Interactive fuzzy programming using partial information about preference through genetic algorithms for multiobjective two-level integer programming problems.

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Abstract —**In this paper, focusing on multiobjective two-level integer programming problems where two decision makers with integer decision variables have multiple objective functions, we present interactive fuzzy programming for them using partial information about preference. In the proposed method, after introducing the fuzzy goals of objective functions for the decision makers, a solution, which minimizes the difference between the membership function values and the reference membership levels for the decision maker at the lower level under the condition that the membership function values are greater than or equal to the minimal satisfactory levels for the decision maker at the upper level, is obtained by genetic algorithms. After evaluating the aggregated satisfactory degree for each of the decision makers through the partial information about preference, if the ratio of the aggregated satisfactory degree for the decision maker at the lower level to that for the decision maker at the upper level satisfies a certain condition and the decision maker at the upper level is content with the solution, a satisfactory solution is obtained. Otherwise, the interactive procedures with updating the minimal satisfactory levels and the reference membership levels is going to be continued until a satisfactory solution is found.**

I. Introduction

In a two-level programming problem, there exist a leading decision maker (DM) at the upper level and a subordinate DM at the lower level, and each of them with individual decision variables attempts to optimize his/her own objective function under the effect of the decision making of the other DM. For a case that the DM at the upper level has priority over the DM at the lower level and neither of them has motivation for cooperation, a solution such that the DM at the upper level optimizes his/her own objective function under the assumption that the DM at the lower level makes a decision to optimize his/her own objective function for any given decision of the DM at the upper level, called a Stackelberg solution, has been already considered [24]. However, the Stackelberg solution under the competitive situation seems impractical for a case where the DM at the upper level is a supervisory administration and the DM at the lower level is a subordinate division in a corporation since both the DMs are considered to be intrinsically cooperative. Lai [7] and Shih et al. [23] proposed fuzzy programming for multi-level linear programming problems under the assumption of the cooperative relationship between the DMs, Sakawa et al. [19, 20] developed interactive fuzzy programming for various multi-level programming problems.

On the other hand, in order to take the diversification of social requirements or various values of the DM, theoretical, methodological or applied researches have been developed for multiobjective programming problems which involve multiple objective functions conflicting with each other in ordinary single-level programming problems [3, 13, 25, 26]. Based on these researches, the authors proposed interactive fuzzy programming using partial information about preference for multiobjective two-level linear/linear fractional programming problems [21, 22], and the extension of the method for multiobjective two-level discrete programming problems has been expected.

Genetic algorithms (GAs) were proposed by Holland [6], as a new learning paradigm that models a natural evolution mechanism. GAs were not much known before Goldberg's book [5] had been published; however, many researchers in various fields have recently attracted much attention to GAs as a methodology for optimization, adaptation and learning [9, 18]. Anandalingam, Mathieu, Pittard and Sinha [1] present a genetic algorithm, which employs a representation of individuals in GAs by not zero-one bit strings but astring of base-10 digits, for obtaining the Stackelberg solution to a two-level linear programming problem. Niwa, Nishizaki and Sakawa [10, 11] propose computational methods using GAs for obtaining Stackelberg solution and Stackelberg-Nash solutions to decentralized two-level zero-one programming problem.

Under these circumstances, in this paper, we focus on multiobjective two-level integer programming problems where there exist two cooperative decision makers with integer decision variables and multiple objective functions. After introducing fuzzy goals to represent ambiguous or fuzzy judgments of the DMs on objective functions, we propose interactive fuzzy programming for them using partial information about preference where a satisfactory solution for the

DMs can be obtained by interactions considering the partial information about the DMs' preferences. In the proposed method, we must solve integer programming problems repeatedly. Because the usage of an enumeration-based method such as the branch-and-bound method requires an enormous computation time for large-scale problems, we try to solve them effectively by using the genetic algorithms with double strings based on reference solution updating [14, 15, 17]. Furthermore, through the application to a numerical example, we show the efficiency of the proposed interactive fuzzy programming through genetic algorithms.

II. Mutliobjective two-level integer programming problem

In this paper, we deal with a two-level integer programming problem in which the decision maker at the upper level (DM1) has k_1 objective functions, the decision maker at the lower level (DM2) has k_2 objective functions, and the DM1 and the DM2 can determine their decisions cooperatively. Such a multiobjective two-level integer programming problem is formulated as follows:

$$\left.\begin{array}{ll}
\underset{\text{upper level}}{\text{minimize}} & z_1^1(\mathbf{x}) = \mathbf{c}_{11}^1\mathbf{x}_1 + \mathbf{c}_{12}^1\mathbf{x}_2 \\
 & \vdots \\
\underset{\text{upper level}}{\text{minimize}} & z_{k_1}^1(\mathbf{x}) = \mathbf{c}_{k_1 1}^1\mathbf{x}_1 + \mathbf{c}_{k_1 2}^1\mathbf{x}_2 \\
\underset{\text{lower level}}{\text{minimize}} & z_1^2(\mathbf{x}) = \mathbf{c}_{11}^2\mathbf{x}_1 + \mathbf{c}_{12}^2\mathbf{x}_2 \\
 & \vdots \\
\underset{\text{lower level}}{\text{minimize}} & z_{k_2}^2(\mathbf{x}) = \mathbf{c}_{k_2 1}^2\mathbf{x} + \mathbf{c}_{k_2 2}^2\mathbf{x}_2 \\
\text{subject to} & A_1\mathbf{x}_1 + A_2\mathbf{x}_2 \leq \mathbf{b} \\
 & x_{1p} \in \{0,\ldots,\nu_p\}; p = 1,\ldots,n_1 \\
 & x_{2q} \in \{0,\ldots,\nu_q\}; q = 1,\ldots,n_2
\end{array}\right\} \quad (1)$$

where $\mathbf{x}_l = (x_{l1},\ldots,x_{ln_l})^T, l = 1,2$ are the decision variable vectors of DMl, $l = 1,2$; $z_i^l(\mathbf{x})$ are the ith objective function of the DMl; $\mathbf{c}_{i1}^l$, $\mathbf{c}_{i2}^l$, $l = 1,2, i = 1,\ldots,k_l$ are n_l-dimensional constant row vectors of DMl, $A = (A_1,A_2)$ is an $m\times(n_1+n_2)$ coefficient matrix of constraint and $\mathbf{b}$ is an m-dimensional constant column vector. In the following, we denote the feasible region satisfying all of the constraints of problem (1) by X, and $\mathbf{x} = (\mathbf{x}_1^T,\mathbf{x}_2^T)^T$, $\mathbf{c}_i^l = (\mathbf{c}_{i1}^l,\mathbf{c}_{i2}^l)$, $n = n_1+n_2$.

III. Introduction of fuzzy goals and evaluation of solution based on preference

For multiobjective two-level integer programming problem (1), in order to consider the imprecise nature of the DM's judgment for each self minimization objective function, a fuzzy goal such as "the objective function should be substantially less than or equal to some value" (called fuzzy min), "the objective function should be substantially greater than or equal to some value" (called fuzzy max) and "the objective function should be in the vicinity of some value" (called fuzzy equal) are elicited [13, 16]. Problem (1) is represented as follows:

$$\left.\begin{array}{ll}
\underset{\text{upper level}}{\text{fuzzy min}} & z_i^1(\mathbf{x}) \in I_1^1 \\
\underset{\text{upper level}}{\text{fuzzy max}} & z_i^1(\mathbf{x}) \in I_2^1 \\
\underset{\text{upper level}}{\text{fuzzy equal}} & z_i^1(\mathbf{x}) \in I_3^1 \\
\underset{\text{lower level}}{\text{fuzzy min}} & z_i^2(\mathbf{x}) \in I_1^2 \\
\underset{\text{lower level}}{\text{fuzzy max}} & z_i^2(\mathbf{x}) \in I_2^2 \\
\underset{\text{lower level}}{\text{fuzzy equal}} & z_i^2(\mathbf{x}) \in I_3^2 \\
\text{subject to} & \mathbf{x} \in X
\end{array}\right\} \quad (2)$$

where, $I_1^l \cup I_2^l \cup I_3^l = \{1,2,\ldots,k_l\}, l = 1,2$ and $I_i^l \cap I_j^l = \emptyset, i, j = 1,2,3, i \neq j, l = 1,2$. Problem (2) can be quantified by eliciting a corresponding membership function $\mu_i^l(z_i^l(\mathbf{x}))$ which is a strictly monotone decreasing function with respect to $z_i^l(\mathbf{x})$. By using membership function $\mu_i^l(z_i^l(\mathbf{x}))$, $i = 1,2$, problem (2) is shown as follows:

$$\left.\begin{array}{ll}
\underset{\text{upper level}}{\text{maximize}} & \mu_1^1(z_1^1(\mathbf{x})) \\
 & \vdots \\
\underset{\text{upper level}}{\text{maximize}} & \mu_{k1}^1(z_{k1}^1(\mathbf{x})) \\
\underset{\text{lower level}}{\text{maximize}} & \mu_1^2(z_1^2(\mathbf{x})) \\
 & \vdots \\
\underset{\text{lower level}}{\text{maximize}} & \mu_{k2}^2(z_{k2}^2(\mathbf{x})) \\
\text{subject to} & \mathbf{x} \in X.
\end{array}\right\} \quad (3)$$

Then the linear membership functions $\mu_i^l(z_i^l(\mathbf{x})), i = 1,\ldots,k$ for the fuzzy goals of the DMl are defined by

$$\mu_i^l(z_i^l(\mathbf{x})) = \begin{cases} 1, & z_i^l(\mathbf{x}) \leq \underline{z}_i^l \\ \dfrac{z_i^l(\mathbf{x}) - \overline{z}_i^l}{\underline{z}_i^l - \overline{z}_i^l}, & \underline{z}_i^l < z_i^l(\mathbf{x}) \leq \overline{z}_i^l \\ 0, & z_i^l(\mathbf{x}) > \overline{z}_i^l. \end{cases} \quad (4)$$

where $\overline{z}_i^l$ denotes a maximum value of unacceptable levels for $z_i^l(\mathbf{x})$ and $\underline{z}_i^l$ denotes a minimum value of unacceptable levels for $z_i^l(\mathbf{x})$, respectively.

We assume that each DM evaluates a solution by taking the weighted sum of all the membership functions with which fuzzy goal for each objective function is prescribed. In other words, a weight coefficient to membership function $\mu_i^l(z_i^l(\mathbf{x}))$ of the ith fuzzy goal of the DMl is assumed as $\lambda_i^l, l = 1,2$. The weighted coefficient vector λ^l which reflected on the preference to the fuzzy goals of the DM1 and the DM2 is denoted as

$$\begin{array}{rcl}
\lambda^l & = & (\lambda_1^l,\ldots,\lambda_{k_l}^l) \\
 & \in & \{\lambda^l \in R^{k_l} \mid \sum_{i=1}^{k_l}\lambda_i^l = 1, \lambda_i^l \geq 0, i = 1,\ldots,k_l\}
\end{array} \quad (5)$$

Then, the DMl evaluates an obtained solution by using the aggregated membership function with the weight coefficient vector.

$$\sum_{i=1}^{k_l}\lambda_i^l\mu_i^l(z_i^l(\mathbf{x})), i = 1,2. \quad (6)$$

Generally, because it is difficult to fix suitably a weight coefficient vector in the weighted sum of the fuzzy goals at a certain value appropriately, in this paper, we assume the weighted coefficient vector that reflects on the preferences of the DMs is given as a set. Especially, we suppose that the DM1 and the DM2 possess two types of such partial information on preference [8, 12] as follows:

- The upper bound UB_i^l and the lower bound LB_i^l are specified for the weight λ_i^l to the fuzzy goal for the ith objective function as follows:

$$LB_i^l \leq \lambda_i^l \leq UB_i^l \tag{7}$$

- An order relation between ith fuzzy goal and kth one $(i \neq k)$ is specified as follows:

$$\lambda_i^l \geq \lambda_k^l + \varepsilon, i \neq k \tag{8}$$

where ε is a small positive constant or zero.

Let $\Lambda^l, l = 1,2$ denote sets of weighted coefficient vectors with the partial information on the preference of the DMl satisfying conditions (7) and (8) as well as condition (5). Each weighted coefficient is restricted in a limited range by condition (7), and condition (8) gives the importance of two fuzzy goals an order. The DMs express the partial information on preference and evaluate the solution through aggregated fuzzy goals by combining these.

IV. Interactive fuzzy programming problem

On the basis of the introduction of fuzzy goals to the multiobjective two-level integer programming problem and the evaluation method of a solution by using the partial information about the DMs' preferences, in this section, we present an interactive decision making method to derive a satisfactory solution by repeatedly solving integer programming problems to obtain a solution such that $\mu_j^2(z_i^2(\mathbf{x})), j = 1,\ldots,k_2$ become closest in the minimax sense to reference membership levels $\overline{\mu}_j^2, j = 1,\ldots,k_2$ which represent aspiration levels for fuzzy goals of the DM2, under the condition that each of $\mu_j^1(z_i^1(\mathbf{x}))$ exceeds the corresponding minimal satisfactory level $\hat{\delta}_i^1$ specified by the DM1.

First, we solve the following maximin problem so as to obtain the information to specify the initial minimal satisfactory level $\hat{\delta}_i^1, i = 1,\ldots,k_1$ and the initial reference membership levels $\overline{\mu}_j^2, j = 1,\ldots,k_2$.

$$\underset{\mathbf{x}\in X}{\text{maximize}} \quad \min_{\substack{l=1,2\\ i=1,\ldots,k_l}} \mu_i^l(z_i^l(\mathbf{x})) \tag{9}$$

Next, a solution such that $\mu_j^2(z_j^2(\mathbf{x})), j = 1,\ldots,k_2$ become closest in the minimax sense to reference membership levels $\overline{\mu}_j^2, j = 1,\ldots,k_2$ under the condition that $\mu_i^1(z_i^1(\mathbf{x}))$ exceed $\hat{\delta}_i^1, i = 1,\ldots,k_1$ is obtained by solving the following minimax problem:

$$\left.\begin{array}{ll} \text{minimize} & \max\limits_{i=1,\ldots,k_2} \{\overline{\mu}_i^2 - \mu_i^2(z_i^2(\mathbf{x}))\} \\ \text{subject to} & \mu_i^1(z_i^1(\mathbf{x})) \geq \hat{\delta}_i^1, i = 1,\ldots,k_1 \\ & \mathbf{x} \in X \end{array}\right\} \tag{10}$$

Then, if neccessary, the reference membership levels are updated according to the preference of the DM2. Furthermore, the solution to problem (10) is evaluated by using the weighted sum of the membership function values in consideration of the DMs to check if $\mathbf{x}^*$ is a satisfactory solution. To be more specific, after calculating membership function values $\mu_j^1(z_j^1(\mathbf{x})), j = 1,\ldots,k_1$ and $\mu_j^2(z_j^2(\mathbf{x}^*)), j = 1,\ldots,k_2$ for $\mathbf{x}^*$, the following four linear programming problems are solved for Λ^l which is a set representing the partial information about the DMl's preference.

$$\underset{\lambda^1\in\Lambda^1}{\text{minimize}} \quad \sum_{i=1}^{k_1} \lambda_i^1 \mu_i^1(z_i^1(\mathbf{x}^*)) \tag{11}$$

$$\underset{\lambda^1\in\Lambda^1}{\text{maximize}} \quad \sum_{i=1}^{k_1} \lambda_i^1 \mu_i^1(z_i^1(\mathbf{x}^*)) \tag{12}$$

$$\underset{\lambda^2\in\Lambda^2}{\text{minimize}} \quad \sum_{i=1}^{k_2} \lambda_i^2 \mu_i^2(z_i^2(\mathbf{x}^*)) \tag{13}$$

$$\underset{\lambda^2\in\Lambda^2}{\text{maximize}} \quad \sum_{i=1}^{k_2} \lambda_i^2 \mu_i^2(z_i^2(\mathbf{x}^*)) \tag{14}$$

In those problems, the optimal objective function values $S_{\min}^1$ and $S_{\max}^1$ to problem (11) and problem (12) represented as the lower bound and the upper bound of the aggregated satisfactory degree of the fuzzy goals for the DM1, while the optimal objective function values $S_{\min}^2$ and $S_{\max}^2$ to problem (13) and problem (14) represented as the lower bound and the upper bound of the aggregated satisfactory degree of fuzzy goals for the DM2. We define s^l and σ^l as follows:

$$\begin{aligned} s^l &= \frac{S_{\min}^l + S_{\max}^l}{2}, \\ \sigma^l &= \frac{S_{\max}^l - S_{\min}^l}{2}. \end{aligned}$$

Then, the satisfactory degree of the aggregated fuzzy goals for the DMl is expressed by following L-L fuzzy number [3],

$$\tilde{S}^l = (s^l, \sigma^l)_{LL}$$

.

In this paper, we suppose that the characteristic function of the L-L fuzzy number is a linear function.

In order to take account of the overall satisfactory balance between both levels, we define the ratio of satisfactory degrees between the DM1 and the DM2 as a fuzzy numbers $\tilde{\Delta}$ [4]:

$$\tilde{\Delta} = \tilde{S}^2 \oslash \tilde{S}^1 \simeq \left(\frac{s^2}{s^1}, \frac{\sigma^2 s^1 + \sigma^1 s^2}{s^1 \cdot s^1}\right)_{LL} \tag{15}$$

where $\oslash$ means division of L-L fuzzy numbers.

The DM1 is guaranteed to have satisfactory degrees larger than or equal to the minimal satisfactory levels for all of the fuzzy goals because the corresponding constraints are involved in problem (10). If we assume that the DM1 takes into account not only the satisfactory degree of aggregated fuzzy goals for the DM1 but also the balance between the satisfactory degree of aggregated fuzzy goals for the DM1 and that for the DM2, the DM1 is supposed to have a fuzzy goal $G_{\tilde{\Delta}}$ such as "the ratio of satisfactory degrees $\tilde{\Delta}$ should be in the vicinity of m". This fuzzy goal is set to that the membership function takes the maximum value 1 in a certain value m, and follows in deviating from m, and value decreases for adjust the ratio of satisfaction of the DM1 and the DM2. To take into account the overall satisfactory balance between both levels as well as the satisfactory degrees of self, it is assumed that the DM1 has a fuzzy goal $\tilde{G}$ for the ratio $\tilde{\Delta}$ of satisfactory degrees.

$$\alpha = \max_{p} \min\{\mu_{\tilde{\Delta}}(p), \mu_{G_{\tilde{\Delta}}}\} \geq \delta_{G_{\tilde{\Delta}}} \tag{16}$$

An interactive fuzzy programming problem for multiobjective two-level integer programming problems stated above is summarized as follows:

Algorithm of the interactive fuzzy programming

Step 0 Formulate a multiobjective two-level integer programming problem. Ask the two DMs about partial information of their preference

Step 1 The DM1 and the DM2 decide membership function with which fuzzy goal for each objective function is prescribed with subjectively referring to these values after the individual minimum value and maximum value

$$\underset{\mathbf{x}\in X}{\text{minimize}} \quad z_i^l(\mathbf{x}), l = 1, 2, i = 1, \ldots, k_l \tag{17}$$

$$\underset{\mathbf{x}\in X}{\text{maximize}} \quad z_i^l(\mathbf{x}), l = 1, 2, i = 1, \ldots, k_l \tag{18}$$

of each objective function of multiobjective two-level integer programming problems was obtained from the genetic algorithms based on reference solution updating which proposed by Sakawa et al. [14, 15, 17].

Step 2 By using genetic algorithms based on reference solution updating, problem (9) is solved. Referring to membership function value of fuzzy goal for the optimum solution which be obtained, the DM1 specifies the membership function $\mu_{G_{\tilde{\Delta}}}(\cdot)$ which is prescribed for the fuzzy goal for the ratio of satisfactory degrees from the DM1, the permissible level $\hat{\delta}_{G_{\tilde{\Delta}}}$ and the minimal satisfactory levels $\hat{\delta}_i^1, i = 1, \ldots, k_1$ which is prescribed for the fuzzy goals of self objective function. The DM2 also specifies initial membership levels $\overline{\mu}_j^2, j = 1, \ldots, k_2$.

Step 3 We use genetic algorithms based on reference solution updating from Sakawa et al. so as to solve problem (10).

Step 4 If the DM2 is satisfied with an obtained solution to problem (10), go to Step 5. Otherwise, the DM2 updates the membership values $\overline{\mu}_j^2, j = 1, \ldots, k_2$ representing the aspiration levels on the preference of self, return to Step 3.

Step 5 For the solution $\mathbf{x}^*$ to problem (10), four problems (11), (12), (13) and (14) based on the partial information on the preferences of the DM1 and the DM2 are solved by using the simplex method in order to derive $S^1_{\min}$, $S^1_{\max}$, $S^2_{\min}$ and $S^2_{\max}$. By using $S^1_{\min}$, $S^1_{\max}$, $S^2_{\min}$ and $S^2_{\max}$, the aggregated degrees of satisfaction of the fuzzy goals of the DM1 and the DM2 are expressed by the L-L fuzzy numbers $\tilde{S}^1$ and $\tilde{S}^2$. Moreover, the ratio $\tilde{\Delta}$ and the degree α are computed.

Step 6 If the solution shown to the DM1 satisfies the termination condition (16) and the DM1 concludes the solution x* as a satisfactory solution, the solution becomes a satisfactory solution and the algorithm stops. Otherwise, referring to the membership function value $\mu_i^l(z_i^l(\mathbf{x})), l = 1, 2, i = 1, \ldots, k_l$ with respect to the present solution $\mathbf{x}^*$, referring to fuzzy number $\tilde{S}^l, l = 1, 2$ with the aggregated degree of satisfaction of the fuzzy goal and the degree α that the degree of the ratio $\tilde{\Delta}$ of the satisfactory degree satisfies fuzzy goal $G_{\tilde{\Delta}}$, ask the DM1 to update the minimal satisfactory levels $\hat{\delta}_i^1, i = 1, \ldots, k_1$ which is prescribed for the fuzzy goals of the DM1, and go to Step 3.

The above algorithm is designed so that it can take account the balance of satisfaction degree between both levels into consideration with respecting the intention of the DM1.

V. Genetic algorithms with double strings based on reference solution updating

Problems (9), (10), (17) and (18) expressed with a preceding paragraph are integer programming problems. Because calculation time increases rapidly as a scale in problem grows large in the way by the enumeration method such as a branch-and-bound method, an approximate solution method using genetic algorithms with double strings based on reference solution updating which was proposed by Sakawa et al. [14, 15, 17] is adopted by this thesis.

index of variable	$s(1)$	$s(2)$	...	$s(n)$	= $\boldsymbol{S}$
integer value	$g_{s(1)}$	$g_{s(2)}$	...	$g_{s(n)}$	

Figure 1: the structure of double string

Sakawa et al. proposed a double string representation. The double string was represented in Fig. 1, in which an element $s(j)$ in the upper row denotes the index of an element in a solution vector and an element $g_{s(j)}$ in the lower denotes the corresponding value of the variable. By using the genetic algorithm which represents individual with double strings by Sakawa et al., the decoding algorithm enable to decode each of the individuals to the corresponding feasible solution using preference solution updating, if it necessary to update the reference solution preventing local of exploratory, operator of genetic such as reproduction, crossover and mutation is repeated to evolve the group of individuals. The algorithm describes in the following.

Algorithm of genetic algorithms with double strings based on reference solution updating

Step 0 (Initialization) Initialize the various parameters used in the genetic algorithm. Generate N individuals at random in order to form the initial population.

Step 1 (Evaluation) By using the decoding procedure based on the reference solution, the feasible solution $\mathbf{x}$ is derived. Then, calculate the fitness value of each individual by using $\mathbf{x}$.

Step 2 (Reference solution updating) If the condition for updating the reference solution is satisfied, update the reference solution.

Step 3 (Termination test) If the termination condition is fulfilled, the elitist individual become an approximately optimal solution. Otherwise, go to Step4.

Step 4 (Scaling) A linear scaling is applied to the fitness value of each individual.

Step 5 (Reproduction) Apply the reproduction operator by elitist expected value selection using the fitness of each individual.

Step 6 (Crossover) Apply the crossover operator by the partially matched crossover (PMX) for double string, new individual is produced.

Step 7 (Mutation) For the lower string of double strings, mutation of bit reverse type is adopted and, for the upper string, another genetic operator, an inversion is employed. Return to Step 1.

In the proposed method, so as to solve problems (9), (10), (11) and (12) by using genetic algoritms, we define the fitness functions as follows:

Problem (9):

$$f(\mathbf{s}) = \min_{\substack{l=1,2 \\ i=1,\dots,k_l}} \mu_i^l(z_i^l(\mathbf{x})) \tag{19}$$

Problem (10):

$$f(\mathbf{s}) = 1 - \max_{i=1,\dots,k_2} \{\overline{\mu}_i^2 - \mu_i^2(z_i^2(\mathbf{x}))\} \tag{20}$$

Problem (17):

$$f(\mathbf{s}) = \frac{\mathbf{c}_i^l\mathbf{x} - \sum_{j\in J^+_{c_i^l}} c_{ij}^l}{\sum_{j\in J^-_{c_i^l}} c_{ij}^l - \sum_{j\in J^+_{c_i^l}} c_{ij}^l} \tag{21}$$

Problem (18):

$$f(\mathbf{s}) = \frac{\mathbf{c}_i^l\mathbf{x} - \sum_{j\in J^-_{c_i^l}} c_{ij}^l}{\sum_{j\in J^+_{c_i^l}} c_{ij}^l - \sum_{j\in J^-_{c_i^l}} c_{ij}^l} \tag{22}$$

where, $J^+_{c_i^l} = \{j | c_{ij}^l > 0, 1 \le j \le n\}$, $J^-_{c_i^l} = \{j | c_{ij}^l < 0, 1 \le j \le n\}$.

VI. Conclusion

In this paper, focusing on multiobjective two-level integer programming problems where two decision makers with integer decision variables have multiple objective functions, we present interactive fuzzy programming for them using partial information about preference. In the proposed method, after introducing fuzzy goals of objective functions for the decision makers, a solution, which minimizes the difference between the membership function values and the reference membership levels for the decision maker at the lower level under the membership function values are greater than or equal to the minimal satisfactory levels for the decision maker at the upper level, are obtained by genetic algorithms. After evaluating the aggregated satisfactory degree for each of the decision makers through the partial information about preference, if the ratio of the aggregated satisfactory degree for the decision maker at the lower level to that of at the upper level satisfies a certain condition and the decision maker at the upper level is content with the solution, a satisfactory solution is obtained. Otherwise, the interactive procedures with updating the minimal satisfactory levels and the reference membership levels are continued until a satisfactory solution is found.

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