Fuzzy Natural Color System for Achromatic Colors

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Abstract- **The present study considered a fuzzy natural color system (NCS) in which triangular pyramid membership functions are constructed on the NCS color triangle. This system can process a fuzzy input to an NCS system and output to a center of gravity of three weights associated with respective grades. Triangular membership functions are applied to the hue angle, and triangular pyramid membership functions are applied to the NCS color triangle relationship. By treating three membership functions of blackness, whiteness, and chromaticness on the NCS color triangle, a target color as a center of gravity of the output fuzzy set can be easily obtained. I will show herein the difference between crisp inputs and fuzzy inputs for achromatic colors.**

I. INTRODUCTION

The NCS (Natural Color System) color descriptive system and atlas were developed in Sweden [3]. In the NCS, a method for obtaining hue expressions has been reported by Sivik (1997). A technique for obtaining expressions of the NCS color triangle using the fuzzy set theoretical method has already been studied for chromatic colors in ref. [9].

In this study, fuzzy inputs of triangular membership functions are used on the NCS color triangle corresponding to the HLS tone plane consisted of lightness and saturation [5]-[8]. A vague color for achromatic colors is clarified. Such a system will help us to determine the average color value as a center of gravity of attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

II. THE NATURAL COLOR SYSTEM

NCS color triangle

The graphical illustration of all colors of a given hue is in the NCS (as in Hering's original model) equilateral triangle, where each point represents a certain nuance. (See Fig.2.) The uni-dimensional "dimension" nuance is thus the location for all colors, of all hues, with a specific relationship between the parameters of whiteness, blackness, and chromaticness.

NCS Chromaticness

Chromaticness corresponds to the dimensional direction of Munsell's Chroma (in HVC system), but not to the scaling. Chroma is an open-ended scale, which is defined by color samples. On the other hand, NCS chromaticness has, as do all the NCS parameters, two end points and the scale between these is divided into 100 equal steps defined as "resemblance to the reference points." Chromaticness in the color triangle is the resemblance (in %) to the maximal color," by which is meant the color with the given hue, which is devoid both of whiteness and blackness. In the NCS color triangle, the corner point C in Fig.2 represents this maximal color. See also the highest chroma in ref. [4]

NCS whiteness and blackness

The other corner points W (white) and S (black) represent correspondingly the imaginable elementary colors white and black according to Hering's phenomenological color model. The variable whiteness (w) is defined as the degree of resemblance to the imagination of the elementary white (W), and the blackness (s) is defined as the degree of resemblance to the imagination of the elementary black (S). The scaling is, as mentioned, an equispacing in 100 steps. (See Fig.2.)

NCS color space

If we now combine the hue circle with the color triangles for each of all possible hues we get the three dimensional NCS color solid. (See Fig.1.) Each point within it thus represents the perception of a surface color whose relative resemblance to one or two chromaic elementary colors and to white and black is given by the relative distances to these points in the color solid.

III. METHODS AND RESULTS

Figure 3 illustrates input fuzzy sets of triangular pyramid, fuzzy input, and crisp output on the NCS color triangle, and crisp output on the HLS tone plane. The fuzzy rules are as follows. (See also Figs.3 and 5.)

R^1 : IF X is A_1 THEN Y is B_1
R^2 : IF X is A_2 THEN Y is B_2
R^3 : IF X is A_3 THEN Y is B_3

Rules R^k: if X is A_k , then Y is B_k (k = 1, 2, 3) where k is the rule number, A_k is a fuzzy set of input, and B_k is a crisp set of output. $X=(x_1, x_2, x_3)$ are input parameters (variable), and $Y=(y_1, y_2, y_3)$ are output parameters; X and Y are fixed on these parameters of NCS. A fuzzy set A_k of input shows triangular pyramid form at corner points S_i, W_i, and C_i, and a crisp set B_k of output of rule R^k is shown at corner points S_o, W_o, or C_o (a fuzzy set B_k' with arrow) on the color triangular, and output is B_k if the input is A_k.

The fuzzy inference method is as follows. Let the input be $x_1=x_1'$, $x_2=x_2'$, and $x_3=x_3'$. *i*: the input of rule R^k, grade $\alpha_k = A_k(X')$, where k=1, 2, 3. *ii*: the output of rule R^k, and the α_k level-set is shown as a vertical filled allow. *iii*: $B_k'=\alpha_k B_k$, where B_k is crisp set in Fig.3b (center). *iv*: the complete inference result B' of rules R^1, R^2, and R^3.

$$B'=\alpha_1 B_1 \cup \alpha_2 B_2 \cup \alpha_3 B_3 = B_1' \cup B_2' \cup B_3'$$

The output parameter, $Y'=(y_1', y_2', y_3')$ corresponds to a coordinate of the central axis of the membership function of B', that is a de-fuzzification. Also, in Fig.3c (right), $Y'=(z_1', z_2')$ corresponds to a coordinate of the HLS system. z_1' (on the lightness axis) is calculated from y_1' and y_2', and z_2' (on the saturation axis) is equal to y_3'.

Figure 4a (left) illustrates fuzzy inputs (I_1-I_6) of achromatic colors on the NCS color triangle. The fuzzy inputs are formed by conical membership functions and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. The fuzzy inputs (I_{01}-I_{06}) are formed by a triangular membership function on the W_i-S_i line corresponding to the achromatic colors of the HLS system.

Figure 4b (right) illustrates arranged modifiers corresponding to conical membership functions of Fig.4 a, where a modifier is shown by an area enclosed by a circle (the top of the 0.5 level-set in Fig.4b, right). The achromatic color names and tone modifiers are the same as those used under JIS [1], [2]. Points of difference are as follows. Two membership functions are used in constructing 'grays' of achromatic colors, because these modifiers occupy a large region on the plane. In this case, for instance, $gray^1$ and $gray^2$ mean lighter gray and darker gray, respectively.

Figure 4c (bottom-center) illustrates crisp achromatic colors, fuzzy achromatic colors, and fuzzy *reddish* achromatic colors. *Reddish* achromatic colors are included as achromatic colors [5]-[8]. Namely, cones (No.1-No.6) include triangles (No.01-No.06) on the W_i-S_i line. These color names are from the fuzzy color-naming system described in a previous work [7], [8].

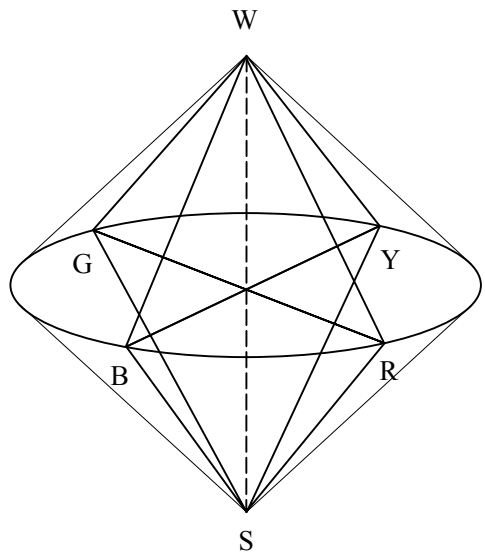

Fig.1. NCS color space and color circle

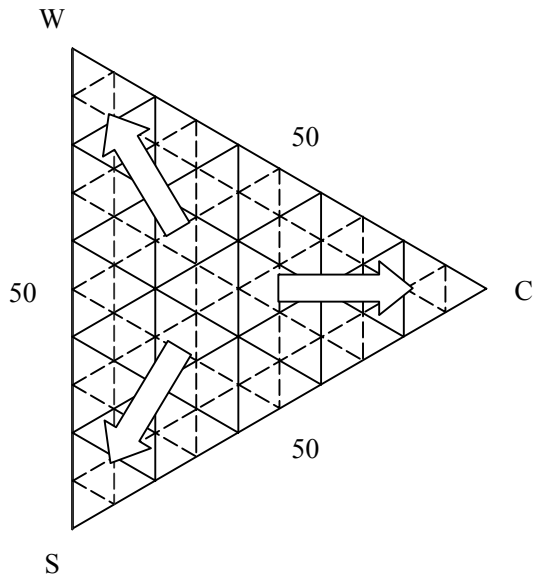

Fig.2. NCS color triangle

Achromatic colors

Figure 5 illustrates fuzzy inputs (I_{01}-I_{06}) on the NCS color triangle. The fuzzy inputs are formed by triangular membership functions, which are made to mutually overlap. The triangular membership functions are included in the conical membership function (Fig.3a and Fig.4a). The triangular membership function Proj(I_{04}) on the unknown axis (not an NCS parameter) is one of six projections of six fuzzy inputs (I_{01}-I_{06}) by the rays from the right side, and the triangular membership function Proj(I_{04}) on the blackness axis is one of six projections of those by the rays from the lower part.

An intersection of A_k for fuzzy sets I_n of input is

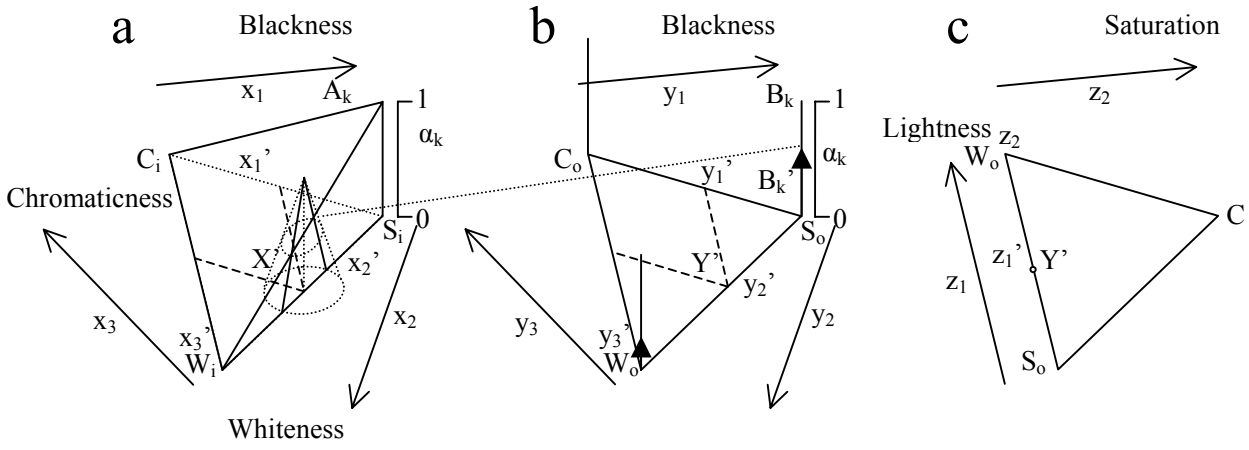

Fig.3. Fuzzy system using the membership function of a triangular pyramid on the NCS color triangle, and the HLS tone plane

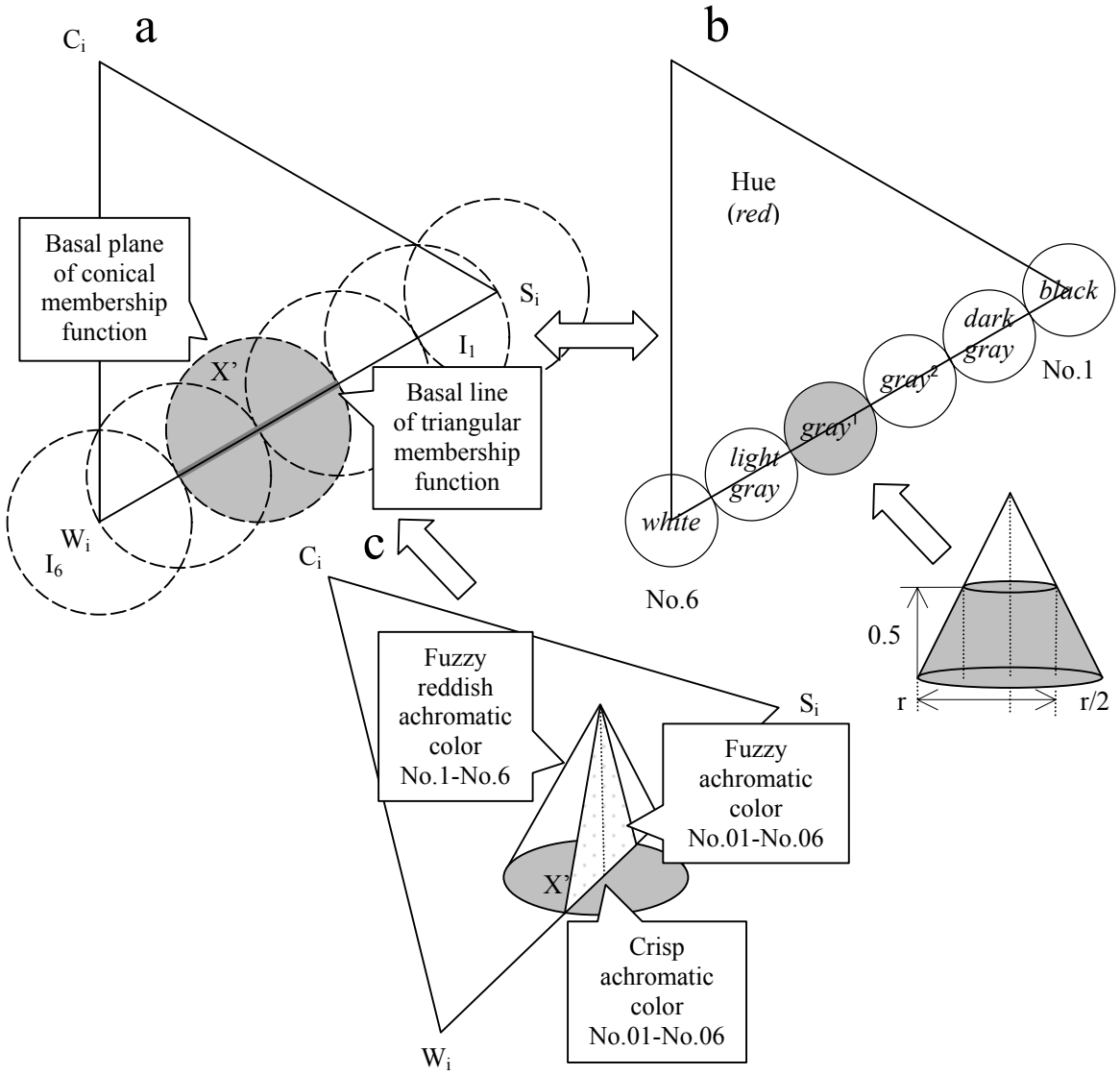

Fig.4. Fuzzy inputs of achromatic colors on the NCS color triangle and top areas of 0.5 level-sets with reddish achromatic color names

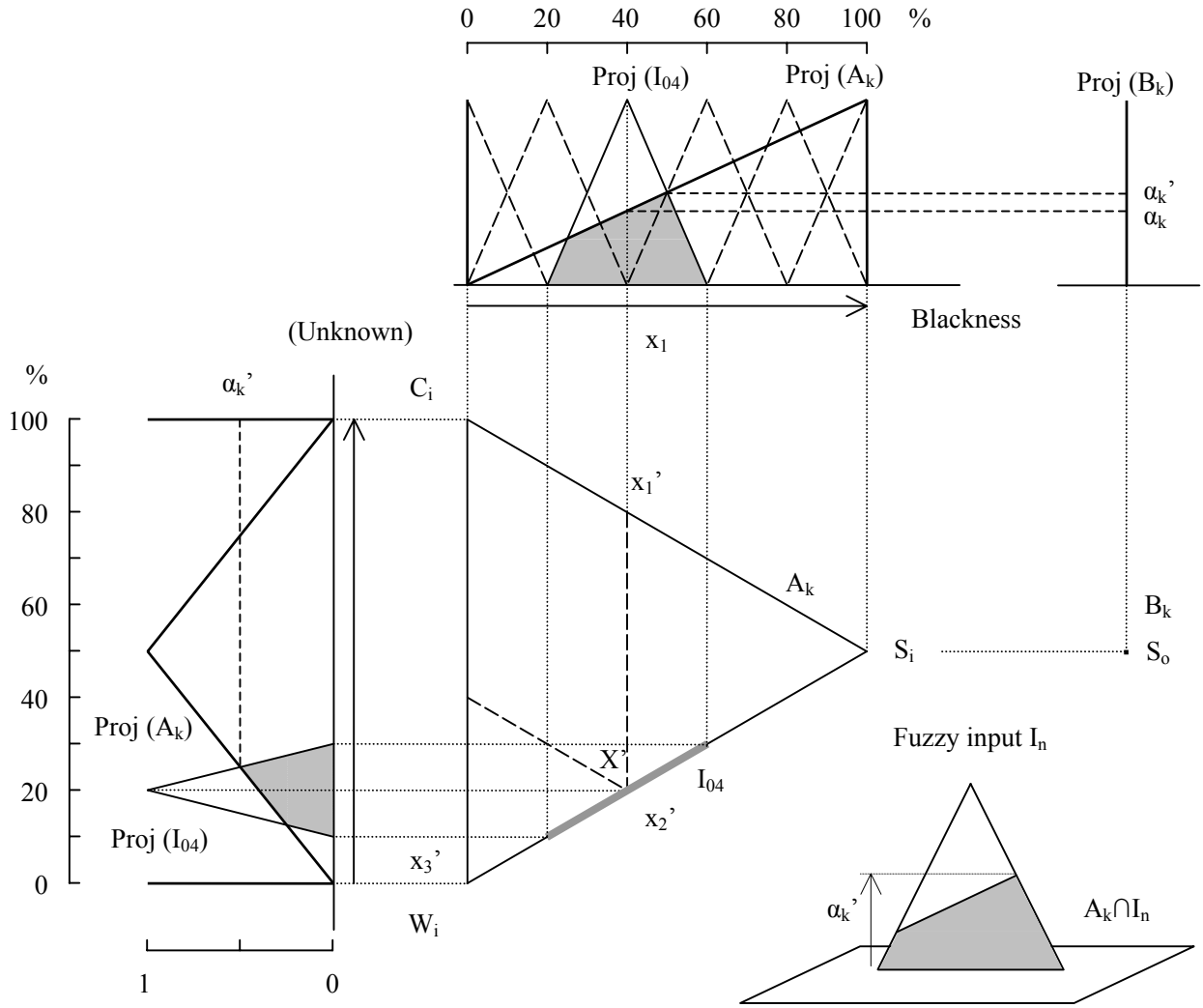

Fig.5. Membership functions of triangular pyramid on the NCS color triangle and one of triangular fuzzy inputs (vague achromatic colors)

$A_k \cap I_n$. (See the shaded area in Fig.5 bottom-right.) Grade α_k' = height ($A_k \cap I_n$). If the input is crisp, α_k' changes to α_k. S_o is the new blackness. Proj(B_k) is a projection of the output fuzzy set at the corner point S_o (Fig.3b, center).

In the present study, for vague achromatic color inputs to the NCS, this system can translate input data X of vague color to output data Y of simple color on the NCS color triangle. The fuzzy input (No.04) on the NCS is constructed by the center X'= (x_1', x_2', x_3') = (40, 60, 0) as percentages and the basal line of the triangle indicating vagueness is $2 \cdot r$ = 46% long.

Table 1 shows the fuzzy inference results for six crisp inputs (x_1', x_2', x_3'). The grades (α_1, α_2, α_3), inference outputs (y_1', y_2', y_3'), and HLS output Y'=(z_1', z_2') were calculated. The relationship between inputs, grades, and outputs are proportional. The achromatic color names are No.01: black, No.02: dark gray, No.03: gray^2, No.04: gray^1, No.05: light gray, and No.06: white. These numbers correspond to the suffixes of fuzzy input I.

Table 2 shows the fuzzy inference results for six fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1', x_2', x_3') in the same manner as in Table 1 and the basal line of the triangle-indicated vagueness is 46% of the total length. The inputs and outputs are not proportional, which indicates nonlinear information processing in this fuzzy system.

Table 3 shows the fuzzy inference results for six fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1', x_2', x_3') in the same manner as the crisp inputs in Table 1 and the radius (r = 23%) of the basal plane (circle) of the cone indicated vagueness. (See Fig.4.) The inputs and outputs are not proportional, which also indicates the nonlinear information processing in this fuzzy system.

Figure 6 illustrates the relationship between the lightness z_1 and the saturation z_2 obtained from data (z_1', z_2') in Tables 1, 2, and 3. Filled squares (FC) are outputs for fuzzy inputs of chromatic colors, and filled circles (CC) are outputs for crisp inputs of chromatic colors

Table 1. Inference results for crisp inputs of achromatic colors

No.	Crisp input			Grade for crisp input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1	α_2	α_3	y_1'	y_2'	y_3'	z_1'	z_2'
01	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	0.0	0.0
02	80	20	0	0.80	0.20	0.00	80.0	20.0	0.0	20.0	0.0
03	60	40	0	0.60	0.40	0.00	60.0	40.0	0.0	40.0	0.0
04	40	60	0	0.40	0.60	0.00	40.0	60.0	0.0	60.0	0.0
05	20	80	0	0.20	0.80	0.00	20.0	80.0	0.0	80.0	0.0
06	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	100.0	0.0

Table 2. Inference results for fuzzy inputs of achromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1'	α_2'	α_3'	y_1'	y_2'	y_3'	z_1'	z_2'
01	100	0	0	1.00	0.17	0.00	85.7	14.3	0.0	14.3	0.0
02	80	20	0	0.83	0.33	0.00	71.4	28.6	0.0	28.6	0.0
03	60	40	0	0.67	0.50	0.00	57.1	42.9	0.0	42.9	0.0
04	40	60	0	0.50	0.67	0.00	42.9	57.1	0.0	57.1	0.0
05	20	80	0	0.33	0.83	0.00	28.6	71.4	0.0	71.4	0.0
06	0	100	0	0.17	1.00	0.00	14.3	85.7	0.0	85.7	0.0

Table 3. Inference results for fuzzy inputs of *reddish* achromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1'	α_2'	α_3'	y_1'	y_2'	y_3'	z_1'	z_2'
1	100	0	0	1.00	0.17	0.17	75.0	12.5	12.5	18.8	12.5
2	80	20	0	0.83	0.33	0.19	61.5	24.6	13.9	31.5	13.9
3	60	40	0	0.67	0.50	0.19	49.2	36.9	13.9	43.8	13.9
4	40	60	0	0.50	0.67	0.19	36.9	49.2	13.9	56.2	13.9
5	20	80	0	0.33	0.83	0.19	24.6	61.5	13.9	68.5	13.9
6	0	100	0	0.17	1.00	0.17	12.5	75.0	12.5	81.3	12.5

already shown in ref.[9]. In addition, open squares (FA) are outputs for fuzzy inputs of achromatic colors, and open circles (CA) are outputs for crisp inputs of achromatic colors. Circles indicate the corresponding coordinate for the input points in the NCS. Squares indicate the centers of tone triangles. That is, circles move toward square points using fuzzy inference.

The relationship between crisp inputs (centers of large circles), conical fuzzy inputs (large circles), top areas of 0.5 level-sets of each modifier (medium circles), and fuzzy inference outputs (circles and squares) on the NCS color triangle is examined.

Figure 7 shows a schematic illustration, which focuses on one light grayish area. The center of modifier "light gray" as the coordinate of conical fuzzy input moves to the area (middle circle) of neighboring "light grayish", because the vagueness of modifier is inferred. The center of modifier "light gray" as the coordinate of triangular fuzzy input moves to the *blackness* direction. The center of modifier "white" as the coordinate of conical fuzzy input moves to 75% whiteness direction. The center of modifier "black" as the coordinate of conical fuzzy input also moves to 75% blackness direction. The center of modifier "white" as the coordinate of triangular fuzzy input moves to the area of neighboring "light gray". The center of modifier "black" as the coordinate of triangular fuzzy input also moves to the area of neighboring "dark gray".

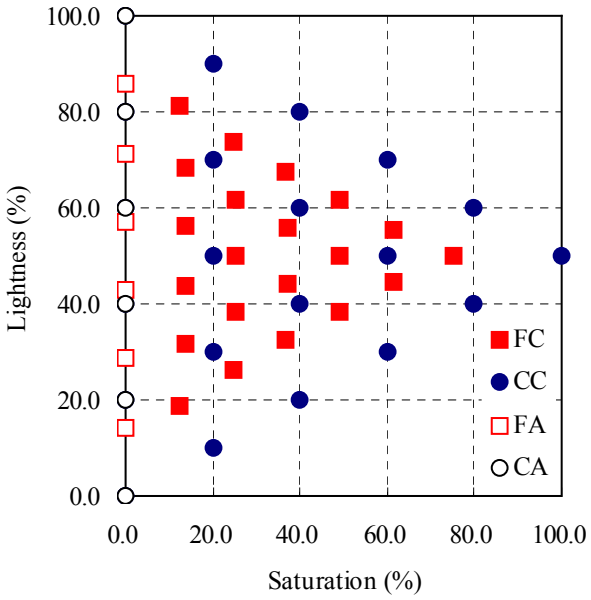

Fig.6. Fuzzy inference results for crisp (circle) and fuzzy (square) inputs on the HLS tone plane

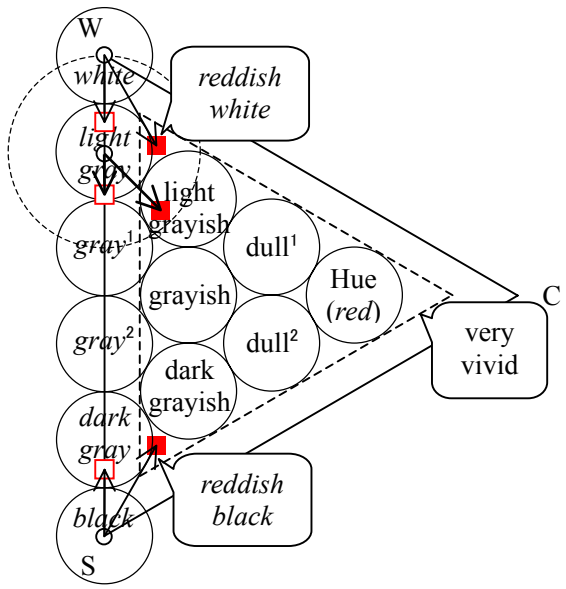

Fig.7. Schematic gathering effects of outputs for triangular and conical fuzzy input

IV. Conclusions

The present paper proposes a fuzzy system that, for a known hue name (e.g., red), can extract crisp outputs of the NCS (which is available for use in fuzzy set theory) and HLS (which is easy to show via graphs). It is difficult to construct such a fuzzy system on the tone plane in the HLS system, because the membership function of the lightness is quite complicated on the tone triangle. The system also extracts, in a simple manner, the membership grades from the projection of a triangular (or conical) membership function of a vague achromatic color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the NCS does not have a 'lightness' attribute.

In the near future, this system will help to ensure important color information (e.g. vagueness, shadings of color) of goods and arts by reducing the confusion between colors often experienced by people.

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References

[1] Japanese Industrial Standards Committee: *Names of non-luminous object colours JIS Z8102*, Japanese Standard Association, 1985, in Japanese.

[2] Japanese Industrial Standards Committee: *Colours specification: Specification according to their three attributes JIS Z 8721*, Japanese Standard Association, 1993, in Japanese.

[3] L., Sivik: *Color systems for cognitive research. In Color categories in thought and language*, C. L. Hardin and Luisa Maffi, Ed., Cambridge University Press, New York, pp.163-193, 1997.

[4] M. D., Fairchild: *Color appearance models*, Reading, Massachusetts, Addison Wesley Longman, Inc., 1998.

[5] N., Sugano: Color-naming system using fuzzy set theoretical method. *Proc. of the Fourth Asian Fuzzy Systems Symposium*, vol.1, pp.42-47, 2000.

[6] N., Sugano: Fuzzy color-naming system using conical membership function of tone modifier, *Biomedical Soft Computing and Human Sciences*, vol. 7, No. 1, pp. 47-52, 2001.

[7] N., Sugano: Color-naming system using fuzzy set theoretical approach. *Proc. of the 10th IEEE International Conference on Fuzzy Systems*, pp.13-16, 2001.

[8] N., Sugano: Evaluation and improvement of fuzzy color-naming system using neighboring JIS modifiers, *Biomedical Soft Computing and Human Sciences*, vol. 8, No. 1, pp.21-27, 2002

[9] N., Sugano: Natural color system using fuzzy set theoretical approach, *Proc. of the International Workshop on Fuzzy Systems & Innovational Computing*, pp.69-74, 2004