# The Explication of Similarity, Compromise, and Attraction Effects by the REGAL Model 

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#### Abstract

Adding a new option in a two-alternative choice situation sometimes changes relationships among the alternatives and choice will be altered accordingly. In this paper it is shown that the well-known similarity, compromise, and attraction effects in the trinary choice can be explained within the framework of the REGAL model [1]. The REGAL model is a probabilistic choice model whose decision is based upon satisfaction computation which involves repeated generation of evaluation feature sets and dominance structuring.


## I. INTRODUCTION

Introducing a third option in a two-alternative choice set often systematically restructures relationships between the options and changes choice probabilities. Among such empirical findings, similarity, attraction, and compromise effects have been drawn much attention and there are several models that try to explain them. In this paper, it is shown that the REGAL model [1] can give a principled explanation of these effects as well as a response time prediction.

## II. THREE CHOICE-SET EFFECTS

Let $\mathrm{P}[\mathrm{X} \leftarrow\{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}]$ be the probability that Option X is chosen from the set of alternatives: $\{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$. The following three choice-set effects are robust ones and have been examined in many studies. They are the target phenomena of the present paper.
Similarity effect Suppose that there is a binary choice set $\{\mathrm{A}$, $\mathrm{B}\}$ and $\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}\}]=\mathrm{P}[\mathrm{B} \leftarrow\{\mathrm{A}, \mathrm{B}\}]$. Then a new Option C that is similar to A is introduced to the original choice set (Fig.1). If Option C reduces the probability that similar Option A is chosen more than dissimilar Option B, that is,
$\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}]<\mathrm{P}[\mathrm{B} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}]$, we say similarity effect occurred $[2,3,4]$. Apparently this effect violates the property of independence from irrelevant alternatives.
Attraction effect (Decoy or asymmetric dominance effect)
When $\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}\}]=\mathrm{P}[\mathrm{B} \leftarrow\{\mathrm{A}, \mathrm{B}\}]$, addition of Option D that is dominated by A but not by B (Fig.3) increases the probability that the asymmetrically dominating Option A is chosen $[5]: \mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{D}\}]>\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}\}]$. This effect violates the regularity principle that the probability of choosing an option should not increase if the choice set is enlarged. An explanation of this effect is that dominance provides a strong reason for choosing the dominating option [6]
Compromise effect This refers to the effect that if we add Option E between two extreme Options A and B (Fig.4), people choose Option E more often than other options. More precisely, under the condition
$\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}\}]=\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{E}\}]=\mathrm{P}[\mathrm{B} \leftarrow\{\mathrm{B}, \mathrm{E}\}]=.5$, this effect implies
$\mathrm{P}[\mathrm{E} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{E}\}]>\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{E}\}]=\mathrm{P}[\mathrm{B} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{E}\}]$, which again violates independence from irrelevant alternatives. The
effect shows that people tend to avoid extreme options and make compromise choices. Simonson [6] and Simonson and Tversky [7] interpreted this effect on the basis of reduction of attribute conflict and ease of justification.

## III. MODELS THAT TRY TO EXPLAIN THE THREE CHOICE-SET EFFECTS

The EBA model $[2,3]$ can account for similarity effect but cannot account for attraction and compromise effects [4]. Componential context model [8] that is based on the principle of loss aversion can explain attraction and compromise effects but cannot similarity effect. (See [4] for a proof). Decision field theory can account for all three effects [4].

## A. The EBA and REGAL Models

The REGAL model was originally proposed as an extension of the EBA model. Thus the REGAL model is explained with reference to the EBA model. According to the EBA model an aspect is probabilistically selected and options that do not have the aspect are discarded. (In this paper, 'attribute', 'aspect', and 'feature' are used interchangeably). Then another aspect is selected and options that do not have the second aspect are discarded. Proceeding in this way, the EBA process terminates when only one option remains. Notice that the last option has always dominated the other options during the elimination process in the sense that it has always been a co-winner with respect to the aspect that was picked up at that time. The aspects that have not been selected in the EBA process play no direct role in the choice.


Fig. 1 Similarity effect

The EBA is insensible to the effects of common aspects which are totally ignored in the process and in the final choice.

## B. Elaboration and Extension of the EBA

It is well recognized that the EBA is sensible especially for simpler decision in the every day life and often leads to less than optimal decisions.

Not surprisingly, as an actual process model of decision making the EBA model shows some short-comings. First, in the EBA model, an aspect that is once considered and an alternative once discarded should never be reconsidered, whereas in actual decisions reconsideration of aspects and alternatives is undeniable. Second, the EBA model does not permit conjunctive aspects, rendering the model rather implausible: in the EBA model aspects should be picked up one at a time, whereas a decision maker often has a bundle of minimum requirements in mind that can be represented by a conjunction of aspects. Third, features that are common to alternatives are totally ignored whereas there is evidence that common features affect choice [9].
The following three stories will clarify the points:
Story 1 Adams was trying to choose either of
$X^{\prime}=\{$ Paris, good hotel $\}$
$Y^{\prime}=\{$ Siberia, good hotel, $\$ 5$ bonus $\}$.
Unfortunately, he picked up the aspect of $\$ 5$ bonus as the EBA model permits and went to Siberia with great reluctance.
The EBA model predicts that a person who prefers Paris may choose $\$ 5$ aspect with some non-zero probability and thus choose Siberia trip. Actually, however, people who happened to have selected $\$ 5$ aspect would never be so hasty and unwise as to finalize her/his choice based on this aspect. They will reconsider the options to avoid immoderate decisions.
Story 2 Bill was trying to choose either of
$X^{\prime}=\{$ Paris, good hotel, good cuisine $\}$
$Y^{\prime}=\{$ Siberia, bad hotel, awful cuisine $\}$.
He picked up the hotel aspect and chose Paris trip, because hotel was superior, totally ignoring the other aspects.
If hotel aspect is sampled, the EBA predicts $X$ should be chosen because $u$ (good hotel) $>u$ (bad hotel). In other words, X is chosen because hotel is superior. But actually this would not be the case, since he chooses $X$ with greatest confidence and satisfaction because PLACE and HOTEL and CUISINE are superior. Apparently it should be the conjunction of features that determine the satisfaction for choices.
Story 3 Charles was trying to choose either of
$X^{\prime}=\{$ Paris, good hotel, good cuisine, $\$ 100$ bonus, whoppering discount, gorgeous free souvenir\}
$Y^{\prime}=\{$ Siberia, good hotel, good cuisine, $\$ 100$ bonus,
whoppering discount, gorgeous free souvenir\}.
He wavered but finally chose $Y$ with great satisfaction.
In the story both alternatives are enriched by fine and lovely common features and the situation virtually involves an approach-approach conflict that is known to be quite easy to resolve without a feeling of regret [10] whereas in the EBA model common aspects should merely be ignored and thus the model cannot explain Charles' contentment.

The three stories seem to claim that if we want to elaborate the EBA model, we need to introduce a mechanism that incorporates A) reconsideration processes, B) more flexible aspect selection processes that permit conjunctions, and explains C) the effects of common features.

## IV. THE OUTLINE OF REGAL MODEL

The REGAL model is a generalization of the EBA model with ingredients of dominant structuring [11] and the race (counter) model [12]. The REGAL is a dynamic and probabilistic model that predicts both choice probabilities and decision times in closed-form expressions and its main objective was to model deliberation, vacillation, and wavering of decision makers. This model permits conjunction of attributes, flexible reconsideration of alternatives and attributes, and is able to explain common feature effects.

The outline of the model is as follows (see also Fig. 2 and Appendix A):
(1) Let $T$ be the set of alternatives and $\Omega$ be the set of attributes used to represent the alternatives. The set $x^{\prime} \subseteq \Omega$ is the feature representation of Alternative $x$.
(2) A decision maker probabilistically samples a set of attributes $\Psi$ from $\Omega$, called an evaluation set or aspect lineup.
(3) The degree of satisfaction for each alternative is evaluated on the current evaluation set. Specifically,
$S_{\Psi}(x)=$ Goodness of Alternative $\mathrm{x} \times$ Structural dominance
of x over the others on evaluation set $\Psi$
(4) A decisional criterion $\theta$ that fluctuates probabilistically is assumed. If $S_{\Psi}(x)-S_{\Psi}(y)>\theta$ for all $y \in T-\{x\}$ then Alternative x is in the state of decisional dominance. If no option is dominant the process goes back to (2).


Fig. 2 THE REGAL MODEL
(5) A unit increment is added to confidence counter of $\mathrm{x}: C(x)=C(x)+1$. The REGAL assumes that confidence for options accumulates in confidence counters with a choice being made by the first to reach the confidence criterion $L$. Therefore, If $C(x)=L$ then Option x is chosen and the process is over. Otherwise the process resume from (2).
REGAL is the abbreviation for "REpeated Generation of Aspects Lineup", where aspects lineup stands for the evaluation set (conjunction of attributes) generated in (2) and repeated generation refers to the loop between (2) and (4) or (2) and (5) which represents people's reconsideration processes. Specifically, the former loop represents people's vacillation and hesitation and the latter loop deliberation in which confidence accrual for options occurs. Detailed presentation of the model is given in Appendix A.

## V. EXPLICATION OF THREE CHOICE-SET EFFECTS

## A. Choice Probabilities

As shown in Appendix $\mathrm{A}, \mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}]$ is given by
$\sum_{l=0}^{v}{ }_{L+l-1} C_{L-1}\left(\frac{M_{A}}{1-R}\right)^{L}\left(\sum_{\substack{l=l_{B}+l_{C}, l_{B} L, l_{C}<L}} \frac{l!}{l_{B}!l_{C}!}\left(\frac{M_{B}}{1-R}\right)^{l_{B}}\left(\frac{M_{C}}{1-R}\right)^{l_{C}}\right)$
$=P(A \leftarrow\{A, B, C\})$
where
$L$ : Confidence criterion
$v=(L-1) \times($ Number of options -1$)$
$R$ : The probability that no options is dominant
$l_{B}<L, l_{C}<L$ : The times Option B or C became dominant in the process.
$M_{A}$ : The probability that Option A becomes dominant, etc,
$M_{A}=\sum_{\Psi \in 2^{2}} P\left(S_{\Psi}(A)-S_{\Psi}(B)>\theta, S_{\Psi}(A)-S_{\Psi}(C)>\theta \mid \Psi\right) P(\Psi)$
$\theta \sim N\left(\mu, \sigma^{2}\right):$ Decisional criterion that fluctuates
$\Psi$ : Evaluation feature set
$P(\Psi)=P\left(\Psi \leftarrow 2^{\Omega}\right)=\prod_{\tau \in \Psi} P(\tau) \prod_{\tau \notin \Psi}(1-P(\tau))$
$P(\tau)$ : The probability that feature $\tau$ is included in $\Psi$.
The free parameters of the model are $L, \mu, \sigma^{2}$ and $P(\tau), \tau \in \Omega$. By adjusting them, we can examine whether REGAL can mimic the three choice-set effects.

In the sequel $\mathrm{P}[\mathrm{A} \leftarrow\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}]$ 's are written as $\mathrm{P}[\mathrm{A}]$ 's unless otherwise specified. Define $\mathbf{c}=(\mathrm{x}, \mathrm{y})$ be the position of Option C in the attribute space. Then the three choice-set effects can be framed as empirical observation on $\mathrm{P}[\mathrm{A}], \mathrm{P}[\mathrm{B}]$, and $P[C]$ given $\mathbf{c}$ under the condition that Options $A$ and $B$ are fixed and placed as in the Figures. Accordingly introduction of functions $\mathrm{P}[\mathrm{A} \mid \mathbf{c}], \mathrm{P}[\mathrm{B} \mid \mathbf{c}]$, and $\mathrm{P}[\mathrm{C} \mid \mathbf{c}]$ is convenient to examine the behavior of the choice probabilities given $\mathbf{c}$. In order to compute choice probabilities, REGAL needs to determine $L, \mu, \sigma^{2}$ and $P(\tau), \tau \in \Omega$. Specifically, principled determination of sampling probability $P(\tau)$ is essential.

## B. Determination of $\Omega$

Observe that the options in Fig. 1 are represented as $A^{\prime}=\left\{\alpha_{1}, \alpha_{2}, \gamma, \delta\right\}, B^{\prime}=\left\{\alpha_{1}, \alpha_{2}, \beta, \delta\right\}, C^{\prime}=\left\{\alpha_{1}, \gamma, \delta, \varepsilon\right\}$ and we have
$\Omega=\left\{\alpha_{1}, \alpha_{2}, \beta, \gamma, \delta, \varepsilon\right\}$. First of all, following several influential studies $[8,13]$, continuous attributes are divided into several parts in this representation. Further, notice that how to divide continuous attributes into parts depends upon the position of Option C in the space. For instance, $A^{\prime}=\left\{\alpha_{1}, \alpha_{2}, \gamma, \delta\right\}$ in Fig. 1 and $A^{\prime}=\left\{\alpha_{1}, \alpha_{2}, \gamma, \delta_{1}, \delta_{2}\right\}$ in Fig.3. Representational difference of this type is a source of choice-set effect.

## C. Sampling Probabilities

If attribute set $\Omega$ is established in this way, our next job is to set up sampling probabilities $P(\tau), \tau \in \Omega$. The determination is in theory empirical one. But we can use two conditions to arrange the probabilities that are to be used in the Numerical Demonstration section.

1) The more a feature has attribute value, the larger the probability it is sampled from $\Omega$. As an example, in Fig.1, it should be that $P(\gamma)>P\left(\alpha_{1}\right)$ because $\gamma$ is larger than $\left.\alpha_{1} .2\right)$ The more a feature is shared by options, the more the probability it is attended to. In Fig. $1 \alpha_{1}$ is included in all options whereas $\varepsilon$ is included in only one option, and thus it should be $P\left(\alpha_{1}\right)>P(\varepsilon)$. Whereas the first condition seems no problematic, the second one might be arguable because in some decision models it is postulated that common features should be ignored $[2,10]$. Theoretically the idea that common features are processed but do not change choice probability has some ground (Appendix B). Empirically there is evidence that common features do affect choices [9].

Considering those two conditions, as a rule of thumb, .90 probability per half unit length of each attributes was assigned as the sampling probability of that attribute. At the same time the probability of .30 per option was assigned to the sampling probability of attribute shared by the options. Then aggregated probability $P(\tau)$ is defined as the minimum of the two probabilities derived from options and attributes.

## D. Numerical Demonstration

Because attributes in Figs.1,3, and 4 have clear numerical values, a modified version of satisfaction function was used:
$S_{\Psi}(x)=\frac{\sum_{\tau \in Y^{\prime} \cap \Psi} u(\tau)}{\sum_{\tau \in \Omega} u(\tau)} \frac{\sum_{y \in T} \sum_{\tau \in x^{n} \cap \Psi-y^{\prime} \cap \Psi} u(\tau)}{(\operatorname{Card}(T)-1) \sum_{\tau \in \Psi} u(\tau)}$
where $u(\cdot)$ is a utility function whose value is corresponding attribute length. The other parameters are set at $\mathrm{L}=20, \mu=.10$, and $\sigma=.20$. As shown in Figs.1, 3, and 4 different feature representations of options are used for each quadrant where the position of Option A is serving as the origin. Option C might be labeled Option D if $\mathbf{c}$ is in the third quadrant and Option E if in the fourth quadrant.
Then Using (1), $\mathrm{P}[\mathrm{A} \mid \mathbf{c}], \mathrm{P}[\mathrm{B} \mid \mathbf{c}]$, and $\mathrm{P}[\mathrm{C} \mid \mathbf{c}]$ were computed changing the position of $\mathbf{c}$ on a grid of the attribute space and Fig. 5 depicts where the three effects occurred. The three choice-set effects are defined as follows:
Similarity effect 1: $\mathrm{P}[\mathrm{B} \mid \mathbf{c}]>.45, \mathrm{P}[\mathrm{A} \mid \mathbf{c}]<.3, \mathrm{P}[\mathrm{C} \mid \mathbf{c}]<.3$
Similarity effect 2: $\mathrm{P}[\mathrm{A} \mid \mathbf{c}]>.45, \mathrm{P}[\mathrm{B} \mid \mathbf{c}]<3, \mathrm{P}[\mathrm{C} \mid \mathbf{c}]<.3$
Compromise effect: $|\mathrm{P}[\mathrm{A} \mid \mathbf{c}]-\mathrm{P}[\mathrm{B} \mid \mathbf{c}]|<.1$,
$\mathrm{P}[\mathrm{C} \mid \mathbf{c}]>\operatorname{Max}(\mathrm{P}[\mathrm{A} \mid \mathbf{c}], \mathrm{P}[\mathrm{B} \mid \mathbf{c}])$, and $\mathrm{P}[\mathrm{C} \mid \mathbf{c}]<.5$
Attraction effects: $\mathrm{P}[\mathrm{A} \mid \mathbf{c}], \mathrm{P}[\mathrm{B} \mid \mathbf{c}] \mid, \mathrm{P}[\mathrm{C} \mid \mathbf{c}]>.5$
Notice that reading of Fig. 5 is somewhat confusing in that it is based upon the probabilities as a function of $\mathbf{c}$, that is, the
position of Option C on the attribute space.
The three effects as defined as above are often incompatible. As an instance, observe that in Fig. 3 Option D is located lower left of Option A. Attraction effect will increase P[A] while similarity effect will do the opposite. On the other hand the effects often co-occur as in the center and bottom right of Fig. 5 Composition of those "forces" would be the source of the complex pattern in Fig. 5 The position of similarity effects 1 and 2 are quite comprehensible, whereas compromise effect at upper left is somewhat puzzling possibly showing complex interaction between options. Overall the model seems to capture the essential portion of the choice-set effects.

## VI. DISCUSSION

Three distinctive components of REGAL are as follows. First, the very idea of evaluation set formation is the key to model people's flexible and even inconsistent evaluation process. It tries to represent the fact that our criteria for evaluating options are ever changing. When it is very hard to find a dominant alternative, there tends to be a conflict among alternatives. This is particularly so when there are competitive and similar alternatives. The REGAL model explains this


Fig. 3 Attraction Effect


Fig. 4 Compromise Effect
effect in terms of the difficulty in searching for good evaluation sets that produce a dominance structure.
Second, computation of satisfaction function which is based upon evaluation set can capture both richness of alternatives supported by common features and dominance relationship between options. Mere existence of a dominant alternative does not necessarily mean that the decision maker can happily make a quick decision. Specifically when all the alternatives are poor, the decision maker will hardly be satisfied, because in such cases only low $S_{\Psi}(x)$ is obtained which degrades decision quality. A decision that ignores important features, common or distinctive, cannot attain high satisfaction. At the same time, low degrees of satisfaction compel us to resample attributes in search for a good evaluation set in the hope that an excellent structure might emerge.

Unfortunately not used in this paper, response time predictions derived from confidence counters will enable us to evaluate people's reconsideration processes for maintaining decision quality because confidence criterion $L$ is strongly related to the importance of the decision.


Fig. 5 Occurrence of Three Choice-Set Effects as a Function of Option C's Location

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## Appendix A Mathematical formulation of REGAL

We use the following symbols and notations.
$\mathrm{T}=\{\mathrm{x}, \mathrm{y}, \mathrm{z}, \ldots\}$ : The finite total set of choice alternatives.
$\Omega=\{\alpha, \beta, \gamma, \rho, \ldots\}$ : The finite total set of features (aspects).
$x^{\prime}, y^{\prime}, z^{\prime}, \ldots \subseteq \Omega$ : Subsets of $\Omega$ representing $\mathrm{x}, \mathrm{y}, \mathrm{z}, \ldots$.
For example, if Alternative $\mathbf{x}$ has features $\alpha, \beta$ and $\gamma$ we write $\mathrm{x}^{\prime}=\{\alpha, \beta, \gamma\} . \quad$ Apparently $\Omega=\underset{x \in T}{\cup} \mathrm{x}^{\prime}$.

## Evaluation set

Consider the power set of $\Omega: 2^{\Omega}$. Any set $\Psi \in 2^{\Omega}$ is called an evaluation set, because evaluation of options will be performed exclusively on this set ignoring features not included in $\Psi$. In other word, $\Psi$ defines a set of features that is attended to by the decision maker and is used for evaluating alternatives during one loop of the REGAL process. REGAL assumes that the probability that $\Psi$ is chosen from $2^{\Omega}$ is given by

$$
P(\Psi)=P\left(\Psi \leftarrow 2^{\Omega}\right)=\prod_{\tau \in \Psi} P(\tau) \prod_{\tau \notin \Psi}(1-P(\tau))
$$

where $P(\tau)$ is the probability that feature $\tau$ is attended to and sampled from the feature set $\Omega$. This expression is essentially a product of independent Bernoulli distributions.
Satisfaction function The satisfaction function $S_{\Psi}(x)$ is defined as

$$
S_{\Psi}(x)=\underbrace{\frac{\operatorname{Card}\left(x^{\prime} \cap \Psi\right)}{\operatorname{Card}(\Omega)}}_{A} \underbrace{\frac{\sum_{y \in T} \operatorname{Card}\left(x^{\prime} \cap \Psi-y^{\prime} \cap \Psi\right)}{(\operatorname{Card}(T)-1) \operatorname{Card}(\Psi)}}_{B}
$$

where $\Psi$ is the evaluation set based on which alternatives are
evaluated and $\operatorname{Card}(\cdot)$ is the cardinality (the number of elements) of a set. Each part of the function has the following meaning: Part A represents the overall degree that Alternative $x$ has the features in $\Psi$ and in turn $\Psi$ has the features in $\Omega$. This part says when both the evaluation set and the option have many features and thus are rich then the function tends to output larger $S$. Part B represents the structural dominance of $x$ over the other alternatives. Part B can take other functional forms according to the nature of features and definition of dominance.

The value $S$ has the range $[0,1]$ and $S=1$ is obtained when $\Psi$ includes all the features in $\Omega, x$ has all the features in $\Psi$, and $y$ has no features in $\Psi$.

## Decision criterion

After the computation of $S_{\Psi}(x), x \in T$, the differences of those values are compared with decision criterion $\theta$. This criterion distributes probabilistically and if $S_{\Psi}(x)-S_{\Psi}(y)>\theta$, $y \in T-\{x\}$ holds, Option x is said to be in the state of decisional dominance. No two options can become decisional dominant simultaneously and there is even a possibility that no option becomes decisional dominant. In the latter case the process resumes from the construction of $\Psi$. Distribution of decision criterion $\theta$ represents accuracy and reliability of simple judgment on a unidimensional scale of $S$, which follows a Thurstonian tradition of psychometrics that internal judgment is fallible [14].

## Confidence counter and confidence criterion

For each Alternative x , there is a confidence counter $C(x)$. The confidence for options accrues in confidence counters during REGAL process and the first option to attain a confidence criterion will be the final choice. If Option $x$ become decisional dominant, then a unit increment is added to $C(x)$. If $C(x)$ reaches confidence criterion $L$, then x is chosen else the process resumes from the construction of $\Psi$.

The confidence criterion represents decision quality: If the decision is a life-and-death matter it will be a high value, whereas if the decision is not so important it tends to be a small value. It is plausible to assume that we feel strong reluctance to the choice when forced to use low confidence criterion.

The flow of the REGAL model can be described as:
Step 1: Construct the evaluation set $\Psi$.
Step 2: Compute the degree of satisfaction $S_{\Psi}(x), x \in T$.
Step 3: If $S_{\Psi}(x)-S_{\Psi}(y)>\theta, y \in T-\{x\}$ then $C(x)=C(x)$ +1 else go to Step 1 .
Step 4: If one confidence counter reaches $L$ then choose the alternative. Else go to Step 1

In this model, there is no guarantee that we could reach a decision. Indeed, especially when $\mu$ is high, there is a possibility that we fall into an infinite loop between Steps 1 and 3, which represents our indecision and vacillation. In order to escape from the infinite loop, $\mu$ might be changed or total restructuring of feature representation might be needed.

## Derivation of Choice probability

Let the probability that, in Step 3, $x$ dominates other alternatives in $T$ be $M_{x}$ :
$M_{x}=\sum_{\Psi \in 2^{\Omega}} P\left(S_{\Psi}(x)-S_{\Psi}(y)>\theta, y \in T-\{x\} \mid \Psi\right) P(\Psi)$
then the probability that no alternatives is dominant is given by:
$R=1-\sum_{x \in T} M_{x}$
Let $N$ be the total number of confidence accumulations (reconsiderations) needed to reach the final decision. This value is a random variable and a rough indicator of decision times. Let $v=(L-1) \times(\operatorname{Card}(T)-1)$. Then we have when $r=0,1, \ldots, v$
$P(N=L, x \leftarrow T)=M_{x}^{L}$
$P(N=L+1, x \leftarrow T)=\sum_{l=0}^{1} \frac{L!}{(L-1)!(1-l)!!!} M_{x}^{L-1} R^{1-l} \Theta(l) \times M_{x}$
$P(N=L+2, x \leftarrow T)=\sum_{l=0}^{2} \frac{(L+1)!}{(L-1)!(2-l)!l!} M_{x}^{L-1} R^{2-l} \Theta(l) \times M_{x}$ $P(N=L+r, x \leftarrow T)=\sum_{l=0}^{r} \frac{(L+r-1)!}{(L-1)!(r-l)!!!} M_{x}^{L-1} R^{r-l} \Theta(l) \times M_{x}$ and when $r=v+1, v+2, \ldots$
$P(N=L+r, x \leftarrow T)$
$=\sum_{l=0}^{v} \frac{(L+r-1)!}{(L-1)!(r-l)!l!} M_{x}^{L-1} R^{r-l} \Theta(l) \times M_{x}$
where

$$
\Theta(l)=\sum_{l=\sum_{t \neq x} l_{t}, l_{t}<L} \frac{l!}{\prod_{t \neq x} l_{t}!} \prod_{t \neq x} M_{t}^{l_{t}}
$$

is a multinomial distribution that does not involve $x$. Because
$P(x \leftarrow T)=\sum_{r=0}^{\nu} P(N=L+r, x \leftarrow T)+\sum_{r=v+1}^{\infty} P(N=L+r, x \leftarrow T)$
$=\frac{M_{x}^{L}}{(L-1)!} \sum_{r=0}^{v} \sum_{l=0}^{r} \frac{(L+r-1)!}{(r-l)!l!} R^{r-l} \Theta(l)$
$+\frac{M_{x}^{L}}{(L-1)!} \sum_{r=v+1}^{\infty} \sum_{l=0}^{v} \frac{(L+r-1)!}{(r-l)!l!} R^{r-l} \Theta(l)$
$=\frac{M_{x}^{L}}{(L-1)!}\left[\sum_{l=0}^{v} \sum_{r=l}^{\infty} \frac{(L+r-1)!}{(r-l)!l!} R^{r-l} \Theta(l)\right]$
setting $\lambda=r-l$ we have
$=\frac{M_{x}^{L}}{(L-1)!}\left[\sum_{l=0}^{\nu} \frac{\Theta(l)}{l!} \sum_{\lambda=0}^{\infty} \frac{(L+l-1+\lambda)!}{\lambda!} R^{\lambda}\right]$
In view of a property of hyper geometric function
$\sum_{\lambda=0}^{\infty} \frac{(L+l-1+\lambda)!}{(L+l-1)!\lambda!} R^{\lambda}=(1-R)^{-(L+l)}$
we proceed

$$
\begin{aligned}
& =\left(\frac{M_{x}}{1-R}\right)^{L}\left[\sum_{l=0}^{v} \Theta(l)\left(\frac{1}{1-R}\right)^{l} \frac{(L+l-1)!}{(L-1)!l!}\right] \\
& =\sum_{l=0}^{v}{ }_{L+l-1} C_{L-1}\left(\frac{M_{x}}{1-R}\right)^{L}\left(\frac{1}{1-R}\right)^{l} \Theta(l)
\end{aligned}
$$

$$
\begin{aligned}
& =\sum_{l=0}^{v}{ }_{L+l-1} C_{L-1}\left(\frac{M_{x}}{1-R}\right)^{L}\left(\frac{1}{1-R}\right)^{l} \sum_{\substack{l \sum_{l \neq x} l_{t, l<L}}} \frac{l!}{\prod_{l \neq x} l_{t} \prod_{l \neq x} M_{t}^{l_{t}}} \\
& =\sum_{l=0}^{v}{ }_{L+l-1} C_{L-1}\left(\frac{M_{x}}{1-R}\right)^{L} \sum_{\substack{l=\sum_{l \neq x} l_{t, l}<L}} \frac{l!}{\prod_{l \neq x} l_{t}!} \prod_{l \neq x}\left(\frac{M_{t}}{1-R}\right)^{l}
\end{aligned}
$$

## Response time prediction

The expectation of $N$ is given by

which can be used to check people's hesitation and vacillation.

## Appendix B An extension of the EBA model that permits common feature selection (Common aspect EBA)

It is often postulated that common features are cancelled out during comparison process. This requirement appears to be too tight. What if the common features are not ignored?
Appendix B presents a modification of the EBA in which common features are processed but does not change choice probabilities. In the choice $x \leftarrow\{x, y, z\}$, if $\mathrm{x}^{\prime}=\{\alpha, \rho, \theta, \omega\}$, $\mathrm{y}^{\prime}=\{\beta, \rho, \sigma, \omega\}$, and $\mathrm{z}^{\prime}=\{\gamma, \sigma, \theta, \omega\}$ with the understanding that $\alpha=u(\alpha)$ etc. where $u(\cdot)$ is a utility function, we have $P(x \leftarrow\{x, y, z\})$
$=\frac{\alpha}{K}+\frac{\rho}{K} P(x \leftarrow\{x, y\})+\frac{\theta}{K} P(x \leftarrow\{x, z\})+\frac{\omega}{K} P(x \leftarrow\{x, y, z\})$
$=\frac{\alpha}{K}+\frac{\rho}{K}\left\{\frac{\alpha+\theta}{\alpha+\theta+\beta+\sigma}\right\}+\frac{\theta}{K}\left\{\frac{\alpha+\rho}{\alpha+\rho+\gamma+\sigma}\right\}+\frac{\omega}{K} P(x \leftarrow\{x, y, z\})$
where $K=\alpha+\beta+\gamma+\rho+\sigma+\theta+\omega$. Notice that this expression is identical to the EBA if $\omega$ is ignored and set to 0 . Setting
$L=\alpha+\rho\left\{\frac{\alpha+\theta}{\alpha+\theta+\beta+\sigma}\right\}+\theta\left\{\frac{\alpha+\rho}{\alpha+\rho+\gamma+\sigma}\right\}$
we can derive that

$$
\begin{aligned}
& P(x \leftarrow\{x, y, z\})=\frac{L}{K}+\frac{w}{K} P(x \leftarrow\{x, y, z\}) \\
& =\frac{L}{K}+\frac{w}{K}\left[\frac{L}{K}+\frac{w}{K} P(x \leftarrow\{x, y, z\})\right] \\
& \vdots \\
& =\frac{L}{K}+\frac{L}{K} \frac{w}{K}+\frac{L}{K}\left(\frac{w}{K}\right)^{2}+\ldots+\left(\frac{w}{K}\right)^{n} P(x \leftarrow\{x, y, z\})
\end{aligned}
$$

Therefore when $n \rightarrow \infty$,
$P(x \leftarrow\{x, y, z\})=\frac{L}{K} \frac{1}{1-\omega / k}=\frac{L}{K-\omega}$
$=\frac{\alpha+\rho\left\{\frac{\alpha+\theta}{\alpha+\theta+\beta+\sigma}\right\}+\theta\left\{\frac{\alpha+\rho}{\alpha+\rho+\gamma+\sigma}\right\}}{\alpha+\beta+\gamma+\rho+\sigma+\theta}$
The last expression is identical with the EBA. The generalization to $n$-option situation is straightforward, which shows how the idea that common features are processed but indifferent to the final choice can be justified.

