Ambiguous Comparative Judgment:
Fuzzy Set Model and Data Analysis

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Abstract – **In this paper, two types of fuzzy set model for ambiguous comparative judgment which does not always hold transitivity and comparability properties were proposed. The first type of the model is a fuzzy theoretic extension of the additive difference model for preference for explaining ambiguous preference strength. The second type of the model is a fuzzy logistic model for explaining ambiguous preference in which preference strength is bounded such as probability measure. In both the models, multi-attribute weighting parameters and all attribute values are assumed to be asymmetric fuzzy L-R numbers. For each model, the parameter estimation method using the fuzzy regression analysis was proposed. Numerical examples for comparative judgments were also demonstrated. Lastly, theoretical and practical implications for the proposed models were discussed.**

I. INTRODUCTION

There are two types of human judgment procedures. One is referred to as absolute judgment in which an evaluator is asked attractiveness of the object for evaluation (e.g., "How much do you like this brand on a 0-100 scale"). The other procedure is referred to as comparative judgment in which an evaluator is asked which alternative is preferred (e.g., " Do you prefer Brand A to Brand B ?" or "How do you estimate a probability of choosing Brand A over Brand B when you compare Brand A with Brand B ? ").

The aim of this paper is to establish a model of ambiguous comparative judgment and to provide the data analysis method for the model. Comparative judgments in social situations often involves ambiguity concerning the confidence. People may not be able to make judgments without representing some confidence intervals. In order to measure the ambiguity (or vagueness) of human judgment, the fuzzy rating method has been proposed and developed [1]. In the fuzzy rating method, respondents select a representative rating point on a scale and indicate lower or upper rating points if they wish depending upon the relative ambiguity of their judgment. For example, the fuzzy rating method would be useful for measuring perceived temperature indicating the representative value and the lower or upper values. This rating scale allows for asymmetries, and overcomes the problem, identified by Smithson[2], of researchers arbitrarily deciding the most representative value from a range of scores. By making certain simplifying assumptions (not uncommon within fuzzy set theory), the rating can be viewed as a L-R fuzzy number, hence making possible the use of fuzzy set theoretic operations [1,3].

In this study, fuzzy set models which explains ambiguous comparative judgment are proposed. Since ambiguous comparative judgment may not always hold transitivity and comparability properties, parameters due to biased responses which may not hold transitivity and comparability properties were assumed in the proposed model. The author proposes two types of the fuzzy set model for ambiguous comparative judgment. The first type of the model is a fuzzy theoretic extension of the additive difference model for preference for explaining ambiguous preference strength which does not always assume boundaries of a judgment scale such as a WTP measure (Measure for Willing To Pay). The second type of the model is a fuzzy logistic model of the additive difference preference for explaining ambiguous preference in which preference strength is bounded such as probability measure (e.g., a certain interval within a bounded interval from 0% to 100%). Since judgment of a bounded scale such as probability judgment causes a methodological problem when the fuzzy linear regression method is used, the fuzzy logistic function is proposed to prevent the problem. In both the models, multi-attribute weighting parameters and all attribute values are assumed to be asymmetric fuzzy L-R numbers. For each model, the parameter estimation method using the fuzzy regression analysis was proposed. That is, the fuzzy linear regression model using the least square method [4,5] is proposed for the analysis of the former model, and the fuzzy logistic regression model[6] is proposed for the analysis of the latter model. Numerical illustrations of psychological experiments are provided for examining both types of the models and the analyses.

Ⅱ. MODEL OF AMBIGUOUS COMPARATIVE JUDGMENT

A. Definition 1 Set of Multidimensional Alternatives:

Let $\mathcal{X} = X_1 \times X_2 \times .... \times X_n$ be a finite set of multidimensional alternatives with elements of the form $X_j = (X_{j1}, X_{j2},...,X_{jn})$, $X_k = (X_{k1}, X_{k2},...,X_{kn})$,..., $X_o = (X_{o1}, X_{o2},...,X_{om})$ where X_{jm} *(l=1,..,o; m=1,..,n) is the value of alternative* X_j on dimension m. Note that the components of X_j may be ambiguous linguistic variable rather than crisp numbers.

B. Definition 2 Classic Preference Relation:

Let $\succ$ be a binary relation on $\mathcal{X}$, i.e., $\succ$ is a subset of $\mathcal{X}\times\mathcal{X}$.

The relational structure $< \mathcal{X}, \succ >$ is a weak order if and only if for all X_j, Xk, X_l, the following two axioms are satisfied.

1). Connectedness (Comparability): $X_j \succ Xk$ *or* $Xk \succ X_j$,

2). Transitivity: If $X_j \succ Xk$ *and* $Xk \succ X_l$, then $X_j \succ X_l$.

However, the weak order relation is not always assumed in this paper. That is, transitivity or connectedness may be violated in the preference relations.

C. Definition 3 Fuzzy Preference Relation:

Since a classical preference relation $\succ$ is a subset of $\mathcal{X}\times\mathcal{X}$, $\succ$ is a classical set often viewed as a characteristic function c from $\mathcal{X}\times\mathcal{X}$ to {0,1} such that

$$c(X_j \succ X_k) = \begin{cases} 1 & iff \quad Xj \succ Xk \\ 0 & iff \quad not(Xj \succ Xk) \end{cases}$$

(Note. "iff" is short for "if and only if"). {0,1} is called valuation set. If the valuation set is allowed to be real interval [0,1], $\succ$ is called a fuzzy preference relation. That is, the membership function μ f is defined as

$\mu f : \mathcal{X}\times\mathcal{X} \rightarrow [0,1]$.

D. Definition 4 Ambiguous Preference Relation:

Ambiguous preference relations are defined as a fuzzy set of $\mathcal{X}\times\mathcal{X} \times S$, where S is subset of one dimensional real number space . S is interpreted as domain of preference strength. S may be bounded, for example, S= [0,1]. The membership function μ_a is defined as:

$\mu_a : \mathcal{X}\times\mathcal{X} \times S \rightarrow [0,1]$.

Ambiguous preference relation is interpreted as a fuzzified version for a classical characteristic function $c(X_j \succ X_k)$. Therefre, ambiguous preference relation for $X_j \succ X_k$ is represented as a fuzzy set $v(X_j \succ X_k)$. For simplicity, $v(X_j \succ X_k)$ will be assumed as an asymmetrical L-R fuzzy number (See Figure 1).

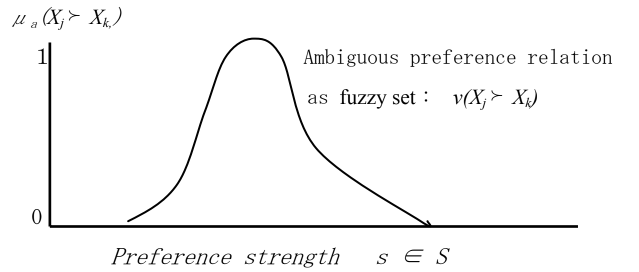

Figure 1 An example of ambiguous preference relation

E. Additive Difference Model of Ambiguous Comparative Judgment

Ambiguous preference relation $v(X_j \succ X_k)$ for $X_j \succ X_k$ is represented as the following additive difference model using L-R fuzzy number.

$$v(X_j \succ X_k) = A_{jk0} + A_{jk1} \otimes (X_{j1} - X_{k1}) + \ldots + A_{jkm} \otimes (X_{jm} - X_{km}) \qquad \ldots(1)$$

where $\otimes$, $+$, and $-$ is product , additive, and difference operation based on the extension principle for fuzzy set, respectively .

A parameter A_{jk0} involves a response bias due to presentation-order , context effects and a scale parameter of a dependent variable. The parameter A_{jk0} would be larger than A_{kj0} if X_j is more salient than X_k. This model can be reduced to Fuzzy Utility Difference Model [7] if multi-attribute weighting parameters are assumed to be crisp numbers, and to Additive Difference Model [8] if multi-attribute weighting parameters and values of multi-attributes are assumed to be crisp numbers.

F. Logistic Model of Ambiguous Comparative Judgment

Let an ambiguous preference relation which is bounded (e.g., fuzzy probability in [0,1]) be $p(X_j \succ X_k)$ for $X_j \succ X_k$. $p(X_j \succ X_k)$ is represented as the following logistic model using L-R fuzzy number.

$$Log\,(p(X_j \succ X_k)\ \Theta\ (1 - p(X_j \succ X_k))) = A_{jk0} + A_{jk1} \otimes (X_{j1} - X_{k1}) + \ldots + A_{jkm} \otimes (X_{jm} - X_{km}) \qquad (2)$$

where Log, Θ, $\otimes$, $+$, and $-$ is logarithmic, division, product , additive, and difference operation based on the extension principle for fuzzy set, respectively .

G. Explaining Non-Comparability and Intransitivity

Non comparability and intransitivity properties are explained if a threshold of comparative judgment is assumed and necessity measure of fuzzy comparative relation since the existence of threshold indicates intransitivity and a necessity measure for fuzzy relation does not always lead comparability. That is,

$X_j \succ X_k$ iff $Nes(\nu(X_j \succ X_k) > \theta)$ …(3).

or

$X_j \succ X_k$ iff

$Nes(p(X_j \succ X_k)\ \Theta\ (1 - p(X_j \succ X_k)) > P\theta)$ …(4).

where $Nes(.)$ is a necessity measure, and $\theta, P\theta$ is threshold parameter for the additive difference model and the logistic regression model, respectively.

Assuming the above relation of (3) or (4), it is clear that intransitivity and non- comparability hold in the comparative judgment.

Ⅲ. FUZZY DATA ANALYSIS FOR THE AMBIGUOUS COMPARARIVE JUDGMENT MODEL

A. Analysis of the Additive Difference Type Model

A set of fuzzy input-output data for i-th observation is defined by:

$$\left(Y_{ijk}; X_{ijk\,0}, X_{ij1}, X_{ij2}, \ldots, X_{ijm;} X_{ik1}, X_{ik2}, \ldots, X_{ikm;}\right) \ldots (5).$$

where Y_{ijk} indicates ambiguous preference represented by fuzzy L-R number, and X_{ijk} is X_{jk} for observation i.

Let $X_{ijk\cdot}$ be $X_{ij\cdot} - X_{ik}$, where − is difference operator based on the fuzzy extension principle, and denote $X_{i.}$ for abbreviation of $X_{ijk\cdot}$ in the following section.

Therefore a set of fuzzy input-output data for i-th observation is re-written by:

$$\left(Y_i; X_{i0}, X_{i1}, X_{i2}, \ldots, X_{im}\right),\quad i = 1,2,\cdots,n \qquad \ldots\ldots\ldots\ldots (6).$$

where Y_i is a fuzzy dependent variable, and X_{im} is a fuzzy independent variable represented by L-R fuzzy numbers. For simplicity, we assume that Y_i and X_{im} are positive for any membership value, $\alpha \in (0,1)$.

The fuzzy linear regression model (where both input and output data are fuzzy numbers) is represented as follows:

$$\overline{Y}_i = A_0 + A_1 \otimes X_{i1} + \ldots + A_m \otimes X_{im} \qquad \ldots\ldots\ldots (7).$$

where $X_{i0} = 1$, $A_j\ (j = 0,1,\cdots,m)$ is fuzzy regression parameter represented by L-R fuzzy number, and $\otimes$ is the product operator based on the extension principle.

It should be noted that although the explicit form of the membership function of $\overline{Y}_i$ can not be directly obtained, the α-level set of $\overline{Y}_i$ can be obtained from the result of Nguyen's theorem[9].

Let $z^L_{i(\alpha)}$ be a lower value of $\overline{Y}_i$, and $z^R_{i(\alpha)}$ be an upper value of $\overline{Y}_i$. Then,

$$Z_i = \left[z^L_{i(\alpha)}, z^R_{i(\alpha)}\right],\quad \alpha \in (0,1] \qquad \ldots\ldots\ (8).$$

where

$$z^L_{i(\alpha)} = \sum_{j=0}^{m}\left\{\min\left(a^L_{j(\alpha)}x^L_{ij(\alpha)}, a^L_{j(\alpha)}x^R_{ij(\alpha)}\right)\right\} \quad \ldots (9).$$

$$z^R_{i(\alpha)} = \sum_{j=0}^{m}\left\{\max\left(a^R_{j(\alpha)}x^L_{ij(\alpha)}, a^R_{j(\alpha)}x^R_{ij(\alpha)}\right)\right\} \quad \ldots. (10).$$

In above equation (9), $a^L_{j(\alpha)}x^L_{ij(\alpha)}$ is a product between lower value of α-level fuzzy coefficient for j -th attribute and α-level set of fuzzy input data X_{ij}. $a^L_{j(\alpha)}x^R_{ij(\alpha)}$, $a^R_{j(\alpha)}x^L_{ij(\alpha)}$ or $a^R_{j(\alpha)}x^R_{ij(\alpha)}$ is defined in the same manner respectively.

To define the dissimilarity between a predicted value and an observed value of the dependent variable, we adopt the following indicator $D_i(\alpha)^2$.

$$D_i\ (\alpha)^2 = (y^L_{i(\alpha)} - z^L_{i(\alpha)})^2 + (y^R_{i(\alpha)} - z^R_{i(\alpha)})^2 \quad \ldots.. (11).$$

Definition by equation (11) can be applied to interval data as well as L-R fuzzy number. That is, the equation (11) represents a sum of squares for distance between interval data.

To generalize, a dissimilarity indicator representing a square of distance for L-R fuzzy numbers can be written as follows:

$$D_i^2 = \sum_{j=0}^{k} w_j((y^L_{i(\alpha j)} - z^L_{i(\alpha j)})^2 + (y^R_{i(\alpha j)} - z^R_{i(\alpha j)})^2) \quad (12).$$

where $\alpha_j = jh/k,\ j=0,..,k,\ h$ is an equal interval, and w_j is a weight for j-th level.

In the case of triangular fuzzy number with $w_j = 1$, the above equation is approximately represented as:

$$Di^2 = (y^L_{i(0)} - z^L_{i(0)})^2 + (y^L_{i(1)} - z^R_{i(1)})^2 + (y^R_{i(0)} - z^R_{i(0)})^2 \quad \ldots .(13).$$

The proposed method is to estimate fuzzy coefficients using minimization of sum of D_i^2 respecting i. That is,

Objective function: $$Min \sum_{i=1}^{n} D_i^{\ 2} \qquad (14)$$

Subject to: $a^L_{j(h)} \geq 0,\ j \in J_1$ (15)

$$a^L_{j(h)} \leq 0,\ a^R_{j(h)} \geq 0,\ j \in J_2 \quad (16)$$

$$a^R_{j(h)} \leq 0,\ j \in J_3 \quad (17)$$

$$-a^L_{j(h)} + a^R_{j(h)} \geq 0 \quad (18)$$

where

$$j \in \{0,\cdots,m\} = J_1 \cup J_2 \cup J_3, \quad J_1 \cap J_2 = \phi,\quad J_2 \cap J_3 = \phi,\quad J_3 \cap J_1 = \phi, \quad (19)$$

$$z^L_{i(\alpha)} = \sum_{j \in J_1} a^L_{j(\alpha)} x^L_{ij(\alpha)} + \sum_{j \in J_2 J_3} a^L_{j(\alpha)} x^R_{ij(\alpha)}$$

$$z^R_{i(\alpha)} = \sum_{j \in J_1, J_{12}} a^R_{j(\alpha)} x^R_{ij(\alpha)} + \sum_{j \in J_3} a^R_{j(\alpha)} x^L_{ij(\alpha)} \quad (20).$$

The estimated coefficients can be derived through the quadratic programming method. The proposed fuzzy least square method is also shown in Figure 2.

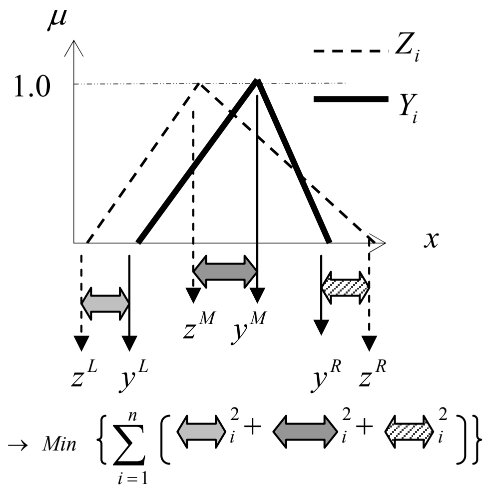

Figure 2. Fuzzy least square regression analysis for input and output data

B. Analysis of the Logistic Type Model

A set of fuzzy input-output data for i-th observation is defined by:

$$\left(P_{ijk}; X_{ijk\,0}, X_{ij\,1}, X_{i\,j2}, \ldots, X_{ij\,m;} X_{ik\,1}, X_{i\,k2}, \ldots, X_{ik\,m;}\right) \quad (21).$$

where P_{ijk} indicates ambiguous preference represented by fuzzy L-R number, and X_{ijk} is X_{jk} for observation i.

Let $X_{ijk\cdot}$ *be* $X_{ij\cdot}$ — X_{ik} (where − is difference operator based on the fuzzy extension principle), and denote $X_{i.}$ for abbreviation of $X_{ijk\cdot}$ in the following section. Therefore, a set of fuzzy input-output data for i-th observation is re-written by:

$$\left(P_i; X_{i\,0}, X_{i1}, X_{i\,2}, \ldots, X_{i\,m}\right),\quad i = 1,2,\cdots,n \quad (22).$$

where P_i is a fuzzy dependent variable, and X_{im} is a fuzzy independent variable represented by L-R fuzzy numbers. For simplicity, we assume that P_i and X_{im} are positive for any membership value, $\alpha \in (0,1)$.

The fuzzy logistic regression model (where both input and output data are fuzzy numbers) is represented as follows:

$$\overline{\log(P_i \Theta (1-P_i))} = A_0 X_{i0} + A_1 \otimes X_{i1} + \ldots + A_m \otimes X_{im} \quad (23).$$

where $\overline{\log(P_i \Theta (1-P_i))}$ is estimated fuzzy log odds, Θ is the division operator based on the extension principle, $X_{i\,0} = 1$, $A_j\ (j = 0,1,\cdots,m)$ is fuzzy regression parameter represented by L-R fuzzy number, and $\otimes$ is the product operator based on the extension principle.

It should be noted that although the explicit form of the membership function of $\overline{\log(P_i \Theta (1-P_i))}$ can not be directly obtained, the α-level set of $\overline{\log(P_i \Theta (1-P_i))}$ can be obtained from the result of Nguyen's theorem [9].

Let $P^L_{i(\alpha)}$ be the lower bound of dependent fuzzy variable, $P^R_{i(\alpha)}$ be the upper bound of the fuzzy dependent variable. Then, α level set of the fuzzy dependent variable P_i can be represented as $P_{i_\alpha} = \left[P^L_{i(\alpha)}, P^R_{i(\alpha)}\right]$ $,\alpha \in (0,1]$.

Therefore, the α level set of the left term in (23) is as follow:

$$[\,\overline{\log(P_i \Theta (1-P_i))}\,]_\alpha$$

$$= [\min(\overline{\log(P^L_{i(\alpha)}/(1-P^L_{i(\alpha)})}, \overline{\log(P^R_{i(\alpha)}/(1-P^R_{i(\alpha)})}),$$

$$\max(\overline{\log(P^L_{i(\alpha)}/(1-P^L_{i(\alpha)})}, \overline{\log(P^R_{i(\alpha)}/(1-P^R_{i(\alpha)})})], \quad (24).$$

Let $z^L_{i(\alpha)}$ be a lower value of $[\,\overline{\log(P_i \Theta (1-P_i))}\,]_\alpha$, and $z^R_{i(\alpha)}$ be a upper value of $[\,\overline{\log(P_i \Theta (1-P_i))}\,]_\alpha$.

Then,

$$Z_i = \left[z^L_{i(\alpha)}, z^R_{i(\alpha)}\right],\quad \alpha \in (0,1] \quad (25).$$

where

$$z^R_{i(\alpha)} = \sum_{j=0}^{m} \left\{\max\left(a^R_{j(\alpha)} x^L_{ij(\alpha)}, a^R_{j(\alpha)} x^R_{ij(\alpha)}\right)\right\}$$

$$z^L_{i(\alpha)} = \sum_{j=0}^{m} \left\{\min\left(a^L_{j(\alpha)} x^L_{ij(\alpha)}, a^L_{j(\alpha)} x^R_{ij(\alpha)}\right)\right\} \quad (26).$$

The parameter estimation method is basically as same as the fuzzy linear regression method and more concrete procedure is described in [6].

Ⅳ. NUMERICAL EXAMPLE OF THE DATA

ANALYSIS METHOD

A. An Example of the Additive Difference Model

Subject and Procedure A subject is a 43 years old adult. The subjects rated differences of WTP (willing to pay) for any two different computers (DELL) which have three types of attributes information (Hard disk: 100 GB and 60 GB; Memory: 2.80 GHz and 2.40 GHz; New Product or Used Product). The subject compared a certain alternative with seven different alternatives. The subject answered representative values, lower values, and upper values for WTP values (Price (Japanese yen) of the willing to pay for the difference) using fuzzy rating method. The subject also rated desirability of each attribute information (Hard disk: 100 GB and 60 GB; Memory: 2.80 GHz and 2.40 GHz; New Product or Used Product) using the fuzzy rating method. The fuzzy rating scale of desirability ranged from 0 point to 100 point .

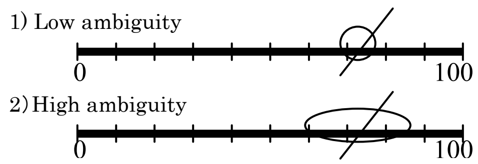

Figure 3. Example of fuzzy rating

Analysis and Results The fuzzy coefficients were obtained by the fuzzy linear regression analysis using the least squares under constraints, as shown in Table 1 and Table 2. The dependent variable of Table 1 was same as in Table 2. However, the independent variables are objective values measured by crisp numbers in Table 1, but, the independent variables were fuzzy rating values measured by a L-R fuzzy number in Table 2 . According to Table 1 and 2, the preference strength concerning comparative judgment was greater influenced by whether the target computer was new or used . The impact of attribute for hard disk was smaller than new-used dimension.

B. An Example of the Logistic Model

Subject and Procedure A subject is a 43 years old adult. The subjects rated ambiguous probability of preferring of a certain computer (DELL) with seven different computers.

Table 1. Results by Fuzzy Regression Analysis
(The independent variables are crisp numbers)

	Attribute	Value	
Fuzzy Coefficient	Hard Disk(L)	Lower	78.5
	Hard Disk (M)	Representative	85.7
	Hard Disk (R)	Upper	986.8
	Memory(L)	Lower	0.0
	Memory(M)	Representative	0.0
	Memory(R)	Upper	0.0
	New or Used(R)	Lower	22332.5
	New or Used (M)	Representative	22332.5
	New or Used(L)	Upper	22332.5
	A_{jk0} (L)	Lower	25450.8
	A_{jk0} (M)	Representative	29420.1
	A_{jk0} (R)	Upper	33111.2

Table 2. Results by Fuzzy Regression Analysis
(The independent variables were fuzzy L-R numbers)

	Attribute	Value	
Fuzzy Coefficient	Hard Disk(L)	Lower	33.9
	Hard Disk (M)	Representative	33.9
	Hard Disk (R)	Upper	33.9
	Memory(L)	Lower	0.0
	Memory(M)	Representative	0.0
	Memory(R)	Upper	0.0
	New or Used(R)	Lower	446.1
	New or Used(M)	Representative	446.1
	New or Used(L)	Upper	446.1
	A_{jk0} (L)	Lower	36082.1
	A_{jk0} (M)	Representative	36082.1
	A_{jk0} (R)	Upper	48004.0

Three types of attributes information (Hard disk: 100 GB and60 GB; Memory: 2.80 GHz and 2.40 GHz; New Product or Used Product) were manipulated as the same as in the previous judgment task. The subject answered of representative values, lower values, and upper values of their probabilities using fuzzy rating method.

Analysis and Results The fuzzy coefficients were obtained by the fuzzy linear regression analysis using the least squares under constraints, as shown in Table 3 and Table 4. However, the independent variables are objective values measured by crisp numbers in Table 3, but, the independent variables were fuzzy rating values measured by a L-R fuzzy number in Table 4 . According to Table 3 and 4, the bounded preference strength was greater influenced by whether the target computer was new or used . Interestingly, the impact of attribute for memory was slightly greater than in the finding in Table 1 and 2.

Table 3. Results by Fuzzy Logistic Regression Analysis
(The independent variables are crisp numbers)

	Attribute	Value	
Fuzzy Coefficient	Hard Disk (L)	Lower	0.000
	Hard Disk(M)	Representative	0.000
	Hard Disk (R)	Upper	0.009
	Memory(L)	Lower	1.781
	Memory(M)	Representative	1.781
	Memory(R)	Upper	1.881
	New or Used (R)	Lower	1.791
	New or Used (M)	Representative	2.097
	New or Used (L)	Upper	2.777
	A_{jk0} (L)	Lower	0.847
	A_{jk0} (M)	Representative	1.201
	A_{jk0} (R)	Upper	1.443

Table 4. Results by Fuzzy Logistic Regression Analysis
(The independent variables are fuzzy L-R numbers)

	Attribute	Value	
Fuzzy Coefficient	Hard Disk(L)	Lower	0.000
	Hard Disk (M)	Representative	0.000
	Hard Disk (R)	Upper	0.000
	Memory(L)	Lower	0.008
	Memory(M)	Representative	0.008
	Memory(R)	Upper	0.008
	New or Used (R)	Lower	0.043
	New or Used (M)	Representative	0.043
	New or Used (L)	Upper	0.043
	A_{jk0} (L)	Lower	1.806
	A_{jk0} (M)	Representative	1.806
	A_{jk0} (R)	Upper	1.806

V. Conclusion

In this study, fuzzy set models for ambiguous comparative judgments which do not always hold transitivity and comparability properties were proposed. The first type of the model is a fuzzy theoretic extension of the additive difference model for preference for explaining ambiguous preference strength. This model can be reduced to Fuzzy Utility Difference Model [7] if multi-attribute weighting parameters are assumed to be crisp numbers, and to Additive Difference Model [8] if multi-attribute weighting parameters and values of multi-attributes are assumed to be crisp numbers. The second type of the model is a fuzzy logistic model for explaining ambiguous preference in which preference strength is bounded such as probability measure. In both the models, multi-attribute weighting parameters and all attribute values are assumed to be asymmetric fuzzy L-R numbers. For each model, the parameter estimation method using the fuzzy regression analysis was proposed. Numerical examples for comparative judgments were also demonstrated. Since both the models require different evaluation methods, comparisons of psychological effects of the both methods will be needed in the future study.

In this study, the least square methods were used for data analyses of two models. However, the possibilistic linear regression analysis by Sakawa [10] and the possibilistic logistic regression analysis by Takemura [6] can be applied for the data analysis of the additive difference type model and the logistic type model, respectively. The proposed models and the analyses for ambiguous comparative judgments will be applied to marketing research, risk perception research, and human judgment and decision making research. The empirical research using the possibilistic analysis and the least square analysis will be needed to examine the validities of the models.

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