An Inverse Fuzzy State Space Algorithm for Optimization of Parameters in a Furnace System

[+]R. Ismail, [*]T. Ahmad, [*]S. Ahmad, [^]R.S. Ahmad
[+] Faculty of Information Technology and Quantitatives Sciences, University of Technology MARA, 40450 Shah Alam, Selangor, MALAYSIA.
[*] Dept. of Mathematics, Faculty of Science, University of Technology Malaysia, 81310 Skudai, Johor, MALAYSIA
^ Dept. of Physics, Faculty of Science, University of Technology Malaysia, 81310 Skudai, Johor, MALAYSIA
email: razidah@pd.jaring.my, tahir_22@hotmail.com

***Abstract* - The need for new approaches and philosophies in modeling and control of complex industrial systems are much influenced by the recent advances in information technology, increased market competition, the demand for low cost operation and energy efficiency. In fact, many real-world systems such as power generation plants, are characterized as multivariable, nonlinear and dynamic systems. For such a large complex system, it is useful to decompose the system into subsystems that can be analyzed and understood separately. The objective of this paper is to present the development of an inverse Fuzzy State Space algorithm for the optimization of the input parameters, which is applied to a furnace system of a combined cycle power plant. This algorithm is derived through the formulation of a new approach for solving inverse problems in multivariable dynamic systems. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, which requires excessive computer time and thus can be very costly. In the algorithm presented, it is assumed that the system could be expressed in the state space representation. To take into account of the uncertainties in the model, the uncertain value parameters of the system to be controlled are represented by fuzzy numbers with their membership function derived from expert knowledge. The optimal combination of the input parameters was extracted through defuzzification using an important theorem, Modified Optimized Defuzzified Value Theorem. The proof of this theorem is also presented. The results obtained in this application demonstrate that the proposed approach is reasonable and effective.**

Keywords: Fuzzy State Space Model, fuzzy modeling, multivariable systems, furnace system, Inverse problems

I. INTRODUCTION

Many of the real-world problems arising in the analysis and design of decision and control systems are far from simple, specifically for handling parameter decision problems in multivariable dynamic systems. Furthermore, a great deal of information for many real-world systems is provided by human experts, who describe the system verbally through vague, uncertain or imprecise statements. The fact that humans are often able to manage complex tasks under significant uncertainty has stimulated the search for alternative modeling and control paradigms.

The most relevant information about any system comes in one of three ways, that is, a mathematical model, sensory input and output data or measurement, and human expert knowledge. The common factor in all these three sources is knowledge. For many years, classical control designers began their effort with a mathematical model and did not go any further in acquiring more knowledge about the system. Today, control engineers can use all of the above sources of information. Apart from a mathematical model whose utilization is clear, numerical input-output data can be used to develop an approximate model as well as a controller, based on the best available knowledge to treat uncertainties in the system. A typical example of techniques that make use of human knowledge and deductive processes is fuzzy modeling. Furthermore, fuzzy sets [1] also provide a tool for handling ill-conditioned or ill-posed problems, which exist as a result of combining measurements with engineering models. The inverse problem [2], or more precisely the inverse modeling, is one type of ill-conditioned or ill-posed problems. In inverse modeling, the desired responses are given and a model is used to estimate the input parameters. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, for example [3], [4]. However, this method requires excessive computer time and thus can be very costly. Today, the techniques used for solving inverse problems are as multivariate as the problems themselves. Thus, the interaction between the analysis of the inverse mathematical problem and the measurement of the real system used to solve the problem is very useful for constructing a good model.

The objective of this paper is to present the development of an inverse Fuzzy State Space algorithm for the optimization of the input parameters, which is illustrated by implementing to a furnace system of a combined cycle power plant. This algorithm is derived through the formulation of Fuzzy State Space Model (FSSM), a new modeling approach for solving inverse problems in multivariable dynamic systems [5]. The construction of this model involved the integration of four different kinds of models, namely mental model, verbal model, mathematical model and state space model. Triangular fuzzy numbers [6] are used to represent imprecise or uncertain parameters in the model, with their membership function derived from expert knowledge.

The paper is organized as follows. After this introduction, section II describes the formulation of the inverse Fuzzy State Space algorithm specifically for multiple-input single-output (MISO) system. The Modified Optimized Defuzzified Value Theorem and its corollary, which forms an important part of this algorithm, are derived and proven in section III. The validity of this algorithm is shown by implementing it to the state space model of a furnace system, which is presented in section IV. Finally, section V draws some conclusions from the presented work.

II. FUZZY STATE SPACE ALGORITHM

In formulating the inverse Fuzzy State Space algorithm, the approach introduced in [7] is modified by considering the state space representation of the system. In his work, he had developed a fuzzy algorithm for optimization of geometrical and electrical parameters of microstrip lines using algebraic equations.

Given an input g_i that takes values in set I_i , and let preferences for different values of g_i be expressed by a fuzzy set F_{Ii} on I_i . For each $x \in I_i$, the value F_{Ii} (x) designates the degree of desirability of using the particular value x within the given set of values I_i. Thus, set F_{Ii} is referred as the set of desirable values of parameter I_i, and F_{Ii} (x) is viewed as the grade of membership of value x in this set. Index i is used here to distinguish different input parameters. The fuzzy sets expressing preference for all input parameters are employed for calculating the associated fuzzy sets for performance parameters. The target values of performance parameters are specified by functional requirements. Performance parameters, resulting from calculations with uncertain or vague input parameters, will also be represented by fuzzy preference functions. Similarly, each of the output parameters is represented by a range and a preference function.

It is assumed that all the fuzzy sets F_{Ii} expressing preferences of all input parameters $g_i \in I_i \subset R^+ (i \in N)$ are determined, normalised and convex. I is a close interval of positive real numbers. S_g is a performance parameter based on the FSSM whereby all input parameters are considered as its variables and can be presented within a fuzzy set F_{Sg} The algorithm to determine a fuzzy set F_{Sg} that is induced on the output parameters by fuzzy sets F_{Ii} through S_g has the following steps:

Step 1: Let $S_g : R^n \rightarrow R$. S_g is the performance parameter such that $r = S_g(g_1, g_2, g_3,\ldots, g_n)$.

Step 2: Select appropriate values for α-cut such that $\alpha_1, \alpha_2, \alpha_3,\ldots, \alpha_k \in (0,1]$ which are equally spaced.

Step 3: To fuzzify the input, determine all the α_k-cuts for all F_{Ip} ($p \in N$).

Step 4: Generate all 2^n combinations of the endpoints of intervals representing α_k-cuts for all F_{Ip} ($p \in N$). Each combination is an n-tuple $(g_1, g_2, g_3,\ldots, g_n)$.

Step 5: Determine $r_j = S_g(g_1, g_2, g_3,\ldots, g_n)$ for each n-tuple, $j \in 1,2, \ldots, 2^n$.

Step 6: Set $F_{Sg} = [\min(r_j), \max(r_j)]$ for all $j \in 1, 2, \ldots, 2^n$.

Step 7: Determine all the α_k-cuts for the preferred output parameter, O_{pref}.

Step 8: Set $[O_{pref} \wedge F_{Sg}]$.

Step 9: Determine $f^* = \sup[O_{pref} \wedge F_{Sg}]$ and find the S_g^*, the S_g value for f^*.

Step 10: Find the endpoints of interval for each input I_p where $p = 1,2,\ldots,n$.

Step 11: Generate all 2^n combinations of the endpoints of intervals representing f^*-cuts for all F_{Ip} ($p \in N$). Each combination is an n-tuple $(g_1^*, g_2^*, g_3^*,\ldots,g_n^*)$.

Step 12: Determine $r^* = S_g^*(g_1, g_2, g_3,\ldots,g_n)_{f^*(opt)}$ by using Modified Optimized Defuzzified Value Theorem or its corollary.

The above algorithm is developed through the three phases of fuzzy system. Step 1 to 7 describe the phase of fuzzification, step 8 and 9 describe the processing of the fuzzified parameter in the fuzzy environment and the phase of defuzzification is described in step 10 to 12. The values determined in the final step of the algorithm are the approximate optimal value of input parameters that will produce the desired value of the output parameters as determined by applying the Modified Optimized Defuzzified Value Theorem or its corollary. It has been shown that all normal and convex fuzzy sets F_{Ii} , expressing preferences of all input parameters $g_i \in I_i \subset R^+ (i \in N)$ are mapped by the FSSM into the normal and convex induced fuzzy sets [8].

III. MODIFIED OPTIMIZED DEFUZZIFIED VALUE THEOREM

This theorem forms the important part of the final phase of defuzzification and covers the whole algorithms and hence the FSSM. It is a modification of the Optimized Defuzzified Value Theorem proposed in [7]. The proof of this theorem and its corollary are shown below.

Theorem:

Let $S_g : R^n \rightarrow R$ where S_g is a performance parameter based on the Fuzzy State Space Model. If $S_g^* = r_j^* = \max r_j$ such that $\mu(r_j^*) = f^*$ for all $(r_j, f^*) \in F_{ind}$, then $r_j^* = S_g^* = \max[S_g(g_1^*, g_2^*,\ldots, g_m^*)]$ where $\mu(g_p^*) = f^*$.

Proof:
Suppose $S_g^{\ *} = r_j^{\ *} = \max\ r_j$ such that $\mu(r_j^{\ *}) = f^{\ *}$ for all $(r_{j,}\ f^{\ *}) \in F_{ind}$.
Determine all the $f^{\ *}$-cuts of all F_{Ip} $(p \in N)$ to create all *n*-tuples of $(g_1^{\ *}, g_2^{\ *},..., g_n^{\ *})$ such that $\mu(g_p^{\ *}) = f^{\ *}$ and $(g_p^{\ *}, f^{\ *}) \in F_{Ip}$
Set $r_j^{\ *} = \max[S_g(g_1^{\ *}, g_2^{\ *},..., g_n^{\ *})]$, therefore $(r_{j,}\ f^{\ *}) \in F_{ind}$
However, since F_{ind} is normal and convex, this imply that $r_j = r_j^*$.

Corollary:
Let $S_g : R^n \to R$ where S_g is a performance parameter based on the Fuzzy State Space Model. If $S_g^{\ *} = r_j^{\ *} = \min\ r_j$ such that $\mu(r_j^{\ *}) = f^{\ *}$ for all $(r_{j,}\ f^{\ *}) \in F_{ind}$, then $r_j^{\ *} = S_g^{\ *} = \min[S_g(g_1^{\ *}, g_2^{\ *},..., g_m^{\ *})]$ where $\mu(g_p^{\ *}) = f^{\ *}$.

Proof:
Suppose $S_g^{\ *} = r_j^{\ *} = \min\ r_j$ such that $\mu(r_j^{\ *}) = f^{\ *}$ for all $(r_{j,}\ f^{\ *}) \in F_{ind}$.
Determine all the $f^{\ *}$-cuts of all F_{Ip} $(p \in N)$ to create all *n*-tuples of $(g_1^{\ *}, g_2^{\ *},..., g_n^{\ *})$ such that $\mu(g_p^{\ *}) = f^{\ *}$ and $(g_p^{\ *}, f^{\ *}) \in F_{Ip}$
Set $r_j^{\ *} = \min[S_g(g_1^{\ *}, g_2^{\ *},..., g_n^{\ *})]$, therefore $(r_{j,}\ f^{\ *}) \in F_{ind}$
However, since F_{ind} is normal and convex, this imply that $r_j = r_j^*$.

This theorem indicates that if the preferred fuzzy intersects on the maximum side of the fuzzy induced, then the set of optimized parameters is the set for the maximum of the induced values. On the other hand, the corollary indicates that if the preferred fuzzy intersects on the minimum side of the fuzzy induced, then the set of optimized parameters is the set for the minimum of the induced values.

IV. IMPLEMENTATION ON A FURNACE SYSTEM

Energy systems in power plants are one of the most frequently mentioned areas for thermal energy consideration. Our interest is the furnace system of a combined cycle power plant, which is regarded as constituents in a heat treatment system. The analysis of such system is often very complicated. The characterizing equations are generally a set of partial differential equations, with nonlinearity arising due to convection of momentum in the flow, variable properties and radiatives transport. However, approximation and idealizations are used to simplify these equations, resulting in algebraic and ordinary differential equations for many practical situations. Thus, it is assumed that the system can be represented by a lumped-parameter model. The state space model of the furnace system developed in [9] is as follows.

$$\left(\frac{d}{dt}\rho_{EG}\right) = \left(-\frac{k_F R_{EG} T_g}{V_F}\right)\rho_{EG} + \left(\frac{1}{V_F} \quad \frac{1}{V_F} \quad \frac{1}{V_F}\right)\begin{pmatrix} w_F \\ w_A \\ w_G \end{pmatrix} \qquad (1)$$

$$\begin{pmatrix} Q_{ir} \\ Q_{is} \\ Q_{rs} \\ Q_{es} \\ p_G \end{pmatrix} = \begin{pmatrix} \frac{\theta T_g R_{EG} k_F}{(1-\theta) w_{EG}}\left(Q_{gs} + w_{EG} c_{gs} T_{gr} - w_{EG} c_{gs} T_g\right) \\ \frac{T_g R_{EG} k_F}{w_{EG}}\left(Q_{gs} + w_{EG} c_{gs} T_{gr} - w_{EG} c_{gs} T_g\right) \\ k_F R_{EG} T_g c_{gs}\left(T_{gr} - T_{ge}\right) \\ k_F R_{EG} T_g c_{gs}\left(T_{ge} - T_{gl}\right) \\ R_{EG} T_g \end{pmatrix} (\rho_{EG}) \qquad (2)$$

Thus, based on equations (1) and (2), the furnace system is modeled as a first-order system with three-input and five-output. The problem of fuzzy-based system modeling consists in developing a fuzzy multivariable model with *n* input and *m* output parameters characterizing as good as possible a certain system behaviour. However, those multiple-input multiple-output (MIMO) models can be split into *m* equivalent MISO models [10] with all of the *n* input parameters of the system but only *one* of the *m* output parameters being subject to fuzzy modeling. Thus, the furnace modeling problem with *three* inputs and *five* outputs can be reduced to the development of fuzzy MISO models with *three* inputs and *one* output. The procedures in the implementation of the inverse Fuzzy State Space algorithm are repeated for each output parameter. Hence, we define $S_g : I_1 \times I_2 \times ... \times I_p \to R$ where S_g is the performance parameter based on the FSSM such that $r = S_g(g_1, g_2, g_3,..., g_n)$. A semi-automated approach using Matlab® m-file is used for the computations involved in this algorithm. The implementation of this algorithm is discussed according to the three phases of fuzzy system.

Phase 1: Fuzzification

Each of the input parameter of the furnace system is fuzzified. The desired value for each input parameter has a value $\alpha = 1$ whereas the domain or the extreme values are specified as $\alpha = 0$ as shown in Table 1.

Table 1 Input parameters specification

input parameters	$\alpha = 0$	$\alpha = 1$	$\alpha = 0$
w_F	10	12	16
w_A	60	65	70
w_G	20	22	25

In this illustration, α-cuts with increment of 0.2 are used to calculate F_{ind} , the fuzzy values of induced output or performance parameters S_g. Each output parameters can be expressed as a linear combination of the input parameters. From the state space model of the furnace system and using the steady state operating data, the input-output relationship are determined to be

$$Q_{ir} = 2.593765 \times 10^5 (w_F + w_A + w_G) \quad (3)$$
$$Q_{is} = 3.523226 \times 10^4 (w_F + w_A + w_G) \quad (4)$$
$$Q_{es} = 1.2053 \times 10^4 (w_F + w_A + w_G) \quad (5)$$
$$p_G = 9.7993 \times 10^2 (w_F + w_A + w_G) \quad (6)$$
$$Q_{rs} = 3.07209 \times 10^4 (w_F + w_A + w_G) \quad (7)$$

Combinations of the endpoints of intervals for all input parameters with respect to each particular value of α-cut are determined. The number of combinations increases with a smaller value of the α-cut. Each of these values is substituted in equations (3) – (7) so as to obtain the corresponding performance parameter. The induced performance parameter F_{Sg} is determined by taking the maximum and minimum value of each performance parameter. These values are used to plot the graph of F_{Sg}.

As for the output parameters, each of the desired output parameter is set to the values published in [3], which are obtained through forward calculations and simulation. The desired value and its domain are shown in Table 2 and are used to calculate the preferred or desired output parameters. α-cuts with increment of 0.2 as in the fuzzification of input parameters are used to calculate O_{pref}, the fuzzy values of preferred or desired output parameters. Combinations of the endpoints of intervals for all output parameters with respect to each particular value of α-cut are determined. These values are used to plot the graph of O_{pref}.

Table 2 Output parameters specification

Output Parameters	Domain	Desired value
Q_{ir}	$[2.5 \times 10^7, 2.8 \times 10^7]$	2.6846×10^7
Q_{is}	$[3.2 \times 10^6, 3.8 \times 10^6]$	3.6417×10^6
Q_{es}	$[1.2 \times 10^6, 1.4 \times 10^6]$	1.2465×10^6
p_G	$[9.0 \times 10^4, 1.2 \times 10^5]$	1.013×10^5
Q_{rs}	$[3.0 \times 10^6, 3.3 \times 10^6]$	3.1749×10^6

Phase 2: Fuzzy Environment

The intersection of the fuzzy preferred output parameter and the fuzzified performance parameter is determined by superimposing the two graphs in order to obtain the f^*- value. The fuzzy value obtained by considering each of the output parameters is shown in Figure 1 - 5. If there are more than one intersection points, the largest fuzzy membership value, f_j^* is taken as the intersection point. Similar plots can be obtained for each of the output parameters.

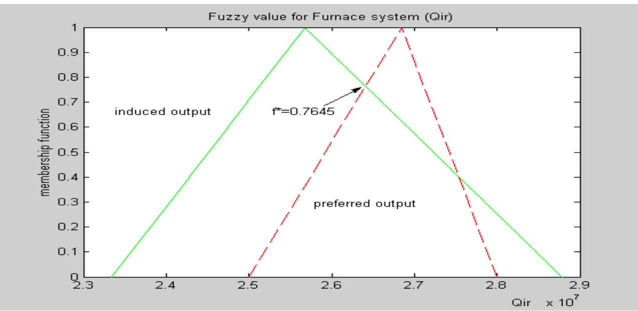

Figure 1 Fuzzy value for Furnace system(Q_{ir})

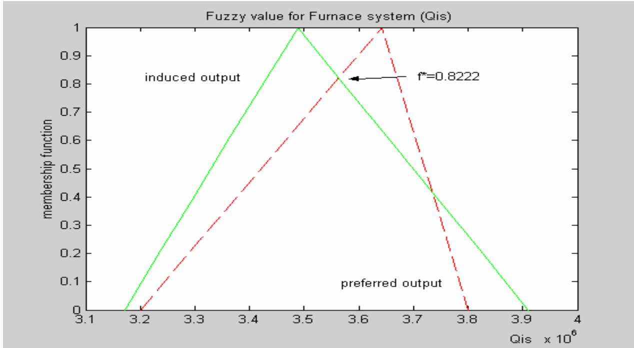

Figure 2 Fuzzy value for Furnace system (Q_{is})

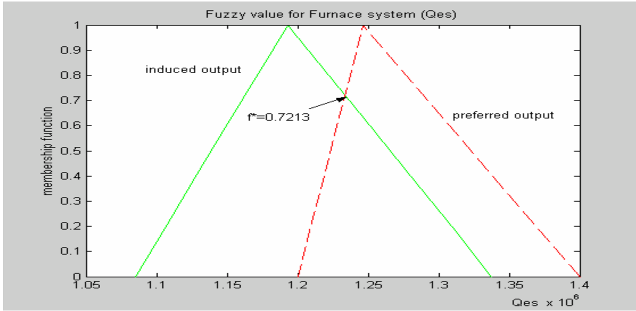

Figure 3 Fuzzy value for Furnace system (Q_{es})

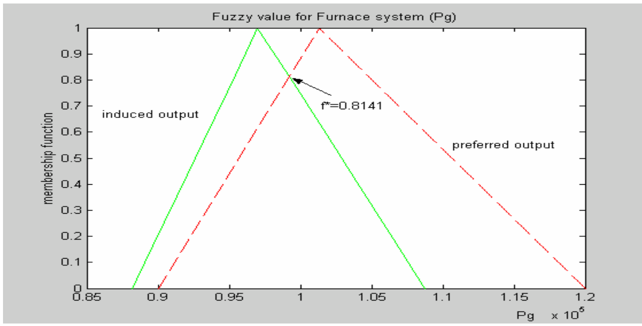

Figure 4 Fuzzy value for Furnace system (p_G)

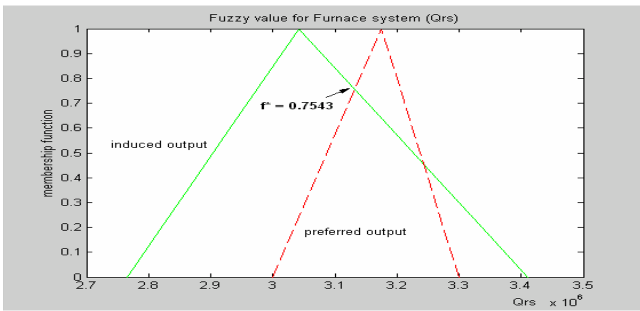

Figure 5 Fuzzy value for Furnace system (Q_{rs})

Phase 3: Defuzzification

With the f^*-value obtained, the steps in the defuzzification process are carried out to calculate the best possible combination of the input parameters in order to accommodate all the constraints defined in the process of fuzzification. Each of the eight combinations of the endpoints of interval are determined and processed by the extension principle, which was first proposed by Zadeh [11]. The selection of the optimal combination for the input parameters is determined by the Modified Optimized Defuzzified Value Theorem. For each output parameter, the fuzzy value and the corresponding optimal input parameters are tabulated in Table 3. Since the membership function designates the degree of desirability, the largest fuzzy value $f^* = 0.8222$ is chosen and used in the rest of the algorithm. Otherwise, the minimum norm value of the input parameters can also be used as an indicator for selection of the f^*- value.

Table 3 Calculated fuzzy value for Furnace system

Output Parameters	Fuzzy value	w_F	w_A	w_G
Q_{ir}	$f^* = 0.7645$	15.2355	68.4710	23.4710
Q_{is}	$f^* =$ **0.8222**	15.1778	68.3556	23.3556
Q_{es}	$f^* = 0.7213$	15.2787	68.5574	23.5574
p_G	$f^* = 0.8141$	15.1859	68.3718	23.3718
Q_{rs}	$f^* = 0.7543$	15.2457	68.4914	23.4914

The results of the implementation of the inverse Fuzzy State Space algorithm for a multivariable furnace system with MISO structure are shown in Table 4, where the optimal combination of the input parameters are w_F = 15.1778 kg/s, w_A = 68.3556 kg/s and w_G = 23.3556 kg/s. These values differ from the desired values with an error of about 26.48%, 5.16% and 6.16% respectively. At the same time, the percentage error for each of the output parameters of the furnace system is computed as shown in Table 5. It is interesting to note that the calculated values obtained using this algorithm are very close to the desired target values of the system.

Table 4 Optimized input parameters

$f^* = 0.8222$	Calculated Values	Desired Values	Error (%)
w_F	15.1778	12	26.48
w_A	68.3556	65	5.16
w_G	23.3556	22	6.16

Table 5 Calculated output parameters for Furnace System

$f^* = 0.8222$	Calculated Values	Desired Values	Error (%)
Q_{ir}	2.7724×10^7	2.6846×10^7	3.27
Q_{is}	3.7659×10^6	3.6417×10^6	4.09
Q_{es}	1.2883×10^6	1.2465×10^6	3.35
p_G	1.0474×10^5	1.0130×10^5	3.39
Q_{rs}	3.2837×10^6	3.1749×10^6	3.42

Subsequently, a comparison is made between the optimal input parameters obtained using the inverse Fuzzy State Space algorithm and the result obtained through simulation carried out by [3]. The percentage error is calculated and tabulated in Table 6. The aim of this comparison is to highlight the difference between inverse modeling by utilizing fuzzy sets and a widely accepted forward modeling based on simulation. With the triangular fuzzy number used in modeling the uncertainty, the obtained result should have the same value as the result in [3] with no uncertainty consideration. It is observed that the values of the input parameters w_F (fuel flow to the furnace in kg/s), w_A (air flow to the furnace in kg/s), and w_G (exhaust gas flow from the gas turbine in kg/s) differ with an error of 7.77%, 6.65% and 0.81% respectively. In order to properly model the uncertainties and further improve the results, the parameters of the fuzzy numbers which are used to model uncertainties in this study, need to be adjusted based on the historical data or human experience. For a better resolution, α-cuts with much smaller increment can be used.

Table 6 Comparison of Optimized Input parameters

Input Parameters	Ismail's	Reference [3]	Error (%)
w_F	15.1778	14.083	7.77
w_A	68.3556	64.093	6.65
w_G	23.3556	23.168	0.81

The good results obtained in this application show that this approach may become an interesting tool for decision-makers. It will provide broader and useful information for power generation planning purposes. Besides, it is relatively easy to take into account experts knowledge and considerations for establishing the membership functions. However, we anticipate to obtain better results with a reduction in computation time through the implementation of the inverse Fuzzy State Space algorithm with MIMO structure, which is currently undertaken.

V. CONCLUSIONS

The formulation of an inverse Fuzzy State Space algorithm for multivariable dynamic system was presented. Briefly, the procedure involved fuzzification of all the input parameters to create fuzzy environment. This is then processed to produce the induced output parameters. The best input parameters were extracted through defuzzification using an important theorem, Modified Optimized Defuzzified Value Theorem. Although we have illustrated the implementation of FSSM for the furnace system of combined cycle power plant, it can be applied to any multivariable dynamic system as long as the mathematical model of the system can be expressed in state space representation. In general, this new technique for determination of optimal input parameters gives a broader and useful information and provides a faster and innovative tool for decision-makers.

ACKNOWLEDGEMENT

R.Ismail is financially supported by University of Technology MARA, Selangor, Malaysia. The authors express their sincere gratitude to the anonymous reviewers for their valuable suggestions.

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Nomenclature

c_{gs}	combustion gas specific heat capacity	Js/kg^{o}K
w_{EG}	mass flow of exhaust gas from the boiler	g/s
Q_{rs}	heat transferred to the reheater	J/s
Q_{ir}	heat transferred to the risers	J/s
Q_{gs}	total heat transferred to the superheater	J/s
Q_{es}	heat transferred to the economizer	J/s
T_g	gas temperature at the superheater	^{o}K
T_{gr}	gas temperature at the reheater	^{o}K
T_{ge}	gas temperature at the economizer	^{o}K
T_{gl}	boiler exhaust gas temperature	^{o}K
R_{EG}	initial gas constant for exhaust gases	(-)
V_F	combustion chamber volume	m^3
θ	tilt angle coefficient 0 < theta <1	(-)
p_G	furnace air pressure	Pa
k_F	chimney flow coefficient	ms
ρ_{EG}	density of exhaust gas from the boiler	kg/m^3