

# Extension of $K$ -Nearest Neighbor Classification Using Fuzzy Relation

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**Abstract**—Classification technique is investigated by many researchers for applying to various fields. In classification, there are two methods ; parametric one and nonparametric one. The nonparametric ones are more useful than parametric ones in the real case. In this paper, the  $K$ -nearest neighbor classification which is one of the nonparametric methods is discussed. Three  $K$ -nearest neighbor algorithms with fuzzy relation are proposed and compare with each algorithm through numerical examples.

## I. INTRODUCTION

Classification technique is investigated by many researchers for applying to various fields ; character recognition, speech recognition, information retrieval[1].

In classification, there are two methods ; parametric one and nonparametric one. In parametric method, objects are classified under assumption that the universe of objects are known. But, in the real cases, this assumption is not satisfied.

On the other hand, nonparametric methods are not need the assumption so that the nonparametric ones are more useful than parametric ones.

In this paper, we discuss the  $K$ -nearest neighbor classification which is one of the nonparametric methods. We propose some extended KNN methods and compare each method through numerical examples.

## II. PRELIMINARIES

### A. Fuzzy relation

We introduce mathematical symbols and fuzzy relation [2]. Let us assume that objects  $\mathcal{O} = \{\mathbf{o}_1, \mathbf{o}_2, \dots, \mathbf{o}_N\}$  are given.

Fuzzy relation  $R$  is defined on  $\mathcal{O} \times \mathcal{O}$ .  $R$  is the fuzzy set which is characterized by the following membership function :

$$\mu_R : \mathcal{O} \times \mathcal{O} \rightarrow [0, 1]. \quad (1)$$

In this paper, we abbreviate membership function to  $R(\mathbf{o}_1, \mathbf{o}_2)$  :

$$R(\mathbf{o}_1, \mathbf{o}_2) = \mu_R(\mathbf{o}_1, \mathbf{o}_2), \quad (2)$$

and we assume that  $R$  is reflexive and symmetric :

$$R(\mathbf{o}, \mathbf{o}) = 1 \quad \forall \mathbf{o} \in \mathcal{O}, \quad (3)$$

$$R(\mathbf{o}, \mathbf{o}') = R(\mathbf{o}', \mathbf{o}) \quad \forall \mathbf{o}, \mathbf{o}' \in \mathcal{O}. \quad (4)$$

$R$  means that if  $R(\mathbf{o}, \mathbf{o}')$  is greater,  $\mathbf{o}$  and  $\mathbf{o}'$  are near or similar, and if  $R(\mathbf{o}, \mathbf{o}')$  is smaller,  $\mathbf{o}$  and  $\mathbf{o}'$  are far or dissimilar.

### B. $K$ -Nearest neighbor classification

First of all, we describe notion of supervised classification. Supervised classification is the method which need the labeled classes. we obtain a set of classification rules from the labeled classes and classify the objects that is not classified.

Assume that objects in  $\mathcal{O}$  are divided into two classes of  $\mathcal{C}$  and  $\bar{\mathcal{C}}$  ( $\mathcal{C} \cap \bar{\mathcal{C}} = \emptyset; \mathcal{C} \cup \bar{\mathcal{C}} = \mathcal{O}$ ). Objects in  $\mathcal{C}$  are classified into labeled classes  $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_L$  are in  $\mathcal{C}$ . On the other hand, object in  $\bar{\mathcal{C}}$  are not classified into any classes.

A simple method that classify the unlabeled object is a nearest neighbor classification (NN). NN allocate unlabeled object  $\mathbf{o} \in \bar{\mathcal{C}}$  into the class which has the nearest element. The class  $\mathcal{C}_i$  is given by

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} \max_{\mathbf{o}' \in \mathcal{C}_j} R(\mathbf{o}, \mathbf{o}'). \quad (5)$$

$K$ -nearest neighbor classification (KNN) is straightforward extension of NN. Let us assume that  $J$  objects are in  $\mathcal{C}$  ;  $\{\mathbf{o}_1, \mathbf{o}_2, \dots, \mathbf{o}_J\} = \mathcal{C}$ . We consider classifying the object  $\mathbf{o} \in \bar{\mathcal{C}}$ . Calculate  $\{m_1, m_2, \dots, m_J\} = \{m_j \mid m_j = R(\mathbf{o}, \mathbf{o}'_j), \mathbf{o}'_j \in \mathcal{C}, 1 \leq j \leq J\}$ .  $(m_{i1}, m_{i2}, \dots, m_{iJ})$  is the decreasing sequence obtained from sorting of  $\{m_1, m_2, \dots, m_J\} : m_{i1} \geq \dots \geq m_{iJ}$ . There, we introduce a new notation  $K \max$  ;

$$\{m_{i1}, \dots, m_{iK}\} = K \max\{m_1, \dots, m_J\}. \quad (6)$$

Examine the labels on the  $K$  nearest neighbors and vote each class. Assign  $\mathbf{o}$  the most frequently voted class (figure 1).

### Algorithm KNN

#### Step 1. Calculate

$$\{\mathbf{o}_1, \dots, \mathbf{o}_K\} = \arg K \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}').$$

#### Step 2. Classify $\mathbf{o}$ into

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} |\mathcal{C}_j \cap \{\mathbf{o}_1, \dots, \mathbf{o}_K\}|.$$

**End KNN.**

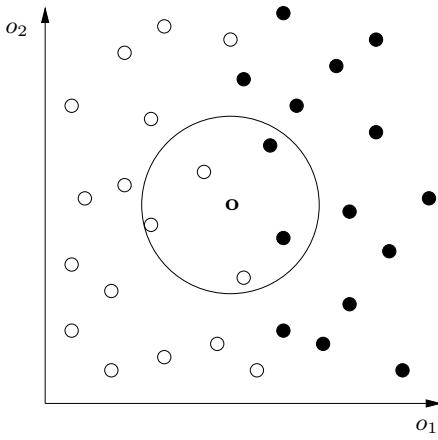


Fig. 1. We consider in the case of classifying the element  $\mathbf{o}$  into the class  $\bullet$  or the class  $\circ$  with  $K = 5$ . Since  $K$ -nearest neighbor is including three elements on the class  $\circ$ , element  $\mathbf{o}$  is classified into the class  $\circ$ .

### III. EXTENSION OF KNN CLASSIFICATION

#### A. FKNN-1

FKNN-1 is one of the methods which is constructed by introducing fuzziness into the KNN [3].

Let us consider classification of  $\mathbf{o} \in \bar{\mathcal{C}}$ . We define a fuzzy  $K_1$  maximum elements:

$$\{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K_1})\} = \arg \text{FKNN-1} \max_{\mathbf{o}' \in \bar{\mathcal{C}}} R(\mathbf{o}, \mathbf{o}'). \quad (7)$$

The cardinality of a set is denoted by

$$|\{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K_1})\}| = \sum_{j=1}^{K_1} R(\mathbf{o}, \mathbf{o}_j). \quad (8)$$

The class of  $\mathbf{o}$  is defined by

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} |\mathcal{C}_j \cap \{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K_1})\}|. \quad (9)$$

#### Algorithm FKNN-1

**Step 1.** Calculate

$$\{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K_1})\} = \arg \text{FKNN-1} \max_{\mathbf{o}' \in \bar{\mathcal{C}}} R(\mathbf{o}, \mathbf{o}')$$

**Step 2.** Classify  $\mathbf{o}$  into

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} |\mathcal{C}_j \cap \{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K_1})\}|.$$

**End FKNN-1.**

#### B. FKNN-2

We propose three methods which are extension of KNN classifications. In the first method, let us introduce new notation over\_FKNN max ;

$$\begin{aligned} & \{m_{j1}, \dots, m_{jK'_1} \mid K'_1 > \sum_{i=1}^{K'_1-1} m_{ji}, \\ & K'_1 \geq \sum_{i=1}^{K'_1-1} m_{ji} + m_{jK'_1}\} \\ & = \text{over\_FKNN max}\{m_1, \dots, m_J\}. \end{aligned} \quad (10)$$

We explain the details. over\_FKNN is minimum into the sets of which the sum of the elements is over  $K'_1$ .

#### Algorithm FKNN-2

**Step 1.** Calculate

$$\begin{aligned} & \{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K'_1})\} \\ & = \text{over\_FKNN max}_{\mathbf{o}' \in \bar{\mathcal{C}}} R(\mathbf{o}, \mathbf{o}'). \end{aligned}$$

**Step 2.** Classify  $\mathbf{o}$  into

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} |\mathcal{C}_j \cap \{R(\mathbf{o}, \mathbf{o}_1), \dots, R(\mathbf{o}, \mathbf{o}_{K'_1})\}|.$$

**End FKNN-2.**

#### C. FKNN-3

In the second method, we selects  $K_{III}$  elements from each class and calculate the average of fuzzy relation in each class. Unlabeled element  $\mathbf{o} \in \bar{\mathcal{C}}$  is assigned to the class which has the highest average of fuzzy relation in all class.

#### Algorithm FKNN-3

**Step 1.** Calculate

$$\begin{aligned} & \{R(\mathbf{o}, \mathbf{o}_1^j), \dots, R(\mathbf{o}, \mathbf{o}_{K_{III}}^j)\} \\ & = \arg \text{FKNN-3} \max_{\mathbf{o}' \in \bar{\mathcal{C}}_j} R(\mathbf{o}, \mathbf{o}') \quad 1 \leq j \leq L. \end{aligned}$$

**Step 2.** Classify  $\mathbf{o}$  into

$$\mathcal{C}_i = \arg \max_{1 \leq j \leq L} |\{R(\mathbf{o}, \mathbf{o}_1^j), \dots, R(\mathbf{o}, \mathbf{o}_{K_{III}}^j)\}| / K_{III}.$$

**End FKNN-3.**

#### D. FKNN-4

In the third method, we selects  $K_{IV} \times |C_j|$  elements from each  $C_j$  ( $1 \leq j \leq L$ ) and calculate average of fuzzy relation in  $C_j$ . Unlabeled element  $\mathbf{o} \in \bar{\mathcal{C}}$  is assigned to the class which has the highest average of fuzzy relation in all class. To describe the algorithm, we define new notation. The number of selected elements are defined by  $S_j$  ;

$$S_j = K_{IV} \times |C_j| \quad 1 \leq j \leq L. \quad (11)$$

**Algorithm FKNN-4**
**Step 1.** Calculate

$$\{R(\mathbf{o}, \mathbf{o}_1^j), \dots, R(\mathbf{o}, \mathbf{o}_{S_j}^j)\} \\ = \arg \text{FS}_j \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}') \quad 1 \leq j \leq L.$$

**Step 2.** Classify  $\mathbf{o}$  into

$$C_i = \arg \max_{1 \leq j \leq L} |\{R(\mathbf{o}, \mathbf{o}_1^j), \dots, R(\mathbf{o}, \mathbf{o}_{S_j}^j)\}| / S_j.$$

**End FKNN-4.**
**IV. NUMERICAL EXAMPLES**

In this paper, we show numerical examples for five algorithms ; KNN, FKNN-1, FKNN-2, FKNN-3, FKNN-4. Let us consider fifty six objects in figure 2. The data set have five labeled class ;  $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4, \mathcal{C}_5$ . We consider the case of classifying unlabeled element  $\mathbf{o}$ . We show fuzzy relation

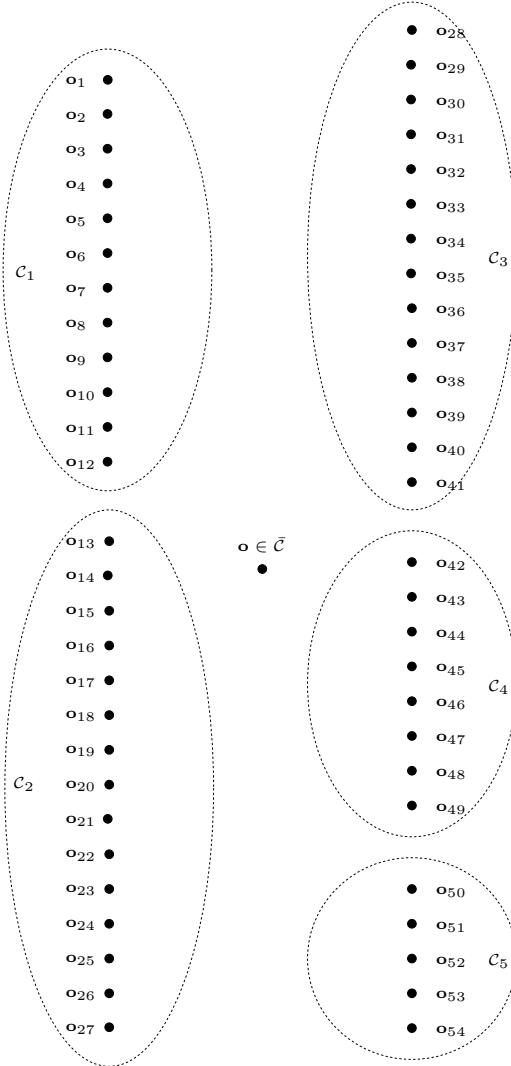


Fig. 2. The data set for numerical example

between  $\mathbf{o}$  and all object in Table I.

**TABLE I**

 FUZZY RELATION  $R(\mathbf{o}, \mathbf{o}')$  IN ALL CLASS.

		class $\mathcal{C}_1$			
		$\mathbf{o}_1$	$\mathbf{o}_2$	$\mathbf{o}_3$	$\mathbf{o}_4$
$R(\mathbf{o}, \mathbf{o}')$	0.9	0.19	0.12	0.1	
	$\mathbf{o}_5$	$\mathbf{o}_6$	$\mathbf{o}_7$	$\mathbf{o}_8$	
$R(\mathbf{o}, \mathbf{o}')$	0.02	0.012	0.0003	0.0002	
	$\mathbf{o}_9$	$\mathbf{o}_{10}$	$\mathbf{o}_{11}$	$\mathbf{o}_{12}$	
$R(\mathbf{o}, \mathbf{o}')$	0.00013	0.00012	0.00011	0.0001	

		class $\mathcal{C}_2$				
		$\mathbf{o}'_1$	$\mathbf{o}'_2$	$\mathbf{o}'_3$	$\mathbf{o}'_4$	$\mathbf{o}'_5$
$R(\mathbf{o}, \mathbf{o}')$	0.8	0.28	0.22	0.21	0.2	
	$\mathbf{o}'_6$	$\mathbf{o}'_7$	$\mathbf{o}'_8$	$\mathbf{o}'_9$	$\mathbf{o}'_{10}$	
$R(\mathbf{o}, \mathbf{o}')$	0.002	0.001	0.0007	0.00065	0.0006	
	$\mathbf{o}'_{11}$	$\mathbf{o}'_{12}$	$\mathbf{o}'_{13}$	$\mathbf{o}'_{14}$	$\mathbf{o}'_{15}$	
$R(\mathbf{o}, \mathbf{o}')$	0.0005	0.0004	0.00015	0.00014	0.000135	

		class $\mathcal{C}_3$				
		$\mathbf{o}'_1$	$\mathbf{o}'_2$	$\mathbf{o}'_3$	$\mathbf{o}'_4$	$\mathbf{o}'_5$
$R(\mathbf{o}, \mathbf{o}')$	0.7	0.29	0.27	0.26	0.03	
	$\mathbf{o}'_{16}$	$\mathbf{o}'_{17}$	$\mathbf{o}'_{18}$	$\mathbf{o}'_{19}$	$\mathbf{o}'_{20}$	
$R(\mathbf{o}, \mathbf{o}')$	0.0015	0.0014	0.0009	0.00089	0.00087	
	$\mathbf{o}'_{21}$	$\mathbf{o}'_{22}$	$\mathbf{o}'_{23}$	$\mathbf{o}'_{24}$	$\mathbf{o}'_{25}$	
$R(\mathbf{o}, \mathbf{o}')$	0.00085	0.0008	0.00067	0.00066		

		class $\mathcal{C}_4$			
		$\mathbf{o}'_1$	$\mathbf{o}'_2$	$\mathbf{o}'_3$	$\mathbf{o}'_4$
$R(\mathbf{o}, \mathbf{o}')$	0.5	0.3	0.01	0.00086	
	$\mathbf{o}'_{46}$	$\mathbf{o}'_{47}$	$\mathbf{o}'_{48}$	$\mathbf{o}'_{49}$	
$R(\mathbf{o}, \mathbf{o}')$	0.00084	0.00083	0.00082	0.00081	

		class $\mathcal{C}_5$				
		$\mathbf{o}'_1$	$\mathbf{o}'_2$	$\mathbf{o}'_3$	$\mathbf{o}'_4$	$\mathbf{o}'_5$
$R(\mathbf{o}, \mathbf{o}')$	0.25	0.24	0.23	0.00064	0.00063	
	$\mathbf{o}'_{50}$	$\mathbf{o}'_{51}$	$\mathbf{o}'_{52}$	$\mathbf{o}'_{53}$	$\mathbf{o}'_{54}$	

**A. KNN**

 In the case of KNN with  $K = 6$ , from

$$K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\} = \{\mathbf{o}_1, \mathbf{o}_{13}, \mathbf{o}_{28}, \mathbf{o}_{42}, \mathbf{o}_{43}\},$$

we have

$$\begin{aligned} |\mathcal{C}_1 \cap K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= |\{\mathbf{o}_1\}| = 1, \\ |\mathcal{C}_2 \cap K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= |\{\mathbf{o}_{13}\}| = 1, \\ |\mathcal{C}_3 \cap K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= |\{\mathbf{o}_{28}\}| = 1, \\ |\mathcal{C}_4 \cap K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= |\{\mathbf{o}_{42}, \mathbf{o}_{43}\}| = 2, \\ |\mathcal{C}_5 \cap K \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= |\{\emptyset\}| = 0. \end{aligned}$$

 In the result,  $\mathbf{o}$  is classified into  $\mathcal{C}_4$ .

**B. FKNN-1**

 In the case of FKNN-1 with  $K_I = 5$ . From

$$\begin{aligned} \arg \text{FK}_I \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_1), R(\mathbf{o}, \mathbf{o}_{13}), R(\mathbf{o}, \mathbf{o}_{28}), R(\mathbf{o}, \mathbf{o}_{42}), R(\mathbf{o}, \mathbf{o}_{43})\}, \end{aligned}$$

we have

$$\begin{aligned} |\mathcal{C}_1 \cap FK_I \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= 0.9, \\ |\mathcal{C}_2 \cap FK_I \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= 0.8, \\ |\mathcal{C}_3 \cap FK_I \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= 0.7, \\ |\mathcal{C}_4 \cap FK_I \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= 0.5 + 0.3 = 0.8, \\ |\mathcal{C}_5 \cap FK_I \max\{\mathbf{o}_1, \dots, \mathbf{o}_{54}\}| &= 0. \end{aligned}$$

There,  $\mathbf{o}$  is classified into  $\mathcal{C}_1$ .

### C. FKNN-2

In the case of FKNN-2 with  $K_{II} = 5$ . From

$$\begin{aligned} \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_1), R(\mathbf{o}, \mathbf{o}_{13}), R(\mathbf{o}, \mathbf{o}_{28}), R(\mathbf{o}, \mathbf{o}_{42}), \\ R(\mathbf{o}, \mathbf{o}_{43}), R(\mathbf{o}, \mathbf{o}_{29}), R(\mathbf{o}, \mathbf{o}_{14}), R(\mathbf{o}, \mathbf{o}_{30}), \\ R(\mathbf{o}, \mathbf{o}_{31}), R(\mathbf{o}, \mathbf{o}_{50}), R(\mathbf{o}, \mathbf{o}_{51}), R(\mathbf{o}, \mathbf{o}_{52})\}, \end{aligned}$$

we have

$$\begin{aligned} |\mathcal{C}_1 \cap \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}')| &= 0.9, \\ |\mathcal{C}_2 \cap \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}')| &= 0.8 + 0.28 = 1.08 \\ |\mathcal{C}_3 \cap \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}')| &= 0.7 + 0.29 + 0.27 + 0.26 \\ &= 1.52, \\ |\mathcal{C}_4 \cap \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}')| &= 0.5 + 0.3 = 0.8, \\ |\mathcal{C}_5 \cap \text{over\_}FK_{II} \max_{\mathbf{o}' \in \mathcal{C}} R(\mathbf{o}, \mathbf{o}')| &= 0.25 + 0.24 + 0.23 \\ &= 0.72. \end{aligned}$$

There,  $\mathbf{o}$  is classified into  $\mathcal{C}_3$ .

### D. FKNN-3

In the case of FKNN-3 with  $K_{III} = 5$ , from

$$\begin{aligned} \arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_1} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_1), R(\mathbf{o}, \mathbf{o}_2), R(\mathbf{o}, \mathbf{o}_3), R(\mathbf{o}, \mathbf{o}_4), R(\mathbf{o}, \mathbf{o}_5)\}, \\ \arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_2} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{13}), R(\mathbf{o}, \mathbf{o}_{14}), R(\mathbf{o}, \mathbf{o}_{15}), R(\mathbf{o}, \mathbf{o}_{16}), R(\mathbf{o}, \mathbf{o}_{17})\}, \\ \arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_3} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{28}), R(\mathbf{o}, \mathbf{o}_{29}), R(\mathbf{o}, \mathbf{o}_{30}), R(\mathbf{o}, \mathbf{o}_{31}), R(\mathbf{o}, \mathbf{o}_{32})\}, \\ \arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_4} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{42}), R(\mathbf{o}, \mathbf{o}_{43}), R(\mathbf{o}, \mathbf{o}_{44}), R(\mathbf{o}, \mathbf{o}_{45}), R(\mathbf{o}, \mathbf{o}_{46})\}, \\ \arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_5} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{50}), R(\mathbf{o}, \mathbf{o}_{51}), R(\mathbf{o}, \mathbf{o}_{52}), R(\mathbf{o}, \mathbf{o}_{53}), R(\mathbf{o}, \mathbf{o}_{54})\}, \end{aligned}$$

we have

$$\begin{aligned} |\arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_1} R(\mathbf{o}, \mathbf{o}')|/K_{III} \\ = (0.9 + 0.19 + 0.12 + 0.1 + 0.02)/5 \\ = 0.226, \\ |\arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_2} R(\mathbf{o}, \mathbf{o}')|/K_{III} \\ = (0.8 + 0.28 + 0.22 + 0.21 + 0.2)/5 \\ = 0.342, \\ |\arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_3} R(\mathbf{o}, \mathbf{o}')|/K_{III} \\ = (0.7 + 0.29 + 0.27 + 0.26 + 0.03)/5 \\ = 0.31, \\ |\arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_4} R(\mathbf{o}, \mathbf{o}')|/K_{III} \\ = (0.5 + 0.3 + 0.01 + \\ 0.00086 + 0.00084)/5 \\ = 0.162, \\ |\arg FK_{III} \max_{\mathbf{o}' \in \mathcal{C}_5} R(\mathbf{o}, \mathbf{o}')|/K_{III} \\ = (0.25 + 0.24 + 0.23 + \\ 0.00064 + 0.00063)/5 \\ = 0.144. \end{aligned}$$

$\mathbf{o}$  is classified into  $\mathcal{C}_2$ .

### E. FKNN-4

In the case of FKNN-4 with  $K_{IV} = 0.5$ , we have

$$\begin{aligned} S_1 &= K_{IV} \times |C_1| = 0.5 \times 12 = 6, \\ S_2 &= K_{IV} \times |C_2| = 0.5 \times 15 = 7, \\ S_3 &= K_{IV} \times |C_3| = 0.5 \times 14 = 7, \\ S_4 &= K_{IV} \times |C_4| = 0.5 \times 8 = 4, \\ S_5 &= K_{IV} \times |C_5| = 0.5 \times 5 = 2. \end{aligned}$$

From

$$\begin{aligned} \arg FS_1 \max_{\mathbf{o}' \in \mathcal{C}_1} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_1), R(\mathbf{o}, \mathbf{o}_2), R(\mathbf{o}, \mathbf{o}_3), \\ R(\mathbf{o}, \mathbf{o}_4), R(\mathbf{o}, \mathbf{o}_5), R(\mathbf{o}, \mathbf{o}_6)\}, \\ \arg FS_2 \max_{\mathbf{o}' \in \mathcal{C}_2} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{13}), R(\mathbf{o}, \mathbf{o}_{14}), R(\mathbf{o}, \mathbf{o}_{15}), \\ R(\mathbf{o}, \mathbf{o}_{16}), R(\mathbf{o}, \mathbf{o}_{17}), R(\mathbf{o}, \mathbf{o}_{18}), R(\mathbf{o}, \mathbf{o}_{19})\}, \\ \arg FS_3 \max_{\mathbf{o}' \in \mathcal{C}_3} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{28}), R(\mathbf{o}, \mathbf{o}_{29}), R(\mathbf{o}, \mathbf{o}_{30}), \\ R(\mathbf{o}, \mathbf{o}_{31}), R(\mathbf{o}, \mathbf{o}_{32}), R(\mathbf{o}, \mathbf{o}_{33}), R(\mathbf{o}, \mathbf{o}_{34})\}, \\ \arg FS_4 \max_{\mathbf{o}' \in \mathcal{C}_4} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{42}), R(\mathbf{o}, \mathbf{o}_{43}), R(\mathbf{o}, \mathbf{o}_{44}), R(\mathbf{o}, \mathbf{o}_{45})\}, \\ \arg FS_5 \max_{\mathbf{o}' \in \mathcal{C}_5} R(\mathbf{o}, \mathbf{o}') \\ = \{R(\mathbf{o}, \mathbf{o}_{50}), R(\mathbf{o}, \mathbf{o}_{51})\}. \end{aligned}$$

we have

$$\begin{aligned}
& |\arg \text{FS}_1 \max_{\mathbf{o}' \in \mathcal{C}_1} R(\mathbf{o}, \mathbf{o}')| / S_1 \\
&= (0.9 + 0.19 + 0.12 + 0.1 + 0.02 + 0.012) / 6 \\
&= 0.223, \\
& |\arg \text{FS}_2 \max_{\mathbf{o}' \in \mathcal{C}_2} R(\mathbf{o}, \mathbf{o}')| / S_2 \\
&= (0.8 + 0.28 + 0.22 + 0.21 + 0.2 + 0.002 + 0.001) / 7 \\
&= 0.224, \\
& |\arg \text{FS}_3 \max_{\mathbf{o}' \in \mathcal{C}_3} R(\mathbf{o}, \mathbf{o}')| / S_3 \\
&= (0.7 + 0.29 + 0.27 + 0.26 + 0.03 + 0.0015 + 0.0014) / 7 \\
&= 0.221, \\
& |\arg \text{FS}_4 \max_{\mathbf{o}' \in \mathcal{C}_4} R(\mathbf{o}, \mathbf{o}')| / S_4 \\
&= (0.3 + 0.5 + 0.01 + 0.0086) / 4 \\
&= 0.204, \\
& |\arg \text{FS}_5 \max_{\mathbf{o}' \in \mathcal{C}_5} R(\mathbf{o}, \mathbf{o}')| / S_5 \\
&= (0.25 + 0.24) / 2 \\
&= 0.245.
\end{aligned}$$

$\mathbf{o}$  is classified into  $\mathcal{C}_5$ .

## V. APPLYING TO FISHER'S IRIS DATA SET

In this section, we show the results of classifying the real data set using five algorithms ; KNN with  $K = 7$ , FKNN-1 with  $K_I = 7$ , FKNN-2 with  $K_{II} = 7$ , FKNN-3 with  $K_{III} = 7$ , FKNN-4 with  $K_{IV} = 0.7$ .

The data set which we use in classification is Fisher's iris data. Fisher's iris data set is one of the general data sets which is used in classification. The iris data set consists of the three iris species ; Setosa, Versicolor, Virginica. The data is represented by four features ; petal width, petal length, sepal width, Sepal length [4].

We show three tables of the results. We show in Table II the informations of the data set Table III, IV and V. In all algorithms, the following fuzzy relation is used ;

$$R(\mathbf{o}_1, \mathbf{o}_2) = \frac{1}{1 + \|\mathbf{o}_1 - \mathbf{o}_2\|},$$

where

$$\|\mathbf{o}_1 - \mathbf{o}_2\| = \left( \sum_{i=1}^4 (o_{1i} - o_{2i})^2 \right)^{\frac{1}{2}}.$$

TABLE II  
THE INFORMATION OF DATA SET ON EACH RESULTS TABLE

Table number	iris species	Number of labeled elements	Number of unlabeled elements
Table III	Setosa	25	25
	Versicolor	25	25
	Virginica	25	25
Table IV	Setosa	15	35
	Versicolor	15	35
	Virginica	15	35
Table V	Setosa	5	45
	Versicolor	5	45
	Virginica	5	45

TABLE III  
RESULTS OF FOUR ALGORITHMS FOR IRIS DATA SET. EACH LABELED CLASS HAS 25 ELEMENTS.

		Setosa	Versicolor	Virginica
KNN	Setosa	25	0	0
	Versicolor	0	24	1
	Virginica	0	2	23
FKNN-1	Setosa	25	0	0
	Versicolor	0	24	1
	Virginica	0	1	24
FKNN-2	Setosa	25	0	0
	Versicolor	0	23	2
	Virginica	0	0	25
FKNN-3	Setosa	25	0	0
	Versicolor	0	24	1
	Virginica	0	3	22
FKNN-4	Setosa	25	0	0
	Versicolor	0	24	1
	Virginica	0	2	23

TABLE IV

RESULTS OF FOUR ALGORITHMS FOR IRIS DATA SET. EACH LABELED  
CLASS HAS 15 ELEMENTS.

		Setosa	Versicolor	Verginica
KNN	Setosa	35	0	0
	Versicolor	0	32	3
	Verginica	0	8	32
FKNN-1	Setosa	35	0	0
	Versicolor	0	32	3
	Verginica	0	4	31
FKNN-2	Setosa	35	0	0
	Versicolor	0	33	2
	Verginica	0	8	27
FKNN-3	Setosa	35	0	0
	Versicolor	0	32	3
	Verginica	0	3	32
FKNN-4	Setosa	35	0	0
	Versicolor	0	32	3
	Verginica	0	3	32

TABLE V

RESULTS OF FOUR ALGORITHMS FOR IRIS DATA SET. EACH LABELED  
CLASS HAS 5 ELEMENTS.

		Setosa	Versicolor	Verginica
KNN	Setosa	45	0	0
	Versicolor	0	42	3
	Verginica	0	12	33
FKNN-1	Setosa	45	0	0
	Versicolor	0	43	2
	Verginica	0	5	40
FKNN-2	Setosa	45	0	0
	Versicolor	0	43	2
	Verginica	0	2	43
FKNN-3	Setosa	45	0	0
	Versicolor	0	43	2
	Verginica	0	2	43
FKNN-4	Setosa	45	0	0
	Versicolor	0	43	2
	Verginica	0	2	43

## VI. CONCLUSION

In this paper, we proposed four classification algorithms. These algorithms are natural extension of KNN with fuzziness. Moreover, we showed the availability of presented algorithms through numerical examples.

In the future works, we will discuss the reason that the extended KNN algorithms has better result than KNN algorithm mathematically.

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