Plant Machinery Optimal Preventive Maintenance Strategy Using Genetic Algorithms

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Abstract— **Most of the mathematical methods to decide the time interval of inspection and the maintenance period are established by reliability theory. However, most of these methods are too complicated to be solved as a ready to use numerical solution for any failure distribution. Moreover, in the practical applications, the machinery plants must satisfy the required conditions of these theoretical models, which is not the case usually. In this paper, we propose a new decision-making method of inspection time interval and maintenance period for plant machinery maintenance by genetic algorithm (GA) when plant machines are preserved by time based maintenance (TBM). The balance between the frequency of inspection and the returns from it is required to decrease unexpected breakdown since faults are detected before they result in costly breakdown. This new method aims to satisfy the reliability model and to minimize the composite maintenance cost; by taking in consideration the cost of losses due to failure and the cost for the preventive maintenance.**

Keywords— Time based maintenance, Inspection Interval, Maintenance period, Reliability theory, Genetic algorithms

I. INTRODUCTION

The problem of choosing and optimizing maintenance strategies is of foremost importance in plant management and operation. An efficient strategy should aim at guaranteeing the level of performance and availability of the system while allowing for a reduction in the resource expenditure. During the last three decades many papers dealing with preventive replacement strategies have been proposed. Barlow and Porschan [1] introduce the basic replacement models either for periodic replacement strategies (age periodic strategy and block periodic strategy). Those models have been extended by many authors (see Cho and Parlar [2], Valdez-Flores, C. [3], Sherif and Smith [4], Pierskalla and Voelker [5]). Zuckerman [6] suggests inspecting the equipment at T, 2T, 3T, etc. If the cumulative damage exceeds a given threshold during a certain period, the equipment is replaced by a new one. Chelbi and Ait-Kadi [7] develop a replacement strategy for non-self announcing failure equipment based on a conditional probability, which increases with the number of inspections. Berenguer et al. [8] use a semi-Markov decision process model to generate the inspection sequence based on some given indicators. Makis et al. [9] develop a Conditional-Based Maintenance in the context of incomplete available information. The optimal strategy uses a control-limit rule based on the age and the degradation level of the equipment at the inspection instant. Most of the proposed mathematical models to decide the maintenance time are still complicated to be implemented as a “ready to use” numerical solution. Moreover, in the practical applications, the machinery plants must satisfy the required conditions of these theoretical models; which is not usually the case.
Considering the effects of system cost and life, the arrangement of preventive maintenance activity becomes an optimization problem. Although this problem can be resolved by completely enumerating the possible answers to the search space, it is exhaust in time and is inefficient for a large space. In the last years, an increasing number of GAs was used to treat the optimization in system reliability ([10] and [11]). The tendency reveals that GAs are an efficient tool to rapidly obtain the optimal solution of preventive maintenance policy. Therefore, GAs are used in this work as a tool to implement and optimize the joint inspection replacement periodicity.

In this paper, we propose a decision-making method of the optimal periodic preventive maintenance and replacement for plant machinery and equipment by genetic algorithms technique. The main objective is to improve the performance of replacement and inspection strategies by considering a joint periodic inspection and replacement strategies, mainly for equipment whose state can only be known through inspection. The mathematical model governing the proposed strategy take into account the failure distribution of the lifetime, the costs incurred to perform each maintenance action (inspection, minimal repair, replacement) and also the cost of production loss due to the idle time between the failure occurrence and the failure detection of the equipment. This model expresses the expected total cost (ETC) per unit time over an infinite horizon. The optimal strategy is the one that minimizes the ETC. The most merit of the method proposed here is that by applying GAs, we can decide the inspection and the maintenance period for a plant machine following a generalized failure rate $\lambda(t)$, in both continuous and discrete time.

II. BASIC THEORY OF THE PROPOSED METHOD

The mathematical models used for the determination of the maintenance inspection and replacement periodicity, are established by the reliability theory. In the present work, the optimization strategy is not only based on the reliability criteria, it is also decided by the composite maintenance cost relating to the cost ratio. Which mainly include the losses due to failure and costs for the preventive maintenance.

In this section, after defining the strategy, the working assumptions and the used notation, the mathematical model will be developed and the existence and uniqueness conditions of an optimal periodic strategy will be established.

A. Notations and Assumptions

The considered equipment is subject to sudden failures and when failure occurs it has to be maintained or replaced. In order to reduce the number of failures, preventive maintenance can be scheduled to occur at specified intervals. However, a balance is required between the amount spent on the preventive actions and their resulting benefits from failure reduction. It will be assumed, not unreasonably, that we are dealing with a long period of time over which the machinery is to be operated and the intervals between the preventive maintenance are relatively short. Moreover, our mathematical model is developed under the following basic assumptions:

- ✓ The failure of the machine can only be detected through inspection;
- ✓ The inspection operation is perfect and the replacement is immediate if failure detected;
- ✓ Failure completely halts production or generates waste products;
- ✓ The time for inspection and repair is negligible;
- ✓ Repair if occurred, makes the system as good as new. Thus, the failure time *pdf* remain the same.

The following notations will be used throughout the paper:

$C(T)$	Expected Total Cost per unit time (ETC)
C_i	Cost of scheduled inspection and preventive maintenance
C_r	Cost of replacement if the machine is found to have failed
C_d	Cost of losses due to failure until inspection
$f(x)$	Probability density function of the machine's life time
$F(x)$	Cumulative distribution function of machine's life time
$\lambda(x)$	Failure rate of the machine,
$R(t), F(t)$	Reliability, Unreliability functions of machine,
T	Inspection interval
T^*	Optimal inspection period (minimizing the ETC)
$\mu = \int_0^\infty t f(t)\,dt$	Mean time to failure of machine
$W_{\alpha,\eta}(x)$	Two-parameters Weibull distribution
α	Scale parameter of Weibull *pdf*
η	Shape parameter of Weibull *pdf*

B. The Reliability model

In this work, the development of the mathematical model and its implementation are based on the reliability theory, and inspired by the first proposed block replacement policy by Barlow and Porschan [1] as well as Osaki [12] who presented important results about the existence of an optimum policy. During the determination of continuous time domain preventive maintenance period, we focus on the case where the reliability function of machinery plant follows the Weibull distribution. The reason for this is that previous practical studies have shown that this distribution fits a large number of machinery. The Weibull distribution is represented in Eq.(1).

$$W_{\eta,\alpha}(x) = \frac{\eta}{\alpha}\left(\frac{x}{\alpha}\right)^{\eta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{\eta}\right],\ x \succ 0 \qquad (1)$$

Where η is the shape parameter which determines the shape of the *pdf* and α is the scale parameter, both parameters are estimated from the life examination data. Depending on the shape parameter the failure rate function is increasing or decreasing. We assume that the aging effect in the machinery develops with a value for the shape parameter $\eta \geq 2$. The effects of machinery component aging are counterbalanced by maintenance actions, performed with period *T*, which rejuvenate them. In practice, during the period *T*, the failure rate increases only slightly. The Weibull distribution shifts the failures to later times, closer to the end.

Preventively replacing or maintaining critical components within the equipment at appropriate times can enhance system's reliability. Deciding the best time depends on the strategy's overall objectives, such as cost minimization or availability maximization. In this work, we consider that after a corrective or a preventive maintenance intervention, the equipment acquire the same reliability as a new one.

C. The composite maintenance cost model

Plant management is inevitably affected by economic constraints. In order to quantify the consequences of a given management action in economic terms it is common practice to introduce a profit (cost) or energy function which contains the factors affecting the plant from an economic point of view. The maintenance cost models are investigated for deciding the preventive maintenance period. The optimal policy once obtained guarantees a decided compromise between maximizing availability and minimizing the expected total cost. When this is the case we need to consider only one cycle of operation and develop its corresponding model. The maintenance policy is one where preventive actions occur at fixed intervals of time, faulty parts replacements occur when necessary, and we want to determine the optimal periodicity of the preventive interventions to minimize the total expected cost per unit of time $C(T)$.

In this inspection problem, it is obvious that the time of inspection is a renewal point. Using the renewal reward theorem, the long run expected cost per unit time is given by

$$C(T) = \frac{\text{The total expected cost in the Cycle}}{\text{The time length of one Cycle}} = \frac{N(T)}{T} \qquad (2)$$

There are two possible cycles of operations. In the first cycle, the machine is found to be in good state upon inspection at time *T*. However, in the second cycle, a failure is detected after inspecting the machine at the end of the inspection period. Then, the average maintenance cost per cycle can be decomposed in two portions as Eq.(3)

$$N(T) = N_1(T) + N_2(T) \qquad (3)$$

It is clear that the average cost of a good cycle is equal to C_i since the machine is found to be in good state after the inspection at time *T,* and also clear that the probability of a good cycle is $P\{X \geq T\} = R(T)$. Eq.(4) shows the average cost of a good cycle.

$$N_1(T) = C_i R(T) \tag{4}$$

However, in a failure cycle, the machine can break down at any time $X < T$. Thus, the average cost would be $E[C_d(T-X)|X<T] + C_i + C_r$, where $E[\cdot]$ denotes the expectation. The probability of a failure cycle is $P\{X<T\} = F(T)$, so

$$N_2(T) = F(T)\left(\frac{\int_0^T C_d(T-x)f(x)dx}{F(T)} + C_i + C_r\right) \tag{5}$$

$$= F(T)(C_i + C_r) + F(T)C_d T - C_d\int_0^T xf(x)dx$$

Using Eq. (4) and (5), we obtain in Eq.(6) the ETC per time unit.

$$C(T) = \frac{1}{T}\left[C_i + C_r(1 - R(T)) + C_d\left(T - \int_0^T R(x)dx\right)\right] \tag{6}$$

The problem is to determine the optimal inspection period $T^*(0 \prec T^* \leq \infty)$ which minimizes the total composite maintenance cost per unit of time. If it exists it will verify $\frac{\partial C(T^*)}{\partial T} = 0$. Thus Differentiating $C(T)$ with respect to *T* in the above Eq.(6) we obtain:

$$\frac{\partial C(T)}{\partial T} = \frac{D(T)}{T^2} \tag{7}$$

Where

$$D(T) = C_d\int_0^T xf(x)dx + C_r(Tf(T) + R(T)) - (C_i + C_r) \tag{8}$$

Theorem: Existence and uniqueness of T^*

Suppose that $\lim_{t\to\infty} tf(t) = 0$, there exists a unique optimal inspection interval T^*, minimizing the total expected maintenance cost per unit of time C(T), verifying Eq.(9)

$$C(T^*) = C_d F(T^*) + C_r f(T^*) \tag{9}$$

Proof: From Eq.(8), it is obvious that $D(0) = -C_i$ has a negative value. By the assumption $\lim_{t\to\infty} tf(t) = 0$, the following result holds $D(+\infty) = \mu C_d - (C_i + C_r)$. We can distinguish two cases:

- ✓ If $C_d\mu - (C_i + C_r) \leq 0$ wich means $C_d\mu \leq C_i + C_r$, then $T^* = \infty$. This is predicable since inspection and repair cost exceeds the cost of downtime due to failure.
- ✓ If $C_d\mu - (C_i + C_r) \succ 0 \Leftrightarrow C_d\mu \succ C_i + C_r$ then, there exists at least one optimal inspection interval T^*, minimizing the total cost per unit of time C(T), and T^* satisfying $D(T^*) = 0$,

$$C_d\mu \succ C_i + C_r \tag{10}$$

$$C_d\int_0^{T^*} xf(x)dx + C_r T^* f(T^*) = C_r R(T^*) - (C_i + C_r) \tag{11}$$

Substituting Eq.(11) to Eq.(6), we obtain Eq.(9). The uniqueness of the solution is also proved for Weibull, Exponential, Log-convex and Log-concave life time density functions.

III. Principle of the Proposed Genetic Algorithm Model

Traditional optimization methods such as gradient descent technologies, Newton's method and various types of mathematical programming, usually require some information about the derivative and possibly second derivative information at each point evaluated in the solution space. These are used to determine the next direction of search. This information is not possible to calculate theoretically when the decision variables are discrete, and analytic approximations would be exceedingly cumbersome, especially in a stochastic framework. A recent study by Goit and Smith has shown that GAs can efficiency solve a wide range of redundancy allocation problem.

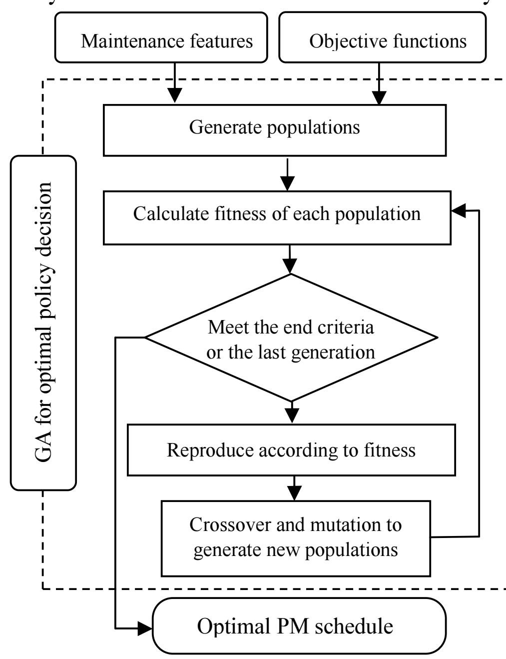

Fig.1 The flow diagram of genetic algorithms

Some of the many advantages of GAs apart from their easy adaptability and ability to solve diverse problem are:

1. GAs search from a population of potential solutions, rather than from a single point. This explicit parallelism helps the GAs in determining close to global optimum solutions without the danger of being trapped in a local optimum. There is also an implicit parallelism (Schema Theorem) that gives the GAs its power in searching very large solution domains.
2. GAs use objective function information directly, with no need for derivatives of other information. The objective function can involve any type of numeric or non-numeric variables, or other data structure, as long as a coding scheme

can be devised to represent the parameter set. GAs take their analogy from the physical world. GAs operate by creating an initial population of solutions, often represented as bit strings, that evolve over successive generation. The solutions with high fitness are mated with other solutions by crossing parts of a solution string with another. In addition, the solution strings are also muted. Over time, operations of weeding out poor fitness solutions and reproducing by crossing high fitness solutions at random points act to randomly-sample in a large part of the huge state space very efficiently. GAs search solution-spaces effectively by recombining and maintaining useful schema (building blocks) in the population. Each population member samples all the possible schema to which their bits belong. For example, the bit-string 10110 samples the region of space 1#### (# represents either 0 or 1). It also samples #0###, etc. In this way, extensive schemas in the space are implicitly sampled. This inherent expletive sampling ability of genetic is called `implicit parallelism'. This refers to the sampling of numerous schemas and the effective resembling of schema since good schema is maintained in the population over generations. GAs have discontinuities for high-dimension stochastic problems, with many non-linearity or discontinuities. They are suited for the characteristics of optimization problems: multi-model domains with some epistasis (one part of the solution or structure is affected by another). The optimum inspection time interval and the optimum preventive maintenance period in our method are the final schema of the designed algorithm (see *Fig.1*). The individuals correspond to the different and possible maintenance periods and inspection schedules. During this process we have to evaluate the fitness for each individual referred also as a genotype. Adapting and giving the significance to the genotypes is done by defining its correspondent phenotype.

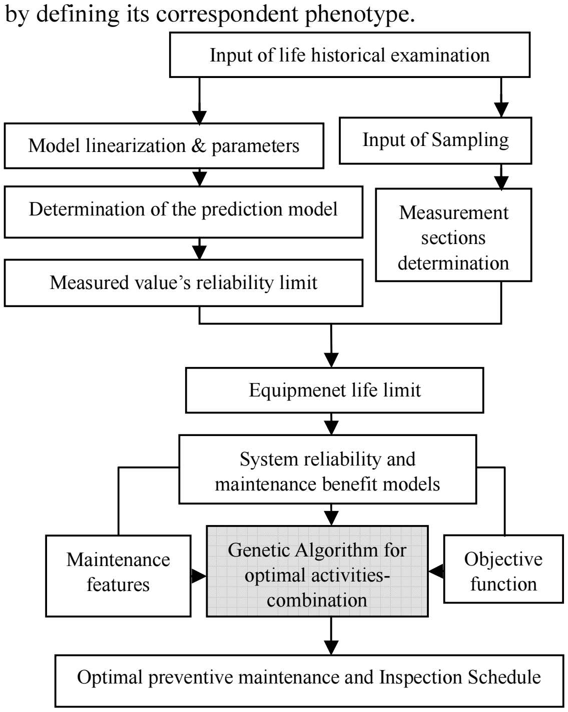

Fig.2 The PM and inspection scheduling procedure

A. Implementation of the genetic algorithm model

As we explained in the previous sections, the decision models for the optimum plant maintenance policy are too complicated to be solved as a numerical expression of solution. The implementation of the presented decision-making method is conducted using genetic algorithms. In this section we present the main steps of this implementation. *Fig.2* presents the PM and inspection scheduling procedure using GAs. This procedure is done taking in consideration the following remarks:

a) If the machinery reaches the end of its life cycle T, than the system is not reliable. Consequently, there is no need for more Maintenance. $R(t) = 0, t \geq T$

b) The life duration of the machinery is divided into a finite and discrete number of equal periods or units of time.

c) The gene length, which is one of the basic parameters in the algorithm, is obtained after the determination of the equipment life limit.

B. Genotype encoding and fitness

In this study, the genotypes used in the genetic algorithms are expressed by a set of binary codes. The transformation into the corresponding phenotypes is generated using the Eq.(12) and Eq.(13) respectively for preventive maintenance period and inspection interval generation.

$$t_k = (k-1) \times \Delta T \times A_k \qquad k = 1, 2 \cdots, N \tag{12}$$

$$t = \sum_{k=1}^{N} A_k \times 2^{k-1} \tag{13}$$

Where A_k is the value of the genotype in the k^{th} bit and N its length. In the GA method, genotypes are evolved by the uniform crossover and mutation with rates proportionally correlated to the system's failure rate. The generation of this process has as result the design of an elite genotype that corresponds to our optimum maintenance period. The evolution of the algorithm is conducted by the maximization of the fitness of each population. In the other side, the objective of the described method is to optimize Maintenance period and inspection time interval mainly by minimizing the composite maintenance cost; we considered then the fitness as an inverse proportional function of the cost.

IV. Simulation Results

In the previous sections, we explained the foundation of the proposed decision-making method and we discussed the different used models. In this section, we apply the proposed method and we present the simulation experiment for the time continuous optimal maintenance period and the discrete time preventive maintenance period. A total of 60 sets of equipment was considered, the life examination of the collected data has shown that failures occurred in 24 sets. The forecasting of the failure time distribution function from the measured values and the corresponding failure rate are shown in figures fig.3 for the continuous time strategy.

Tab1. Input parameters

C_i	C_r	C_d	α	η
6.5 10^3 *JPY*	1.5 10^3 *JPY*	9 10^3 *JPY*	3	4.1918

The analysis and examination of the life data has allowed the determination of the input parameters, shown in table 1. The failure rate in this case study is increasing with time. The termination of GAs is either fitness-error of two successive generations less than 0.0001 or reaching the number of maximum generation.

A. Continuous time Preventive maintenance period

The maximum measured value for reliability limit is 95%, the projection of this limit on the time axis sets the length of the cycle. The range of the calculated preservation cycle is [0, 7853] hour. If we consider plant machinery operating an average of 8 hours/day, 5 days/week, then this life time cycle correspond to duration of 49 months. The final output of the genetic algorithm in this case indicates that the optimum preventive maintenance corresponds to an optimum period of 2355 hours that can be approximated to $T^* = 15 Months$ with an optimum ETC of ¥*5.3 Millions*.

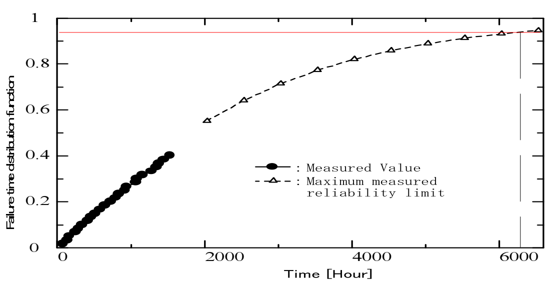

Fig.3. The failure time distribution for the PM period

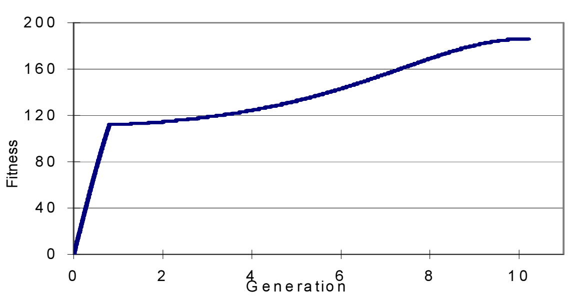

Fig.4. The hill climbing of genetic algorithms

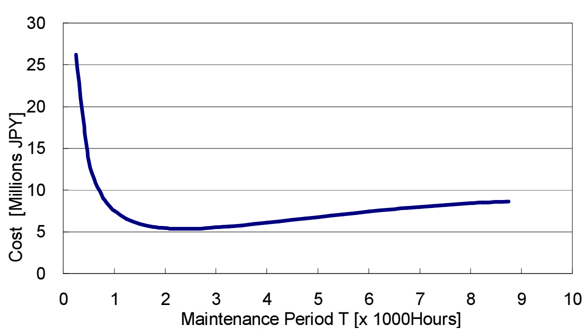

Fig.5. The cost function Continuous time PM

Figure fig.3 shows the failure time distribution function, we can see clearly that the calculated failure times follow the same trend as the measured data. As expected an increasing failure rate.

It can be seen that the incremental differences of the fitness in Fig.4 change suddenly around 112 fitness value, this correspond to periodical inspection/replacement $T \in [750, 1300]$ hours with almost unchangeable ETC (¥*8.9 Millions*). This is explained by the inflection in the failure rate. Finally the effectiveness of this method is verified in Fig.5 for the continuous time preventive maintenance.

B. Discrete time Preventive maintenance period

The study of repairable systems subjected to failures constitutes an essential issue in the reliability literature. Until recently, most of the research on discrete time repairable systems is performed using homogeneous Markov chain. When phase type distributions were used [13], most models assumed exponential distributions for operating and repair times for simplicity. However, the exponential distribution presents some limitations. Neuts et al. [14] study a single unit system with operating and repair times following phase type distributions. However, phase type distributions are still limited to cases with rational Laplace transform.

In this section, the previously presented preventive maintenance strategy and its GA implementation were adapted to simulate a discrete time periodic inspection/replacement policy. The use of the GAs allows us to avoid the mathematical limitations of distributions and transformations.

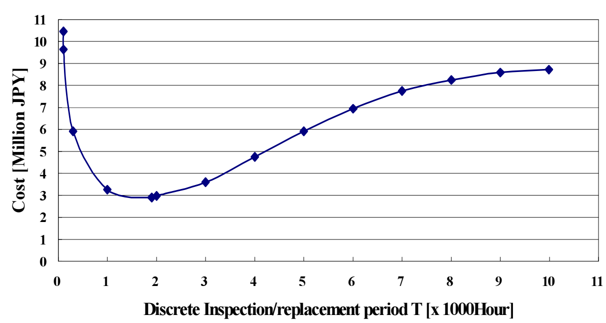

Fig.6. Discrete time Preventive Maintenance cost

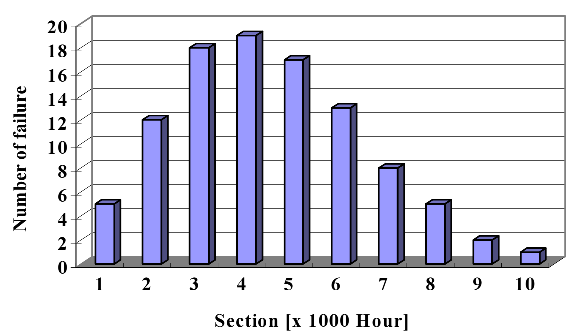

Fig.7. Total number of failure in discrete time

We apply this method to a system represented by a general distribution. The result of the number of failure within each sampling time used in the performed simulation and the corresponding rate of failure are shown in fig.7. All the sampling intervals used for the life examination were made with a sampling period of 1000 hours. As explained previously, since the maximum measured value for reliability limit is 95%, we obtain a $[0 \sim 9999]$[Hour] cycle. The final output of the genetic algorithm in this case indicates an optimum preventive maintenance period of 1900 hours, $T^* = 1 year$ and an optimal *ETC= ¥2.9 Millions*.

As expected the failure rate is increasing and we can see a decrease in the number of failure explained by the optimal maintenance periodicity. Optimizing the maintenance period using GAs allows us not only to deal with Weibull model cases that have any values of m and η, but also with discrete cases in which the optimality is hardly obtained by analytical methods. As shown in Fig 7, As the GA evolutes approaching the optimum, the first time of inspection is occurring. This means that it is better to take the longer interval until first inspection. The reason is that failure rate increases at any time and as time passes, machinery becomes easier to fail as closing the end of inspection time.

The results we obtained by the method proposed in this paper and the effectiveness is also verified in Fig.6. for the discrete time periodically inspection/replacement strategy.

V. CONCLUSIONS

In the economically competitive world it is increasingly important to consider the cost factors associated with safety related systems in addition to those of production systems. The operation and management of a plant requires proper accounting for the constraints coming from reliability requirements as well as from budget and resource considerations.

Most of the mathematical methods to decide the inspection time interval by reliability theory are too complicated to be solved. Moreover, in the practical applications, the machinery plants must satisfy the required conditions of these theoretical models; which is not the case usually. In order to overcome these problems, in this paper, we have proposed a decision-making method for optimizing inspection interval and maintenance period to minimize the maintenance cost by reliability theory and genetic algorithm (GA). The most merit of the method proposed here is that we can decide the preventive maintenance period for plant machinery that have failure rate conforms to any distribution; therefore, we may say the method is more practical. The effectiveness of this method has been verified by simulation resultants in both continuous and discrete times.

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