An Efficient Taboo Search Approach for Competitive Facility Location Problem

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***Abstract*— Facility location problem with considering competitiveness to other facilities has been studied in many literatures. In this study, we consider the model that a firm locates new facilities on a plane which has been already located several facilities. We propose an algorithm to solve the problem in cases that both new and existing facilities are plural. First we show that one of optimal solutions is found by a combination problem, and then we find approximate solution by applying taboo search to the problem. In order to illustrate the efficiency of our algorithm, results for numerical experiments are shown.**

I. Introduction

Competitive facility location problem (CFLP) is an optimal location problem for commercial facilities which have competitiveness with other facilities to customers. A decision maker in CFLP needs to locate her/his own facilities with considering location of other competitive facilities. CFLP has the following assumptions about users of service of facilities, called “customers”:

- each of customers exist at one of several points, defined as “demand points” (DP), and
- each of customers always uses only one facility whose attractive power to her/him is the maximum, and the locater of its facility obtains their buying power (BP).

In many studies of CFLP, an attractive power of a facility to a customer is estimated as the distance from the facility to her/him. Hakimi [3] assumed that there are two types of facilities: one is the type of facilities which has already located, and the other is that which will be located by the decision maker in the future. He considered CFLP located on a network connecting all DPs. Drezner [1] considered CFLP located on the plane including all DPs with the assumption of Hakimi.

On the other hand, Huff [4], [5] assume that an attractive power of a facility to a customer is estimated as a combination of quality level of facility and to the distance. Karkazis considered CFLP in the location model of Hakimi with estimating both quality level of facilities and distance to customers paralelly. Uno [8], [9] considered CFLP in the location model of Drezner with the assumption of Huff.

In this paper, we considered CFLP in the location model of Drezner with the assumption of Huff. In the above studies of Drezner and Uno, strictly solving method for their CFLP are suggested in cases that both of two types of facilities are only singular. First, we show that this CFLP can be reformulated as a combinatorial optimization problem to a set of a finite number of points in the plane. However, because this combinatorial optimization problem is NP-hard from the study of Hakimi [3], we need to construct an efficient algorithm to find an optimal solution for the problem. Taboo search, suggested by Glover [2], is one of valid approximate solving methods; for details, the reader can see the book of Reeves [7]. Secondly, we suggest applying taboo search to the combinatorial optimization problem. Results of numerical experiment for several examples of CFLP confirm the efficiency of taboo search.

The construction of this paper is as follows: In Section II, we formulate CFLP in the location model of Drezner with the assumption of Huff as a maximization problem of obtaining BP for the locater of future facilities. For the formulated problem, we show that this problem can be reformulated as a combinatorial optimization problem to a set of a finite number of points in the plane in Section III. In order to solve the combinatorial optimization problem, in Section IV, we introduce the concept of taboo search and apply the algorithm to the problem. Computational results are given in Section V, and finally we summarize conclusion and future studies of this study in Section VI.

II. Formulation of CFLP

Let k demand points (DPs) be given on the plane $\mathbf{R}^2$, and let $I \equiv \{1, \cdots, k\}$. With each DP $i \in I$, its site denoted by $\mathbf{u}_i \equiv (x_i^D, y_i^D) \in \mathbf{R}^2$ and buying power (BP) denoted by $w_i > 0$ are associated. Let $\mathbf{w} \equiv (w_1, \cdots, w_k)$.

Let m facilities be already located on the plane, and let $J_A \equiv \{-m+1, \cdots, 0\}$. With each facility $j \in J_A$, its site is denoted by $\mathbf{v}_j \equiv (x_i^A, y_i^A) \in \mathbf{R}^2$, and its qualitative value is denoted by $q_j > 0$.

In CFLP, we consider the location of n facilities on the plane. Let $J \equiv \{1, \cdots, n\}$. Similarly to the facilities having already located, with each of the new located facilities $j \in J$, its site is denoted by $\mathbf{v}_j \equiv (x_i^F, y_i^F) \in \mathbf{R}^2$. Let $\mathbf{v} \equiv (\mathbf{v}_1, \cdots, \mathbf{v}_n)$, this is a decision vector of CFLP. In this paper, we assume that qualitative values of the new located facilities are fixed and denoted by $q_1, \cdots, q_n$, where they hold that $q_1 \geq \cdots \geq$

$q_n > 1$. In order to deal with both of these two types of facilities together, we denote $\bar{J} \equiv J_A \cup J$.

We assume that customers on demand points use only one facility according to the following criterion. For DP $i \in \{1, \cdots, k\}$ and facility $j \in \bar{J}$, distance from $\mathbf{v}_i$ to $\mathbf{u}_j$ is represented as Euclid norm, denoted by $||\mathbf{v}_i - \mathbf{u}_j||$. We represent the attractive power of facility j to customers on DP i as the following definition suggested by Huff [4], [5]:

$$c_i(\mathbf{v}_j) \equiv \begin{cases} G\dfrac{q_j}{||\bar{\mathbf{v}}_j - \mathbf{u}_i||^2}, & \text{if } ||\bar{\mathbf{v}}_j - \mathbf{u}_i|| > \varepsilon, \\ G\dfrac{q_j}{\varepsilon^2}, & \text{if } ||\bar{\mathbf{v}}_j - \mathbf{u}_i|| \le \varepsilon. \end{cases} \quad (1)$$

Here, G is a constant value according to the type of service provided by facilities, and in this CFLP, the value of G is common to all facilities, and ε is a positive number meaning the upper of distance that all customers think as no troubles about movement to facilities.

With equation (1), the maximum attracting power in all facilities having already located and in all new facilities are represented as follows, respectively:

$$\bar{c}_i^A \equiv \max_{j \in J_A} c_i(\mathbf{v}_j) \quad (2)$$

$$\bar{c}_i(\mathbf{v}) \equiv \max_{j \in J} c_i(\mathbf{v}_j) \quad (3)$$

It is assumed that customers on DP i use one of new facilities if $\bar{c}_i(\mathbf{v}) > \bar{c}_i^A$ and then the locater of new facilities obtains BP w_i. Then, we denote the following 0-1 variable in order to represent which of type of facilities are used by customers on each DP:

$$\theta_i(\mathbf{v}) = \begin{cases} 1, & \text{if } \bar{c}_i(\mathbf{v}) > \bar{c}_i^A, \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Let $\theta(\mathbf{v}) \equiv (\theta_1(\mathbf{v}), \cdots, \theta_k(\mathbf{v}))$. Then, if the location of new facilities are given as $\mathbf{v}$, sum of BP that the locater of new facilities obtains is represented as follows:

$$f(\mathbf{v}_1, \cdots, \mathbf{v}_n) \equiv \theta(\mathbf{v}) \cdot \mathbf{w}^T. \quad (5)$$

Therefore, CFLP is formulated as the following maximizing problem for obtaining BP:

$$\begin{aligned} &\text{maximize} && f(\mathbf{v}_1, \cdots, \mathbf{v}_n) && (6) \\ &\text{subject to} && \mathbf{v}_j \in \mathbf{R}^2, \quad j = 1, \cdots, n && (7) \end{aligned}$$

Next, we introduce difficulty to solve the above CFLP with using some figures. We consider an example of CFLP for one new facility, where $(k, m, n) = (6, 2, 1)$ and sites of DPs and facilities having already located are given in Fig.1. In this example, we give the situations that all DPs have the same BP, denoted by $\bar{w}$, and that all facilities, including the new facility, have the same quality level. Then, from the latter situation, all customers at DPs use the nearest facility. In Fig.1, before locating new facility, facility -1 obtains $2\bar{w}$ from DP $1, 4$ and facility 0 obtains $4\bar{w}$ from DP $2, 3, 5, 6$.

One of optimal solutions of CFLP in this example is shown in Fig.2. In Fig.2, facility 1 obtains $4\bar{w}$ from DP $2, 3, 4, 6$.

To solve CFLP analytically is generally difficult because CFLP has the following characters:

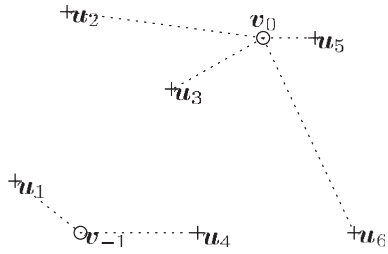

Fig. 1. Example of Competitive facility location problem

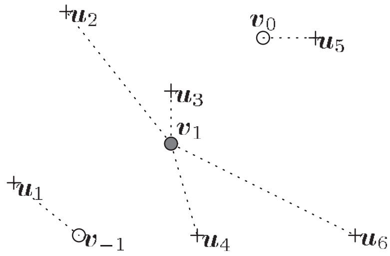

Fig. 2. Optimal location of new facility

- Objective value of CFLP increases or decreases discontinuously. In the above example, objective values can become $0, \bar{w}, \cdots, 4\bar{w}$.
- In general CFLP, region of optimal solution is very narrow because optimal locations are sites which are well-balanced near to many DPs . In Fig.2, if facility 1 is shifted with a few distance, facility 1 can only obtains no more than $3\bar{w}$.

These characters mean that it is difficult to apply general solving methods for non-linear programming problem, e.g. Steepest descent method, to use Kuhn-Tucker conditions, etc to CFLP. In the next section, we reformulate CFLP to a combinatorial optimization problem in order to solve CFLP more easily.

III. Reformulation of CFLP to a combinatorial optimization problem

For simplification of notations in the following part, let $\eta_i \equiv \sqrt{G/\bar{c}_i^A}$. From equation (1), a total set of DPs with customers who never use facility $j \in J$ for any facility location is represented as follows:

$$I_j^{\triangle} \equiv \{i \mid i = 1, \cdots, k, \ \sqrt{q_j}\eta_i \le \varepsilon\}. \quad (8)$$

Let $I_j = \{1, \cdots, k\} \backslash I_j^{\triangle}$. For facility $j \in J$ and subset $\bar{I} \subseteq I_j$, we consider the following maximization problem $(P_j(\bar{I}))$:

$$\begin{aligned} &\text{maximize} && r_j && (9) \\ &\text{subject to} && ||\mathbf{v}_j - \mathbf{u}_i|| \le \sqrt{q_j}\eta_i r_j, \\ & && \qquad \text{for } i \in \bar{I}, && (10) \\ & && \mathbf{v}_j \in \mathbf{R}^2, \ r_j \ge 0 && (11) \end{aligned}$$

Here, r_j is a variable introduced by the following theorem:

Theorem 1: Let $(\mathbf{v}_j^{\bar{I}}, r_j^{\bar{I}})$ denote an optimal solution of $(P_j(\bar{I}))$. Then, if $r_j^{\bar{I}} < 1$, the locater can obtain BP from all DPs in $\bar{I}$ by locating facility j on $\mathbf{v}_j^{\bar{I}}$.

Proof: Constraint inequality (10) is transformed as follows:

$$\bar{c}_i^A \leq G \frac{q_j}{||\mathbf{v}_j - \mathbf{u}_i||^2} \cdot \hat{r}_j, \text{ for } i \in \bar{I}. \tag{12}$$

From equation (1) and (4), the above relation means that the locater can obtain BP from DP i if $\mathbf{v}_j$ holds that $r_j < 1$. Therefore, the locater can obtain BP from all DPs in $\bar{I}$ by locating facility j on $\mathbf{v}_j^{\bar{I}}$. ■

For two DPs $i_1, i_2 \in I$, $LS(i_1, i_2)$ is denoted by the line segment whose edges are u_{i_1} and u_{i_2}. Then, the following definition is useful to find an optimal solution for problem $(P_j(\bar{I}))$ and $CFLP$:

Definition 1: It is assumed that point $p \in \mathbf{R}^2$ satisfies one of the following conditions:

(C1) $p = \mathbf{u}_{i_1}$ for facility $j \in J$ and DP $i_1 \in I_j$,

(C2) For facility $j \in J$ and two DPs $i_1, i_2 \in I_j$, p is an interior division point of $LS(i_1, i_2)$ whose ratio is that $\eta_{i_1} : \eta_{i_2}$.

(C3) For facility $j \in J$ and three DPs $i_1, i_2, i_3 \in I_j$, p is in the convex closure of $\mathbf{u}_{i_1}$, $\mathbf{u}_{i_2}$, and $\mathbf{u}_{i_3}$ which holds the following relation:

$$\frac{||p - \mathbf{u}_{i_1}||}{\eta_{i_1}} = \frac{||p - \mathbf{u}_{i_2}||}{\eta_{i_2}} = \frac{||p - \mathbf{u}_{i_3}||}{\eta_{i_3}}. \tag{13}$$

Then, p is called “a candidate point” (CP). Moreover, if candidate point p made by one of the above conditions holds that $\eta_{i_1} q_j < ||p - \mathbf{u}_{i_1}||^2$ for i_1 and j in this condition, p is “effective” for facility j.

The following lemma is obviously satisfied by the order of qualitative estimations of new facilities:

Lemma 1: If a CP is effective for facility $j > 1$, then the CP is also effective for facility $j - 1$.

The following theorem illustrates a relation between CPs and problem $(P_j(\bar{I}))$:

Theorem 2: Let $(\mathbf{v}_j^{\bar{I}}, r_j^{\bar{I}})$ be an optimal solution of $(P_j(\bar{I}))$. Then, for any facility $j \in J$ and subset $\bar{I} \subseteq I_j$, $\mathbf{v}_j^{\bar{I}}$ is the same point as one of CPs.

Proof: Clearly, problem $P_j(\bar{I})$ has at least one active constraint for an optimal solution of the problem. Now we divide the proof into the following three cases about the number of active constraints.

1) *The case is that there is one active constraint.* This only occurs in cases that $\bar{I}$ is singleton. Let i_1 be the sole element in $\bar{I}$. Then, it is obviously satisfied that $\hat{\mathbf{v}}_j^{\bar{I}} = \mathbf{u}_{i_1}$. This means that $\hat{\mathbf{v}}_j^{\bar{I}}$ is the same point as one of CPs (C1).

2) *The case of two active constraints.* Let $i_1, i_2 \in \bar{I}$ be two DPs whose constraints in (10) are active for $(\mathbf{v}_j^{\bar{I}}, r_j^{\bar{I}})$. Then, problem $P_j(\bar{I})$ can be reduced to the following problem:

$$\begin{aligned} \text{maximize} \quad & r_j && (14) \\ \text{subject to} \quad & \frac{||\mathbf{v}_j - \mathbf{u}_{i_1}||}{\eta_{i_1}} = \frac{||\mathbf{v}_j - \mathbf{u}_{i_2}||}{\eta_{i_2}} && (15) \\ & \mathbf{v}_j \in \mathbf{R}^2, \ r_j \geq 0 && (16) \end{aligned}$$

Clearly, an optimal location for the above problem is represented as follows:

$$\mathbf{v}_j^{\bar{I}} = \frac{\eta_{i_2} \mathbf{u}_{i_1} + \eta_{i_1} \mathbf{u}_{i_2}}{\eta_{i_1} + \eta_{i_2}} \tag{17}$$

This means that $\mathbf{v}_j^{\bar{I}}$ is the same point as one of CPs (C2).

3) *The case of more than three active constraints.* Let $i_1, i_2, i_3 \in \bar{I}$ be three DPs whose constraints in (10) are active for $(\mathbf{v}_j^{\bar{I}}, r_j^{\bar{I}})$. Then $\mathbf{v}_j^{\bar{I}}$ holds the following equation:

$$\frac{||\mathbf{v}_j^{\bar{I}} - \mathbf{u}_{i_1}||}{\eta_{i_1}} = \frac{||\mathbf{v}_j^{\bar{I}} - \mathbf{u}_{i_2}||}{\eta_{i_2}} = \frac{||\mathbf{v}_j^{\bar{I}} - \mathbf{u}_{i_3}||}{\eta_{i_3}} \tag{18}$$

From equation (18), we consider the following three cases about η_{i_1}, η_{i_2}, and η_{i_3}. (iiia) In cases that $\eta_{i_1} = \eta_{i_2} = \eta_{i_3}$, $\mathbf{v}_j^{\bar{I}}$ can be found as a circumcenter of $\triangle \mathbf{u}_{i_1} \mathbf{u}_{i_2} \mathbf{u}_{i_3}$. (iiib) In cases that η_{i_1}, η_{i_2}, and η_{i_3} are the same except one of them, we can discuss the case by regarding as $\eta_{i_1} \neq \eta_{i_2} = \eta_{i_3}$ without generality. Then, $\mathbf{v}_j^{\bar{I}}$ holds the following equation:

$$\left(\mathbf{v}_j^{\bar{I}} - \frac{\eta_{i_2} \mathbf{u}_{i_1} + \eta_{i_1} \mathbf{u}_{i_2}}{\eta_{i_1} + \eta_{i_2}} \right) \cdot \left(\mathbf{v}_j^{\bar{I}} - \frac{\eta_{i_2} \mathbf{u}_{i_1} - \eta_{i_1} \mathbf{u}_{i_2}}{\eta_{i_1} - \eta_{i_2}} \right) = 0 \tag{19}$$

From equation (19), $\mathbf{v}_j^{\bar{I}}$ is on a circumference of circle. Hence, $\mathbf{v}_j^{\bar{I}}$ can be found as the intersection point of the circle and a perpendicular bisector about $\mathbf{u}_{i_2}$ and $\mathbf{u}_{i_3}$. (iiic) In cases that all of η_{i_1}, η_{i_2}, and η_{i_3} are different, similarly to equation (19), a circle about i_2 and i_3 is found. Then we can find $\mathbf{v}_j^{\bar{I}}$ as an intersection point of these two circles. Therefore, we can find $\mathbf{v}_j^{\bar{I}}$ in the three cases (iiia), (iiib), and (iiic).

Next we show that $\mathbf{v}_j^{\bar{I}}$ is in a point of convex full of $\mathbf{u}_{i_1}$, $\mathbf{u}_{i_2}$, and $\mathbf{u}_{i_3}$. On the assumption that $\mathbf{v}_j^{\bar{I}}$ is not included on the convex full, $\mathbf{v}_j^{\bar{I}}$ is in the opposite side to one of the three DPs for the line connecting the other two DPs. Without generality, the latter two DPs can be regarded as i_1, i_2. Let $\mathbf{v}_j'$ be a interior division point on line segment $LS(i_1, i_2)$ whose rate is that $\eta_{i_1} : \eta_{i_2}$. Then, $\mathbf{v}_j'$ can be decrease the value of left side in constraint (10) about all of i_1, i_2, i_3. This contradicts either that $\mathbf{v}_j^{\bar{I}}$ is an optimal solution of problem $P_j(\bar{I})$ or that constraints in (10) about i_1, i_2, i_3 are active. Therefore, it is shown that $\mathbf{v}_j^{\bar{I}}$ is in a point of convex full of $\mathbf{u}_{i_1}$, $\mathbf{u}_{i_2}$, and $\mathbf{u}_{i_3}$.

This means that $\mathbf{v}_j^{\bar{I}}$ is the same point as one of CPs (C3).

Therefore, it is shown that for all of the above three cases, $\mathbf{v}_j^{\bar{I}}$ is the same point as one of CPs. ■

From Theorem 1 and 2, the following theorem about CFLP is shown:

Theorem 3: An optimal solution for CFLP is given by locating each facility on an effective CP for its facility.

Let S_j denote the set of effective CPs for facility j. Then, from Theorem 3, an optimal solution for CFLP can be found by solving the following problem combination problem (P_C):

$$\text{maximize}: \quad f(\mathbf{v}_1, \cdots, \mathbf{v}_n) \tag{20}$$

$$\text{subject to}: \quad \mathbf{v}_j \in S_j, \quad \forall j = 1, \cdots, n \tag{21}$$

The complexity of problem P_C is estimated by the following theorem:

Theorem 4: A complexity for problem P_C is bounded to $O(n^{3n})$.

Proof: From Definition 1, upper bounds of numbers of CPs (C1), (C2), and (C3) are ${}_nC_1$, ${}_nC_2$, ${}_nC_3$, respectively. Then, sum of number of all CPs are bounded to $O(n^3)$. As the number of locating facilities are n, a complexity for problem P_C is bounded to $O(n^{3n})$. ■

From Theorem 4, in cases that the number of locating facilities is large, to find strict optimal solution for problem P_C need enormous computational time and cost. In the next section, we propose an algorithm in order to find an approximate optimal solution for problem P_C by a reasonable time and cost.

IV. Application of taboo search algorithm for CFLP

In this section, we introduce taboo search algorithm and apply the algorithm to problem (P_C). For details of taboo search, the reader can reed the literature of Reeves [7].

Taboo search is regarded as one of local search methods. Let R denote a parameter to represent neighborhood of solution $\mathbf{v} \in S_j^n$. We define "move" for each solution about taboo search algorithm as the following two types:

(a) to transfer one facility located on an effective CP to another effective CP whose distance from the CP is less than R,

(b) to exchange locations of two facilities such that the distance between their locating CPs is less than R and both facilities are located on CPs effective for them.

Moreover, we define "neighborhood" of solution $\mathbf{v} \in \bigcup_{j=1}^n S_j$ as a set of all solutions which can transfer by only one move from $\mathbf{v}$. Let $N(\mathbf{v}) \subset \bigcup_{j=1}^n S_j$ be neighborhood of $\mathbf{v}$. At one local search from now solution in taboo search algorithm, neighborhood of its solution are searched and its solution generally transfer to the best solution in the neighborhood.

The above "move" is decomposed into several attributes. In this section, we distinguish between attributes about before transfer of now searching solution and after that, and define these attribute as "from attribute" and "to attribute", respectively. For example, about two effective CPs $p_1, p_2 \in S_{j_1} \cap S_{j_2}$, if a move from $\mathbf{v}$ is type No.1 and $\mathbf{v}_{j_1} = p_1 \to p_2$, from attribute of this move is $\mathbf{v}_{j_1} = p_1$ and to attribute of that is $\mathbf{v}_{j_1} = p_2$. If its move is type No.2 and $\mathbf{v}_{j_1} = p_1 \to p_2$, $\mathbf{v}_{j_2} = p_2 \to p_1$, from attribute of this move is $\mathbf{v}_{j_1} = p_1, \mathbf{v}_{j_2} = p_2$ and to attribute of that is $\mathbf{v}_{j_1} = p_2, \mathbf{v}_{j_2} = p_1$.

Objective function of problem P_C has many local optimal solutions generally. In taboo search algorithm, in order to search various local optimal solutions without concentration of search for a local optimal solution, history of moves chosen in a past of taboo search are recorded. Then, constraints whose aim is not to choose backward moves for moves in this history, called "taboo constraints", can be used. Taboo constraints are divided into the following two types.

(i) Taboo constraints about recency An aim of this type of taboo constraint is to prevent short-term circulation of solutions chosen at taboo search. In cases that a move is chosen, taboo constraints for from or to attribute in the move are activated in a given term in taboo search. If a taboo constraint for from or to attribute is active, type No.1 of moves including its attribute are taboo that is, not chosen in the term even if one of such moves makes its value of objective function of problem P_C better than that for all solutions in neighborhood. For example, move $\mathbf{v}_{j_1} = p_1 \to p_2$ was chosen in a past of taboo search, in the term such moves whose from attribute is $\mathbf{v}_{j_1} = p_2$ and whose to attribute is $\mathbf{v}_{j_1} = p_1$ are not chosen. About type No.2 of moves, such a taboo constraint is applied if both from and to attribute in their moves are active. Let $T_{from}, T_{to} > 0$ denote terms that such a taboo constraint is active for from and to attribute, respectively. From $k > n$ in general CFLP, number of moves prevented by taboo constraint of activation about from attribute is more than that of to attribute. Then we set $T_{from} < T_{to}$ because of prevention for making neighborhood small too far. In using the above taboo constraints, the terms about activation for each attribute are needed to memorize, called "recency memory".

(ii) Taboo constraints about frequency An aim of this type of taboo constraint is to prevent long-term circulation of solutions chosen at taboo search. In using such taboo constraints, the following penalty functions are defined for from and to attribute.

Let $y_{from}(\mathbf{v}_{j_1} = p_1)$, $y_{to}(\mathbf{v}_{j_1} = p_2)$ denote frequencies of from attribute $\mathbf{v}_j = p_1$ and to attribute $\mathbf{v}_j = p_2$ included in all past chosen moves, respectively. Then, we define penalty functions about from and to attribute for move $\mathbf{v}_j = p_1 \to p_2$ as $g_{from}(y_{from}(\mathbf{v}_{j_1} = p_1))$ and $g_{to}(y_{to}(\mathbf{v}_{j_1} = p_2))$, respectively, where functions g_{from}, g_{to} are non-increasing for frequency of each attribute. We represent an estimate function as the sum of objective function for problem P_C and these penalty functions. Then, in taboo search algorithm, a now solution is transferred to the solution which maximizes the estimate function in all solutions transferred by moves non-taboo by their including active attributes. In using the above taboo constraints, frequency for each attribute included in all past chosen moves are needed to memorize, called "frequency memory".

As the end of this section, we summarize a procedure of taboo search algorithm for problem P_C. Let x_{best} be temporary optimal solution in this algorithm. Let x_{now} be now solution in this algorithm, and x_{next} be next solution which is chosen in $N(x_{now})$.

Algorithm (Taboo search)

1) Based on Definition 1, find S_j for any $j \in J$. Set x_{now} by locating each facility on one of its candidate points randomly, and $x_{best} \leftarrow x_{now}$.
2) Find x_{next} which maximizes the estimate function in all solution transferred by moves non-taboo by their including active attributes.
3) If $f(x_{next}) > f(x_{best})$, then set $x_{best} \leftarrow x_{next}$.
4) If an established terminal condition for taboo search algorithm is satisfied, algorithm is terminated. Obtained approximate optimal solution is x_{best}. Otherwise, update recency memory and frequency memory for the chosen move at the current Step 1, set $x_{now} \leftarrow x_{next}$, and return to Step 1.

V. NUMERICAL EXPERIMENTS

In this section, we apply taboo search algorithm to some examples of CFLP and verify its efficiency. For DPs, we set their sites to $\mathbf{u}_i \in [0,1] \times [0,1]$ randomly, and their BP to $w_i \in \{1, \cdots, 10\}$ randomly. For competitive facilities located already, we also set their sites to points in $[0,1] \times [0,1]$ randomly, and their quality value to $\{3, \cdots, 12\}$ randomly. For new facilities located by the decision maker of CFLP, we set their quality value to $\{1, \cdots, 10\}$ randomly and sort them in order of size.

Next we give parameters about taboo search algorithm. For activated terms, we set $T_{from} = n/2$ and $T_{to} = |S_1|/10$. For penalty functions about frequency memory, we represent $g_{from}(z) = g_{to}(z) = z$. We establish that terminal condition, mentioned at the previous section, is true only if x_{best} does not update more than 100 times at the loop of Step 1 to 3 in the above algorithm.

In this paper, we give five examples of CFLP setting $(k, m, n) = (20, 7, 3)$, $(40, 10, 5)$, $(60, 13, 7)$, $(80, 18, 8)$, $(100, 20, 10)$, and for each example, we set $R = 0.1, \cdots, 0.8$ and $R \to \infty$. Results of implementation for taboo search algorithm at 20 times is given in TABLE I to V. Here, CPU times in these tables mean computational times for all implementations of taboo search algorithm with using DELL Optiplex GX260 (CPU: 2.33 GHz, RAM: 512MB).

TABLE I

TABOO SEARCH FOR $(k, m, n) = (20, 7, 3)$

R	CPU time(s)	Best value	Mean value	Worst value
0.1	0.05	77.00	53.35	33.00
0.2	-	77.00	61.00	54.00
0.3	0.05	77.00	74.20	67.00
0.4	0.10	77.00	75.40	73.00
0.5	0.10	77.00	77.00	77.00
0.6	0.10	77.00	77.00	77.00
0.7	0.15	77.00	77.00	77.00
0.8	0.15	77.00	77.00	77.00
∞	0.15	77.00	77.00	77.00

TABLE II

TABOO SEARCH FOR $(k, m, n) = (40, 10, 5)$

R	CPU time(s)	Best value	Mean value	Worst value
0.1	0.75	140.00	120.25	80.00
0.2	1.05	148.00	131.70	102.00
0.3	1.85	148.00	144.75	135.00
0.4	2.25	148.00	144.70	140.00
0.5	3.35	148.00	147.50	140.00
0.6	4.00	148.00	148.00	148.00
0.7	3.90	148.00	147.60	140.00
0.8	4.20	148.00	148.00	148.00
∞	5.25	148.00	148.00	148.00

First, we verify computational time for taboo search algorithm. The numbers of CPs for examples of CFLP are computed 94, 430, 1041, 1662, and 2430 in order of TABLE. CPU time for computation of all CPs is less than 1 second even by example of $(k, m, n) = (100, 20, 10)$. This means that most of CPU times is due to loops of Step 1 to 3 in the above taboo search algorithm. Such CPU time is estimated by product of the following three elements:

- number of facilities located by the decision maker of CFLP,
- mean value to numbers of move for each facility, and
- number of such loops going through.

The third element is dependent on both numbers of facilities and CPs, so in cases that these numbers are larger than these examples, we need to have terminal conditions based upon upper of CPU time. For some examples, it is shown such cases that some initial points chosen randomly at Step 0 require CPU time which is more than two times as large as mean CPU time. There is not tendency that taboo search from such initial points can be found better solution. This means necessity that various initial points are used in taboo search algorithm. Moreover, taboo search is known as one of the methods which can be applied parallel computing [7]. We think that to taboo search algorithm to CFLP, parallel computing for several initial points also makes more efficient algorithm possible.

Secondly, we verify accuracy for taboo search algorithm. For the above five examples, all best solutions for $R \to \infty$ in TABLE I to V are consistent with strictly optimal solutions found by enumeration of all feasible solutions. Moreover, $R = 0.4$ in TABLE I, II, IV, $R = 0.5$ in TABLE III, and $R = 0.5$ in TABLE V, can obtain sufficiently good objective values. We think that by setting $0.4 \le R \le 0.6$, sufficiently wide neighborhood about taboo search algorithm can be obtained. Moreover, setting $0.4 \le R \le 0.6$ has the advantage of CPU time for another value of $R > 0.6$. These mean that taboo search algorithm by setting $0.4 \le R \le 0.6$ can be expected for finding an accurate solution for problem P_C by a limited CPU time.

TABLE III

TABOO SEARCH FOR $(k, m, n) = (60, 13, 7)$

R	CPU time(s)	Best value	Mean value	Worst value
0.1	4.05	210.00	183.70	155.00
0.2	8.50	215.00	204.45	177.00
0.3	14.60	224.00	214.70	204.00
0.4	21.90	224.00	216.80	201.00
0.5	23.95	224.00	220.35	215.00
0.6	27.60	221.00	218.25	212.00
0.7	36.25	224.00	217.05	203.00
0.8	42.75	224.00	218.95	213.00
∞	40.95	224.00	218.35	213.00

TABLE IV

TABOO SEARCH FOR $(k, m, n) = (80, 18, 8)$

R	CPU time(s)	Best value	Mean value	Worst value
0.1	10.85	242.00	196.85	159.00
0.2	24.60	252.00	215.65	202.00
0.3	36.40	245.00	233.05	219.00
0.4	59.45	249.00	237.75	223.00
0.5	64.70	252.00	241.30	232.00
0.6	78.95	252.00	241.85	233.00
0.7	85.15	249.00	240.95	232.00
0.8	94.65	250.00	241.35	235.00
∞	117.05	252.00	242.65	234.00

VI. CONCLUSION

In this paper, we have considered a location model of several facilities in an environment that there are competitive facilities whose quality levels vary. For a formulated optimal location problem, we proposed that

- we define ”candidate point” in order to reformulate a continuous optimization location problem to a discrete optimization location problem, and
- for the discrete optimization location problem, we suggest applying taboo search algorithm to the problem and show its efficiency by solving several examples of location problem.

In the future of this study, we think that CFLP can be applied in the case of various decisions making in competitive environments. In cases that strategies in Game’s theory can be regarded as location on $\mathbf{R}^2$, we can use the above solving method to a problem formulated as CFLP. However, in such cases except facility location, feasible set for problems are often represented as that of multi-dimensional. Construction of solving algorithm to CFLP on multi-dimensional space is an interesting future study.

TABLE V

TABOO SEARCH FOR $(k, m, n) = (100, 20, 10)$

R	CPU time(s)	Best value	Mean value	Worst value
0.1	28.95	293.00	265.75	206.00
0.2	66.20	314.00	292.50	279.00
0.3	100.05	310.00	300.90	287.00
0.4	126.90	320.00	308.80	294.00
0.5	160.75	317.00	310.00	305.00
0.6	207.10	320.00	314.20	304.00
0.7	243.15	318.00	314.20	302.00
0.8	246.40	320.00	314.50	304.00
∞	283.20	320.00	315.25	304.00

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