GAFIS: Genetic Algorithm with Fuzzy Inference System

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Abstract - **Applications of Genetic Algorithms – GA for optimisation problems are widely known as well as their advantages and disadvantages compared with classical numerical methods. In practical tests, a GA appears a robust method with a broad range of applications. The determination of GA parameters could be complicated. Therefore for some real-life applications, several empirical observations of an experienced expert are needed to define these parameters. This fact degrades the applicability of GA for most of the real-world problems and users.**

Therefore, this article discusses some possibilities with setting a GA. The setting method of GA parameters is based on the fuzzy control of values of GA parameters (GAFIS). The feedback for the fuzzy control of GA parameters is realized by virtue of the behavior of some GA characteristics.

Keywords: Genetic algorithms, GAFIS, FIS, Control surface of GA, GA characteristic.

1 Introduction

The using of genetic algorithms for optimisation the fuzzy systems is known [9]. We are designed the opposite way. The setting of GA parameters is often a seriously complicated procedure because it must meet two contradictory requirements:

- to search up the whole space
- to search some parts of the space in detail

A balance between the utilisation of the whole space and the detailed searching of some parts can be adapted to pressure [1,8] of selection and recombination operators. This balance is critical for a GA behavior. For this reason, it is very important to understand the influence of selection and recombination operators on the GA. The operators have a direct influence on the GA convergence.

For "suitable" behaviour of a GA with respect to the problem to be solved the follow requirements can be formulated:

1. to find an acceptable solution (a global one is the best)
2. an acceptable number of iterations (generations) and time of computation

These requirements will be fulfilled if "suitable" methods (selection and recombination) and "suitable" setting of a GA will be used. Only just in verbal term and mean "suitable" is a problem. When setting GA parameters, the following solutions are available for the standard or less experienced GA user:

- to try set up of GA parameters for a specific problem with the advice of experts [1,2] – can be acceptable if taken in all aspects
- to set up parameters by expert or to become an expert yourself!
- to try to set up parameters randomly by means of common antecedents [1], and make pre-sets in case of a failure – the work after a longer time having some magic and the user getting to be a wizard!

It is necessary to say that the human experts in a given branch could have a problem with "suitable" behaviour of a GA as well. The "suitability" of changes of GA parameters comes during the run of the algorithm and sometimes comes from the experiments and experiences with a GA behaviour [3].

The following criteria and fundamentals can be used to solve the problem. We try to suggest a solution on the basis of given reasons. The basis of the solution is a fuzzy control of GA parameters.

2 Genetic Algorithms

Used GA represents generation model of GA. The short formal description of GA is follows:

$$P_{n+1} = f\left(P_n, \xi_n\right), \quad \xi \in \left(S, C, M\right) \tag{1}$$

Where: P…population
ξ …a set of GA operators: S …selection
C …crossover
M…mutation
n …number of generation

3 Characteristics of GA

Fuzzy Inference System – FIS is used for the fuzzy control of GA parameters. The following must be determined for applications of FIS to control the GA:

- input values
- model of FIS

3.1 Quantitative Characteristics

The input values are extracted from quantitative characteristics of GA. Various types of these characteristics are contained, for instance, in [1, 2, 3, 5].

The following characteristics can be used as the input:

→ Variability of population - *varH*

Also called survival probabilities. This variability is represented by a ratio of identical individuals to all individuals per population (2). Since a binary string represents the individuals, Hamming distances are used to compare these individuals. The Hamming distance is given by the metric system ρ_H, (3):

$$\operatorname{var} H = \frac{\#kind}{\#individual}\ ,\ varH \in (0,1] \qquad (2)$$

$$\rho_H(\vec{a},\vec{b}) = \sum_{i=1}^{n} |a_i - b_i| \qquad (3)$$

Where: $\vec{a} = (a_1,\ldots,a_n) \in \mathrm{B}^n, \vec{b} = (b_1,\ldots b_2) \in \mathrm{B}^n$

→ Coefficient of partial convergence - *cpc*

Value *cpc* (5) determines the average or the weight-average change of a standard value of fitness function (4) per sample of generations.

Let the population be sorted:

z_n^k … n-th fitness value from k-th population

$n = 1$ … the best solution

$k = 1$ … the first population

$$\|\vec{z}_n\| = \frac{z_n^{(k)}}{\sum_{k \in sample} z_n}; z_n \geq 0, \|\vec{z}_n\| \in [0,1], \qquad (4)$$

where a *sample* is a discrete interval [k, k+*sample*] of observations of values z_n related to the generation.

$$cpc = \sum_{i=k}^{k+sample-1} f_w(z_n^{i+1} - z_n^i), \qquad (5)$$

where f_w is a weight function related to the *sample*. If f_w=1, then $pc \in$ [-1,1]. For a small number of samples (about 5÷10 generation) f_w=1 is used. The value of a sample can be constant. We used ten samples in our research (it is sample∈[k,k+9]). For a time behavior of the *sample* the so-called H-characteristics (see below) was used.

→ Furthermore, some characteristic based on cluster analysis, monitoring fitness values (function values z_{min}, z_{mean}, and so on) and another statistical method based on fitness distribution can be used.

→ So-called H-characteristics [3] for determination of a suitable moment to change GA parameters, which is a fuzzy control action. If a function value of H-characteristics is under stagnation, the change is required. Stagnation is tested by *cpc*.

The general formulas of **H-**characteristics are represented by the following expressions (6,7):

- $$H_n^1 = \sum_{\substack{i_1 < i_2 \\ i_1, i_2 \in K}} \sum_{j=0}^{m} \left(x_{n(i_1),j} - x_{n(i_2),j}\right)^2 \qquad (6)$$
- $\Delta H_n^1 = H_n^1 - H_{n-1}^1,\quad H_0^1 = 0$
- $\left|\Delta H_n^1\right|$

- $$H_n^2 = \sum_{\substack{i_1 < i_2 \\ i_1, i_2 \in K}} \left(\sum_{j=0}^{m} \left(x_{n(i_1),j} - x_{n(i_2),j}\right)^2\right) \cdot \ldots$$
$$\ldots \cdot \left(x_{n(i_1)0} - x_{n(i_2)0}\right)^2 \qquad (7)$$
- $\Delta H_n^2 = H_n^2 - H_{n-1}^2,\quad H_0^1 = 0$
- $\left|\Delta H_n^2\right|$

where K is a characterized subset of population,

$x_{n(1)0}$ … $x_{n(p)0}$ represent ordered sequence of fitness values of n-th generation (until now denoted as $\mathbf{z}_n$),

$x_{n(i)j}$ … denotes i-th co-ordinate of domain element from n-th generation with fitness value $x_{n(i)0}$.

Several figures are presented for the case of test and set $K=\{1,2,3\}$, (see figure 1,2). We can see that **H-**characteristics follow the behaviour of z_{min}. Therefore, we observe the **H-**characteristics and adapt the GA parameters when these characteristics are stabilized (look at ΔH and |ΔH|).

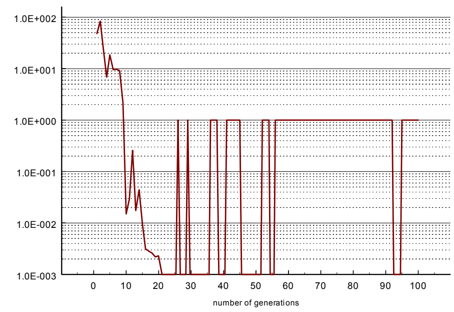


Fig.1. The $\mathbf{H^1}$ characteristic

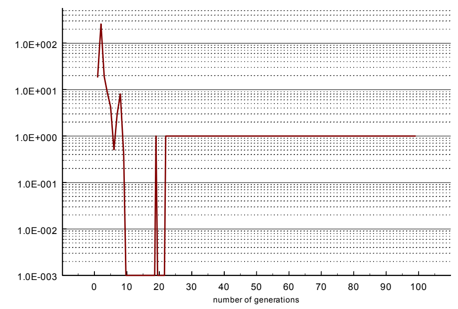


Fig. 2. The $|\Delta\mathbf{H}^2|$ characteristic

3.2 Qualitative-Quantitative Characteristic

This is a verbal characteristic with respect to the common aspect of a GA. It serves for assembling and setting of FIS by an expert, with respect to the rule of fuzzy modelling.

Verbal characteristics represent the instruction for building the fuzzy rules, as e.g (with terms mentioned above):

- if *varH* is "hi" then *mutation* is/set "low" & *selection* is/set "hi"
- if *varH* is "low" & *pc* is "stagnation" then *mutation* is/set "hi"
- and so on …

Next a design for a fuzzy membership function:

- selection intensity = {low, mild, hi}

Where [2]:	"low"	tournament ≈ 1÷3
	"mid'	tournament ≈ 3÷5
	"hi"	tournament ≈ > 4

- and so on …

4 Fuzzy setting of GA

The design of the algorithm for fuzzy setting of GA parameters is based on the classical model GA given by figure 3. Our idea is the adaptation of the GA operators value $\xi \in$(selection; crossover; mutation) during the run of GA.

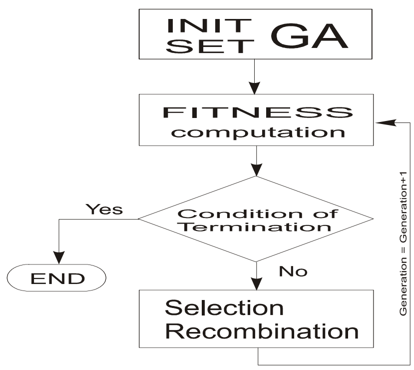


Fig. 3. The Genetic Algorithm

The fuzzy control is applied if the condition of fuzzy adaptation is true. The arguments why not to use the fuzzy control during the complete run of GA are as follows:

1. The time cost of calculating of FIS must be respected.
2. After changing the value of GA parameters it is reasonable to let some time-generations for stabilization of GA process.
3. If convergence of GA is acceptable it is not possible to use fuzzy control action.

These three arguments represent *"Condition of Fuzzy Adaptation"* in our algorithm given by fig. 4.

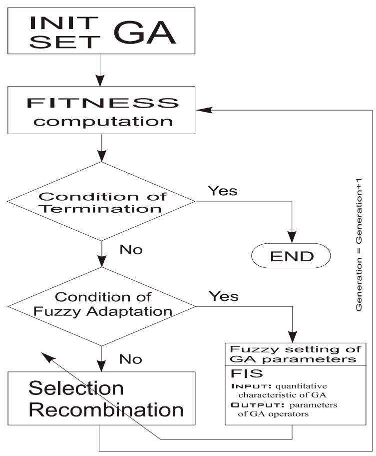


Fig. 4. The GA with Fuzzy control of parameters

A basis of fuzzy control mechanism is Fuzzy Inference System – FIS [4] assembled by an expert. Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. For our work we used Mamdani's inference method. The process of fuzzy inference involves these pieces of fuzzy logic: membership function, fuzzy logic operators, if-then rules, and defuzzification.

The follows a short description of the pieces fuzzy control mechanism and FIS:

Real inputs: Represent numerical values of observed characteristic of GA, see above *varH*, *cpc*, and so on. The inputs are crips (non-fuzzy) numbers.
Fuzzify inputs: Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1. Suitable parameterisation is used for fuzzy sets in antecedent and consequent.
Fuzzy rules: Fuzzy sets and fuzzy operators are subjects and verbs of fuzzy logic. These IF-THEN rule statements (8) are used to formulate the conditional statements that comprise fuzzy logic.

$$R_i: \quad \text{IF } (in_1 \text{ is } A_{1i}) \,\&\ldots\&(in_m \text{ is } A_{mi}) \text{ THEN } (out_1 \text{ is } B_{1i}) \,\&\ldots\&(out_n \text{ is } B_{ni}) \qquad (8)$$

Apply fuzzy operator to multiple part antecedents: If there are multiple parts to the antecedent, apply fuzzy logic operators and resolve the antecedent to a single number between 0 and 1.
Apply implication method: Use the degree of support for the entire rule to shape the output fuzzy set. The consequence of a fuzzy rule assigns an entire fuzzy set to the output.
Aggregation: The output of each rule is a fuzzy set (sets if necessary). The output fuzzy sets B_r for each rule, these are then aggregated into a single output fuzzy set B_o.
Defuzzifcation: The input for the defuzzification process is a fuzzy set μ (the aggregate output fuzzy set) and output is a single number y^*. For defuzzification we used the method of centroid (center-of-sums) by equation (9).

$$y^* = defuzz(B_o) = \frac{\int_y y_n \cdot \sum_{r=1}^{n} \mu B_r(y)dy}{\int_y \sum_{r=1}^{n} \mu B_r(y)dy}, \qquad (9)$$

Defuzzified value is directly acceptable value of GA parameters, like for example:

Output #1 $\equiv y_1^* \equiv$ "power of tournament selection",
Output #2 $\equiv y_2^* \equiv$ "probability of bit-mutation",
$y_1^* = 2$ … represent binary tournament selection,
$y_2^* = 0.03$… represent 3% probability of mutation.

5 Test Problem

For the testing of the performance of evolutionary heuristic algorithms, such as GA, some set of artificial designed optimisation problems was used. Usually it is a set of five famous DeJong's [6,7] functions: F1-F5 and few other testing functions called after their authors or again as F6-F12.

These functions are designed to represent a specific part or set of problems for optimisation algorithms. The test functions are categorised using a taxonomy incorporating:

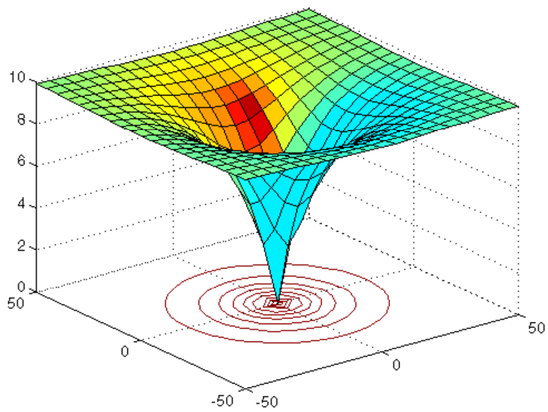
- Continuous v. discontinuous
- Convex v. non-convex
- Unimodal v. multimodal
- Quadratic v. non-quadratic
- Low dimensionality v. high dimensionality
- Deterministic v. stochastic

We used a test function denoted Ackley's function. The function is a continuous, multimodal test function obtained by modulating an exponential function with a cosine wave of moderate amplitude. Originally, it was formulated by Ackley (see [6] and [7]). To facilitate its use for minimization and to achieve a standardisation of the global minimum to an objective function value of zero, the function is formulated as follows:

$$f_{Ackley}(\vec{x}) = -c_1 \cdot \exp\left(-c_2 \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(c_3 \cdot x_i)\right) + c_1 + e \qquad ,(10)$$

$x_i \in [-20, 30]; \quad \min f_{Ackley}(\vec{x}) = 0,$
n…*dimensionality*, $c_1 = 20$, $c_2 = 0.2$, $c_3 = 2\pi$

In order to facilitate an empirical visualisation analysis of this function, a 3D representation, i.e. with two parameters, $n=2$ will be used. Its graph is follows:



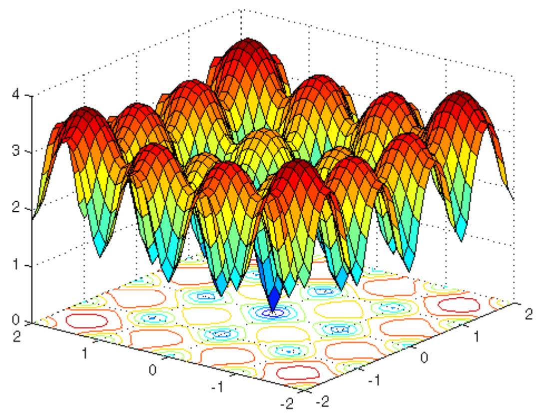


Fig. 5. Ackley's Function

6 Experiment

In our tests we used following setting of GA:

method, operator, parameter	setting per test #1,#2,#3		
Model GA, # of individuals	generation, non-elite, 40		
Parameter encoding, length	gray, 20bit/parameter		
Test problem	min{$F_{Ac}(x_1,x_2)$}, $x_i \in (-25,25)$		
All without elitism!	#1	#2	#3
Selection - [#] tournament	2	3	FIS
Crossover - [%], # of cut-point	60; 2	50; 3	FIS
Mutation per bit - [p_m]	0.02	0.05	FIS

An environment of MatLab®, with our own genetic toolbox was used. All results represent 100 runs of our GA per test #1, #2 and #3.
Setting of GA (S, C, M) per test representatives:

Test #1: Approximate values according to some GA literature. This setting is constant per all runs of GA. The setting corresponds to the workaday user.

Test #2: The setting corresponds to: test problem, size of population, applied operators and making use of tips from experts [1,2 and 3]. This setting is constant for all runs of GA.

Test #3: The base for the setting is FIS, which is given by an expert. For this test the GA with fuzzy control was the value of GA parameters described in figure 4 (in detail see above). Initial setting are as for the test #2. A value of the *sample* is ten.

The short description of designed FIS:

inputs: varH$\in$(0, 1], cpc$\in$[-1,1]

Fuzzy inputs			
varH – in1	low	mid	hi
μ_{BELL} – c,σ,β	[0.3 3.278 0]	[0.238 3.28 0.5]	[0.3 3.278 1]
cpc – in2	descent	stagnation	grow
μ_{BELL} – c,σ,β	[0.862 2.5 -1]	[0.132 1.69 0]	[0.857 2.5 1]

outputs: S$\in$[1, 10], M$\in$[0, 0.1]

Fuzzy outputs			
Selection - S	low	mid	hi
μ_{BELL} – c,σ,β	[2.64 7.12 -1.03]	[1.33 3.57 2.58]	[2.86 2.43 9.05]
Mutation – M	low	mid	hi
μ_{BELL} – c,σ,β	[0.0244 3.28 -0.0106]	[0.0163 3.28 0.0395]	[0.0359 3.28 0.101]

Fuzzy rules:

output #1 – Selection - S				output #2 – Mutation - M			
in1\in2	desc.	stagn.	grow	in1\in2	desc.	stagn.	grow
low		*LOW*		low		*MID*	
mid	*MID*	*MID*	*HI*	mid	*LOW*	*HI*	*LOW*
hi			*MID*	hi			*LOW*

Membership function: by equation (11):

$$\mu_{BELL,i} = \frac{1}{1 + \left| \frac{x_i - c}{\sigma} \right|^{2\beta}} \qquad \textbf{(11)}$$

Of course, this setting of FIS does not stand for the best solution. It presents a subjective view of an expert with respect to the objective characteristics of GA, see above.

The setting is viewable as control surfaces of GA, see figure 6. The change of behaviour (convergence) of GA in accordance with fuzzy control action can be seen from a characteristic in figure 7.

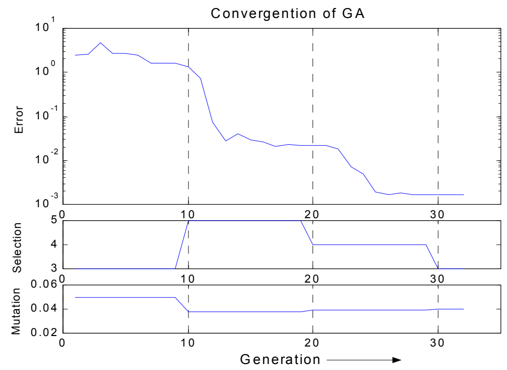


Fig. 7. A behaviour of GA, test #3

7 Conclusion and Further Research

A designed GA with Fuzzy Control of GA parameters – GA-FC, let us say FIS, can be further modified with better designed FIS, and for example, different shapes and number of membership functions with better respect of GA, along with further criteria and characteristics of GA. This work represents a new doorway by an algorithm given in figure 4, and must be understood for the possibilities of Fuzzy Setting of GA Parameters.

Practical results of a GAFIS showed (see figure 8) that the GA-FC algorithm (test #3) for our specific problem demonstrated essentially better behaviour than when used with common setting – test #1.

Further, GAFIS has about 10 % better convergence regarding the number of generations than GA constantly set by an expert – test #2. GAFIS is on average slower per generation with respect to the time cost.

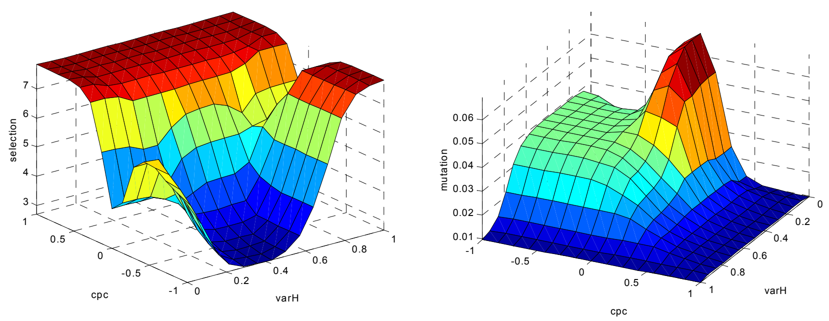


Fig. 6. Control surfaces of GA – Selection, Mutation

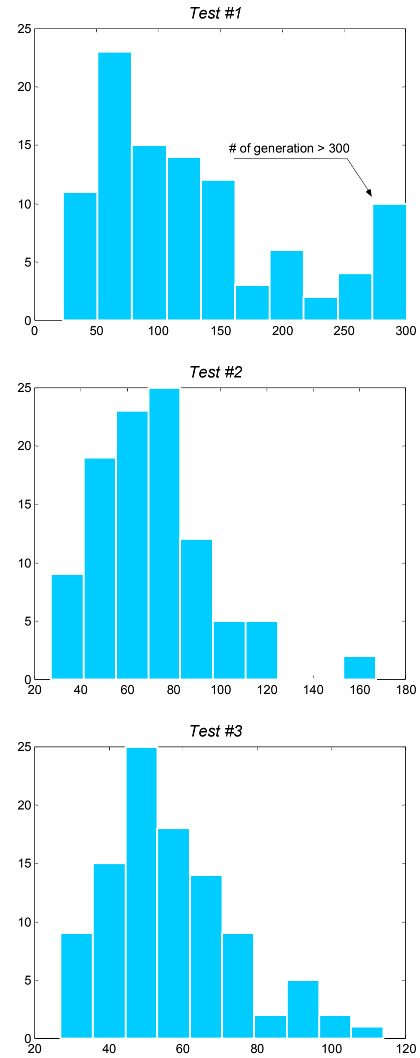


Fig. 8. Histogram of GA for test #1,#2 and #3
x-axis ... number of generation to the solution
y-axis ... frequency

The GAFIS presents a chance for every user who is only able to formulate the fitness and takes already built-up FIS. Another areas of use are the dynamic variable systems where FIS can provide a good description of the system behaviour. Achieved results will be used for further analysis of GAs behaviour and for improvement of the GAs power in the solving of prominent engineering applications.

Acknowledgements

This work has been supported by MŠMT No.: KONTAKT ME 526, MSM 2611 00009 and MSM 2611 00013.

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