# Method for Representation of Complexity Using Curvature Integration and Multi-Resolution Representation

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***Abstract*—In curve design, controlling macroscopic feature that emerges from the total of shape elements is important. However, trial and error is required in order to control the curved profile to be set reflecting macroscopic feature imaged by designer, because there is no useful method for controlling macroscopic feature in the conventional computer aided design system. We proposed the method for representation of macroscopic feature using curvature integration and multi-resolution representation. This method was applied to shape-generation for the design of automobile side-view. As a result, it was confirmed that the control of macroscopic feature was possible. In the present study, it was shown that the possibility of new design-support in curved profile.**

## I. INTRODUCTION

Currently, a curve and curved surface shape are widely used in various industrial products. In considering new curved profiles, controlling the overall shape feature that emerges from the total of shape elements is important. The main reason for this appears to be that human beings tend to perceive the overall shape feature (such as Gestalt) macroscopically as pointed out in cognitive psychology [1]. However, trial and error is required in order to control curved profiles to be set reflecting macroscopic feature imaged by designer, because there is no useful method for controlling macroscopic feature in the conventional computer aided design system.

Quantitative representation of macroscopic feature is important to control macroscopic feature. However, representation of macroscopic feature is difficult using conventional microscopic shape information (such as dimension and curvature), because the microscopic shape information represents the partial feature of curved profile. If quantitative representation of macroscopic feature is realized, a designer can control macroscopic feature indirectly by shape-generation that utilized search algorithm (such as genetic algorithm) including method for representation of it. Therefore, in curve design, the method for representation of macroscopic feature and the design-support system that can control macroscopic feature by designer is desired.

In our past study, we had proposed the method for representation of macroscopic feature "complexity" using curvature integration, and the effectiveness of the method had been confirmed in various curved profiles (such as basic curved profiles and existing automobile side-views) [2-5]. "Complexity" affects evaluation of the important item on design, such as "beauty" and "similarity". Moreover, it is possible that the quantification of "complexity" using the amount of physics computed from curved profile, because there is little individual difference of evaluation for "complexity."

In the present study, firstly, we proposed the method for representation of macroscopic feature "complexity" using curvature integration and multi-resolution representation. Multi-resolution representation was utilized for preventing the generation of curved profiles containing the swell that a man cannot recognize. Next, this method was applied to shape-generation for the design of automobile side-view described by cubic Bézier curve, and the possibility of controlling macroscopic feature "complexity" using the proposed method was verified.

## II. CURVATURE INTEGRATION

In the knowledge of the study about the "complexity" in outline shapes, the number of vertices is cited as one of important factors of the "complexity" [6,7]. A vertex is the feature point for a straight-line profile. The feature point in the curved profile is equivalent to a high curvature point [8]. Therefore, it is considered that the number of high curvature points cause "complexity" in the curved profile. However, in order that curvature changes continuously, the threshold that divides a high curvature point and the other point is needed as a parameter. In our past study, the number of high curvature points was not computed using threshold, but the integration of the absolute curvature was computed as curvature integration. This value is known one of the global properties of curved profile in the differential geometry [9].

Curvature integration from curvature function in the curved

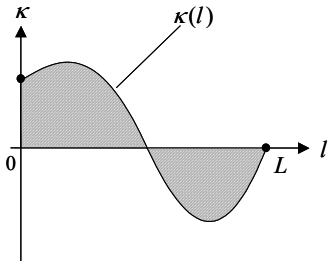


Fig. 1. Extraction of curvature integration based on curvature function.

profile is calculated in the following manner. In Fig. 1, the vertical axis is curvature $\kappa$, the horizontal axis is the curve length $l$, the curvature function is $\kappa(l)$ and the total length of curved profile is $L$. Curvature integration is calculated using the following equation:

$$I = \frac{1}{2\pi}\int_0^L \left| \ \kappa(l) \ \right| \ dl \qquad (I \geq 1) \tag{1}$$

## III. MULTI-RESOLUTION REPRESENTATION

Smoothing was utilized for preventing the generation of curved profiles containing the swell that a man cannot recognize. In this method, parameter controls the size of the swell removed. Moreover, it is called multi-resolution representation of shape to change a parameter to many stages and to acquire the shape of various resolutions [10]. There is study that analyzes the property of shape based on change of the amount of physics in multi-resolution representation [11-13].

The multi-resolution representation in curved profile is based on the view of scale space proposed by Witkin [14]. In this method, Gaussian kernel $G(u,\sigma)$ of width $\sigma$:

$$G(u,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{u^2}{2\sigma^2}\right) \tag{2}$$

is used for smoothing. $\sigma$ is the parameter for smoothing. A two-dimensional planer curve is defined in the following equation:

$$C(u) = (x(u), y(u)) \tag{3}$$

Then, smoothed curve, $X(u,\sigma)$ and $Y(u,\sigma)$, are computed by the convolution of $C(u)$ and $G(u,\sigma)$, and are defined as:

$$\begin{aligned} X(u,\sigma) &= x(u) \otimes G(u,\sigma) \\ &= \int_{-\infty}^{\infty} x(v)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(u-v)^2}{2\sigma^2}\right)dv \end{aligned} \tag{4}$$

$$\begin{aligned} Y(u,\sigma) &= y(u) \otimes G(u,\sigma) \\ &= \int_{-\infty}^{\infty} y(v)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(u-v)^2}{2\sigma^2}\right)dv \end{aligned} \tag{5}$$

In smoothing by Gaussian kernel, no new inflection points are created at higher smoothing [15]. Therefore, the curvature integration function $I(\sigma)$ computed by multi-resolution representation is a monotonically decreasing function.

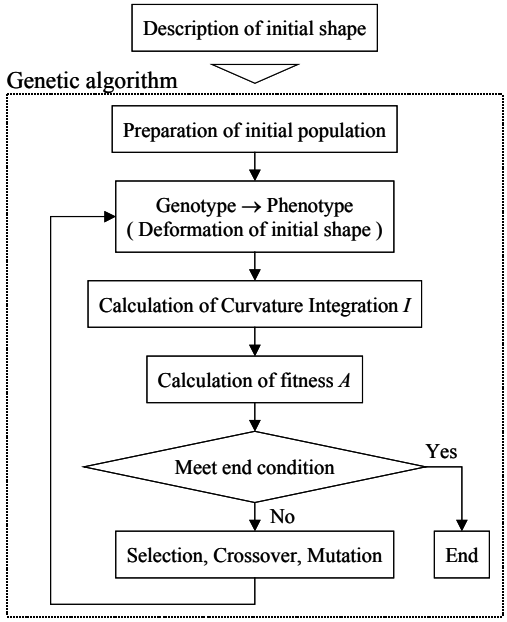


Fig. 2. Algorithm of shape-generation method.

For preventing the generation of curved profiles containing the swell that a man cannot recognize, the following two methods were proposed in the present study. One is that curvature integration is computed after smoothing with arbitrary parameter. In this method, multi-resolution representation is utilized for adjustment of parameter. The other is that index $S$ that shows the robustness of "complexity" represented by curvature integration is newly proposed. In this method, the curved profile with the swell of the size that is hard to be recognized is removed using $S$ as index. $S$ is defined as following equation using $I^*(\sigma)$ by multi-resolution representation.

$$S = \int \ I^*(\sigma) \ \ d\sigma \tag{6}$$

Here, $I(\sigma)$ was standardized as following:

$$I^*(\sigma) = \frac{I(\sigma)-1}{I(0)-1} \tag{7}$$

## IV. APPLICATION TO SHAPE-GENERATION

### *A. Construction of Shape-Generation Method*

The algorithm of proposed shape-generation method is shown in Fig. 2

Based on studies of Tian [16], the automobile side-view was described as a polygonal profile consisting of eight basic points (Fig. 3), and defined junction points (Fig. 4) for description by curved profile. In consideration of the freedom of shape description, and the simplicity of control, cubic Bézier curve was used as description of curved profile in the shape generation method (Fig. 5). The automobile side-view (Sedan) was used as the initial shape for the shape-generation, and the curve control variables (the position of the basic point, the direction of a tangent vector, and the size of a tangent

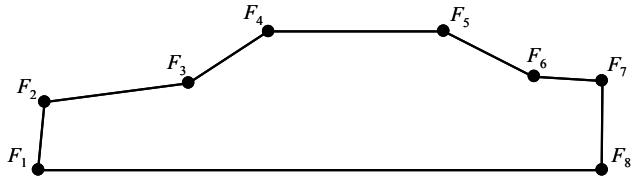


Fig. 3. Basic points.

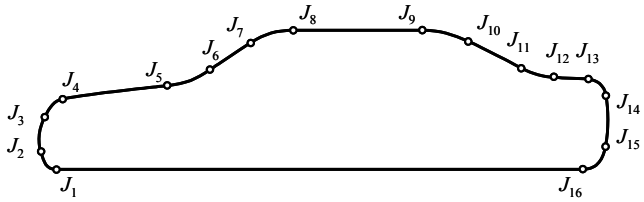


Fig. 5. Interpolation by a cubic Bézier curve.

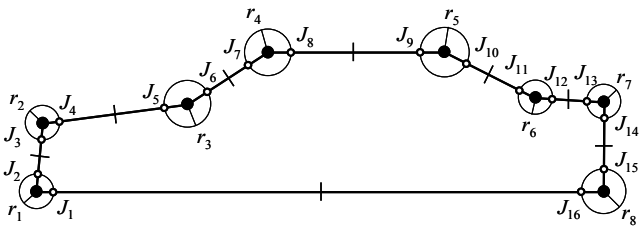


Fig. 4. Junction points.

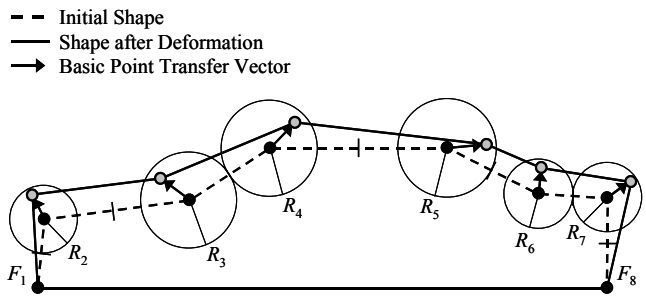


Fig. 6. Movable range of basic points.

vector) were changed in the shape-generation. Then, movable ranges of curve control variables were defined for preventing shape generation from generating a curved profile having a self-intersection and cusp. The movable range of basic points is shown in Fig. 6 as an example.

In the shape generation method, a genetic algorithm (GA) was used as a search algorithm. GA is a search algorithm imitating the evolution process of a living thing. Global search is attained in order that parallel search by many individuals is performed. The curve control variables were manipulated in the shape generation. The chromosome for GA was composed of an arrangement of the numerical values for this manipulation. The fitness was the absolute value of the difference between curvature integration of an individual and curvature integration that the designer set. Crossover was handled in the manner described by Obayashi [17]. The random weighted mean of the real number variable was used. Other GA parameters were referred to DeJong's standard parameter [18].

*B. Shape-Generation*

In shape-generation, the amount of change in curvature integration was set as four levels of -0.25, +0.25, +0.50, and +0.75 based on the result of analysis about the range of change in curvature integration. The presentation of samples is shown in Fig. 7. As a result of analyzing the relationship between curvature integration and "complexity", it was found that both are high correlation. However, in a certain generated

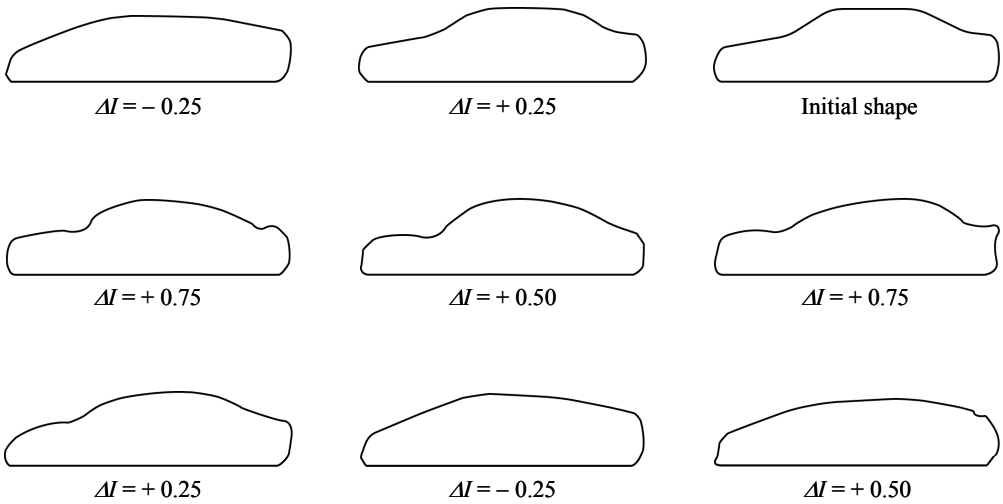


Fig. 7. Presentation of samples.

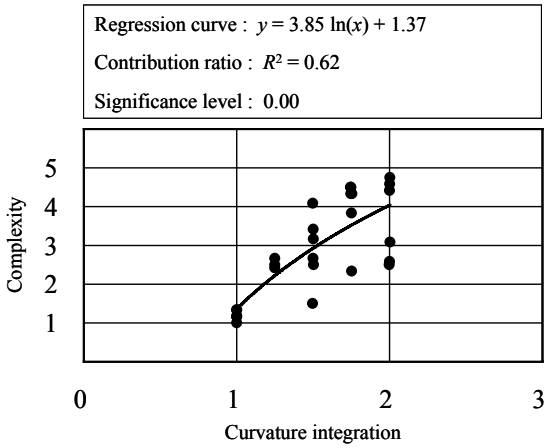


Fig. 8. Relationship between curvature integration and complexity.

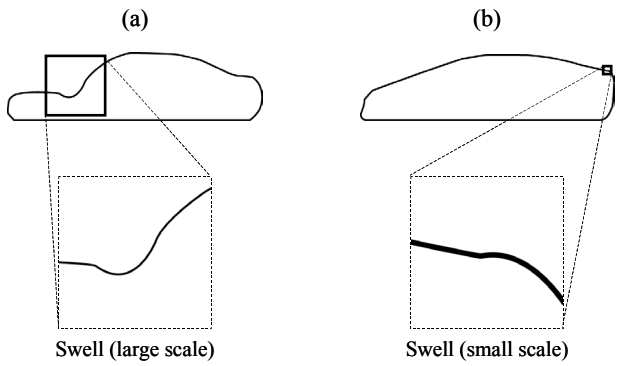


Fig. 9. Swell (large scale and small scale).

shape, although the value of curvature integration was high, the value of "complexity" was low (Fig. 8).

The generated shape (a) to which curvature integration and "complexity" correspond was extracted, and it was compared with the generated shape (b) to which curvature integration and "complexity" not correspond (although curvature integration is equal, evaluations of "complexity" differ greatly). As a result, it was confirmed that the generated shape (a) contained the swell of the size recognized enough, and, on the other hand, the generated shape (b) contained the swell of the size which is hard to be recognized (Fig. 9). This tendency was found in the whole sample. Therefore, it was confirmed that the importance of preventing the generation of curved profiles containing the swell that a man cannot recognize.

*C. Multi-Resolution Analysis*

Multi-resolution representation was applied to the generated shapes (a) and (b) shown in Fig. 9, and $I(\sigma)$ was computed. In Fig. 10, the vertical axis is $I(\sigma)$, the horizontal axis is $\sigma$. Moreover, Fig. 11 shows the situation of smoothing in the range of $\sigma$ to which $I(\sigma)$ decreases greatly. The swell of the size which can be recognized easily is smoothed in the range of $\sigma$=0.01 to $\sigma$=0.1 in the generated shape (a). On the other hand, the swell of the size which is hard to be recognized is smoothed in the range of $\sigma$=0.001 to $\sigma$=0.01 in the generated shape (b). Fig. 12 expands a part of the generated shape (b) in Fig. 11.

About the 1st method proposed in Chapter 3, in order to calculate the value of suitable $\sigma$, the correlation coefficient $R$ between the natural logarithm of $I(\sigma)$ and "complexity" in generated shapes was computed to every $\sigma$ (Fig. 13). As a result, $R$ became the highest when $\sigma$ was set to 0.02, as shown in Fig. 13. However, this knowledge is restricted to the experiment conditions in the present study. In order to utilize this method, it is necessary to newly build the model that outputs the value of suitable $\sigma$.

About the 2nd method proposed in Chapter 3, in order to calculate the value of suitable $S$, the correlation coefficient $R$ between the natural logarithm of $I$ and "complexity" in generated shapes removed in the small order of $S$ was computed to every $S$ (Fig. 14). As a result, it was confirmed that $R$ becomes high as generated shape was removed in order with the small value of $S$. If the value of $S$ is set more highly, the robustness of the "complexity" represented by curvature integration will become higher. Therefore, the adjustment of $S$ for remove is considered to be easy as compared with that of

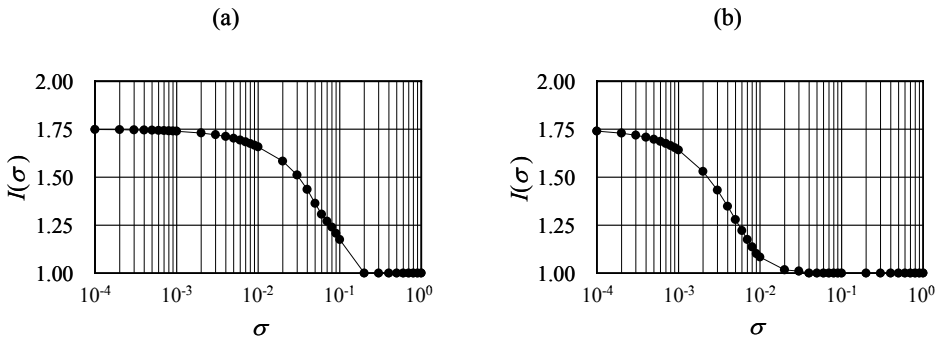


Fig. 10. Comparison of the change of $I(\sigma)$ on generated shape (a), and that on generated shape (b).

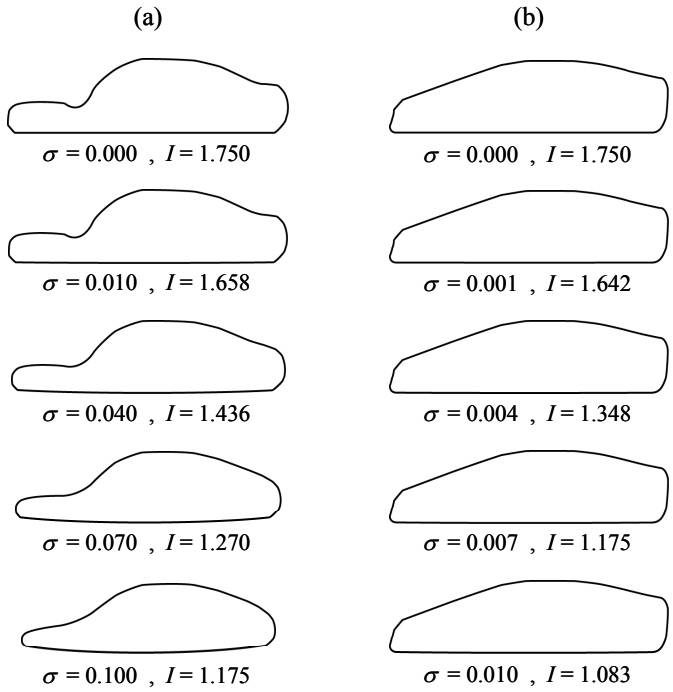


Fig. 11. Situation of smoothing in the range of $\sigma$ to which $I(\sigma)$ decreases greatly.

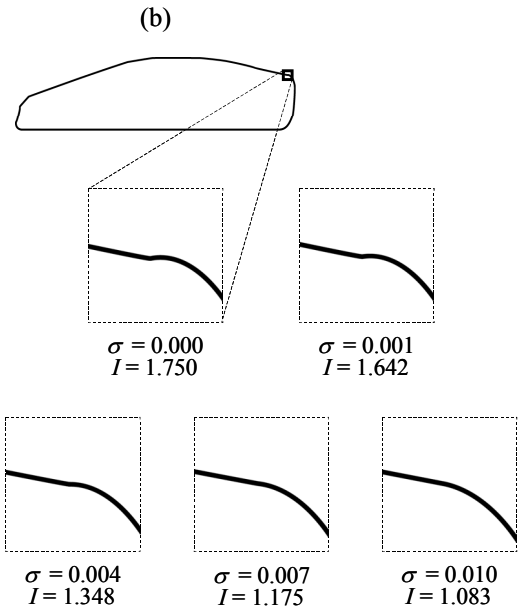


Fig. 12. Part of the generated shape (b) in Fig. 11.

suitable $\sigma$. However, it is necessary to smoothing repeatedly for calculation of $S$, and calculation cost becomes high.

About the comparison of above-mentioned two methods, it will verify in various application from now on. In the present study, the method for preventing the generation of curved profiles containing the swell that a man cannot recognize was proposed, and the effectiveness of this method in the shape-generation was confirmed.

## V. CONCLUSIONS

In the present study, the method for representation of macroscopic feature "complexity" using curvature integration and multi-resolution representation was proposed. And, this method was applied to shape-generation for the design of automobile side-view. As a result, it was confirmed that the

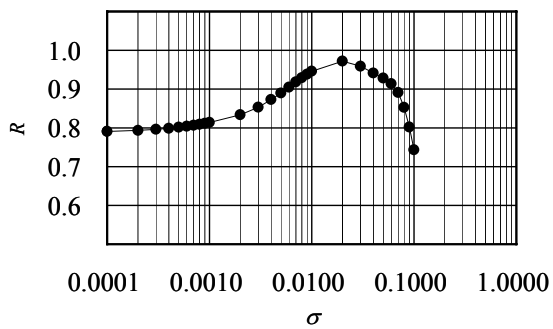


Fig. 13. Relationship between $\sigma$ and correlation coefficient $R$.

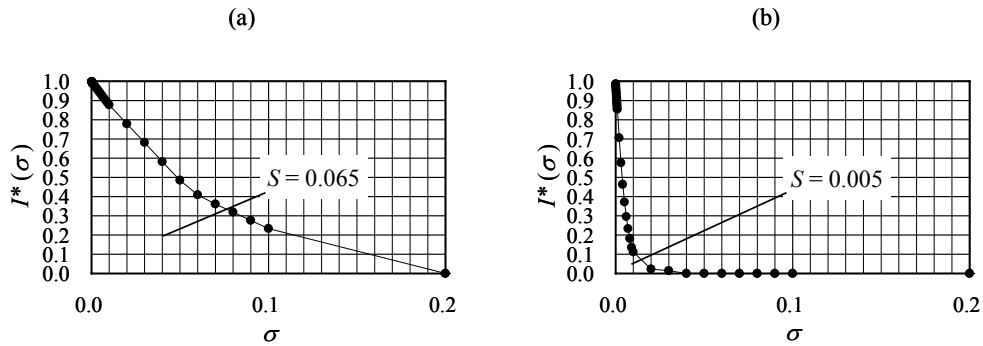


Fig. 14. Comparison of $S$ on generated shape (a), and $S$ on generated shape (b).

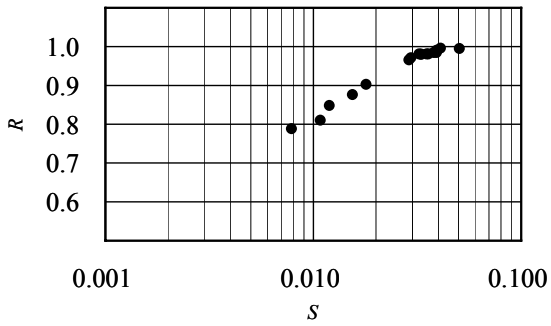


Fig. 15. Relationship between *S* and correlation coefficient.

control of macroscopic feature "complexity" was possible by use of this method as shape-generation index. It was shown that the possibility of new design-support in curved profile.

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