Fig.9 (b)

Fig. 4	Fig. 9 (b)
11 and 16 have isolated circumstance, and 25 and 27 have not enough communication with people.	V_{11}, V_{16}, V_{25}, V_{27} have isolated circumstance.
1, 2, 3, 6, 10, 14, 17, 23 have wide interchange of each other. (pupil have more than four people of which degree of relation is greater than 0.3)	V_1, V_2, V_3, V_6, V_{10}, V_{14}, V_{17}, V_{23} have wide interchange (order of nodes is grater than 4)
3 has strong reliable relationship among friends	V_3 has strong reliable relationship among friends
An interchange of each other is active within the same sex.	An interchange of each other is limited to the same sex.

In table 4, degree of relation under 0.3 is regarded as the degree of relation zero, this does not mean that the method we proposed here can't display the region of week fuzzy relation among nodes.

8. Conclusions

We show a fuzzy graph which is intuitional and understandable.

GA is used for the drawing graph, and for evaluating the drawing rules the genes are evaluated directly.

Drawing method we show is useful not only for a fuzzy graph drawing but also for improving convergence in inversion.

Also, comparing the results in the graph we propose with the results in fuzzy matrix, a fuzzy graph we propose have relatively good understandability and also have good intuitional characteristic.

References

[1]P. Eades: “A Heuristic for graph drawing,” Congress Numerantium, vol.42, pp.149-160, 1984.

[2]K.Sugiyama, S. Tagawa and M. Toda: “Methods for visual understanding of hierarchical system structures”, IEEE Transaction on System, Man and Cybernetics, 11(2); 109-125, 1981.

[3] A. Rosete-Suarez, M. Sebag, A. Ochoa-Rodriguez: “A study of evolutionary graph drawing”, LR1 - RR - 1228, 1999.

[4] Ron Davison and David Harel: “Drawing graphs nicely using simulated annealing,” ACM Transaction on Graphics, Vol.15, No.4, pp301-331, 1996.

[5] K.Sugiyama: ”Graph drawing and applications,” World Scientific, 2002.

[6]T. Kamada: “Visualizing abstract objects and relations”, A Constraint-Based Approach, World Scientific, 1989.

[7] K. Sumi, H. Ebara, H. Nakano, and H. Maeda: “Automatically graph drawing using genetic algorithm,” IEICE-A, Vol. j80-A, No.1, pp187-194, Jar. 1997.

[8] H. Yamashita, H. Sunouchi: “Fuzzy information analysis approach to human sciences,” Kyoritu publishing, pp51-66, 1995.

[9] H. Yamashita, H. Suda: “Introduction to fuzzy mathematic - soft science: fundamental and application - ,” Morikita publishing, 1997.