

# Tracking of feature points in dynamic image with occlusion and appearance by SMC implementation of PHD filter

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**Abstract:** Purpose of this paper is to track feature points in dynamic image of independently moving multiple objects with appearance and occlusion. We propose a state space model having state consisting of position and velocity of feature points on multiple objects. We track feature points by estimating the state of the model using Sequential Monte Carlo (SMC) implementation of Probability Hypothesis Density (PHD) filter. Corresponding to appearance and occlusion due to motion of objects, feature points on the objects have birth and death. Moreover, they are observed with missing and false detection due to feature extraction process. Tracking of multiple targets with birth, death, missing, and false detection requires variable dimension of the state, and representation of the targets by finite random set (FRS) is suitable for the situation. SMC implementation of PHD filter is applied to estimate the state. An advantage of the filtering method is that dimension of the state does not increase when the number of feature points becomes large. So, computational cost is much less than the conventional methods without PHD filter. SMC implementation make PHD filter algorithm be tractable by approximating the PHD with many particles. Numerical simulation and real image experiment show performance of feature points tracking by proposed method.

Keywords: target tracking, dynamic image, Sequential Monte Carlo (SMC), Probability Hypothesis Density (PHD) filter, finite random set (FRS).

## 1. Introduction

In the field of computer vision, there are many approaches and problem settings to give the ability of human eye perception to machines based on computer. Among them, estimating 3D information from dynamic image is important problem and has been studied extensively [1,2,3,4,5,8,9].

There are mainly two approaches to the 3D reconstruction problem from dynamic image, one using optical flow [3], and the other one using feature points. In the latter approach, feature points are chosen as corner or cross on the image which are extracted by lower level image processing based on variation of intensity. We focus on the latter approach in this paper.

There are two methods in the approach of using feature points.

The first method uses “sequential feature points track” where correspondence of feature points between time adjoining frames is determined by low level image processing based on local features of the image. Procedure to get sequential feature points track is as follows: extract feature points at 1st frame of dynamic image, then track the feature points by searching for similar point on neighborhood region at subsequent frames. Typical 3D reconstruction based on this method is that: factorization which decomposes measurement matrix into structure of objects and motion [2,9] and representing structure and motion by state space model [1,5,8]. An advantage of this method is that correspondence of feature points between time adjoining frames is known. However, there are two kinds of errors in this method. One is missing and false detection at extraction in 1st frame. Another one is searching error when tracking of the feature points. Moreover, dealing with appearance and occlusion is difficult in this method.

The second method uses feature points extracted at each frame. This method has an advantage that it can easily be adjusted to appearance and occlusion. However observation may have missing and false detection at each frame. Moreover, correspondence of feature points between time adjoining frames is unknown. Therefore, it is necessary to determine the correspondence by some method. The approach using state space model can determine the correspondence in natural way. There are some researches dealing with this situation in the field of target tracking [4,6] which take into account the correspondence of feature points between time adjoining frames.

Purpose of this paper is to track feature points in dynamic image of independently moving multiple objects with appearance and occlusion. Where missing and false detection in feature point extraction often occur. We expect a tracking method which effectively copes with birth, death, missing and false detection of feature points. To track the feature points in this situation, we use state space modeling approach. By letting state be a set of position of feature points and observation be a set of coordinates of feature points in the image, we formulate a state space model with the state and the observation denoted by finite random set (FRS).

We use sequential Monte Carlo (SMC) implementation of Probability Hypothesis Density (PHD) filter for state estimation. In the PHD filter, joint posterior distribution of feature points are denoted by FRS, and PHD of the FRS is estimated instead of the

FRS itself. PHD is uniquely defined by a property that one yield expected number of feature points in region  $S$  when PHD is integrated over the region  $S$ . Representation of state and observation by FRS is suitable for tracking of multiple feature points with birth, death, missing, and false detection. Moreover, it has advantage about state dimension not increasing as number of feature points increases. So PHD filter can reduce computational cost compared with conventional filters. PHD filter involves computationally intractable integral, which has no closed form solution, but it can be approximately estimated by SMC implementation proposed in [10].

Rest of this paper is as follows: section 2 defines proposed model, section 3 refers estimation of the state by SMC implementation of PHD filter, section 4 shows result of simulation and real image experiment, and section 5 will make conclusion.

## 2. Model

Assume a scene with multiple objects moving independently and the objects having appearance and occlusion. True position of  $j$ -th feature point on object in the image at discrete-time  $k$  is denoted by

$$\mathbf{x}_{k,j} = [x_{k,j}, y_{k,j}]. \quad (1)$$

Let  $N(k)$  be the number of feature points in the scene, then we can from a set of feature points denoted as

$$\mathbf{X}_k = \{\mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,N(k)}\} \subseteq E_s, \quad (2)$$

where  $E_s$  is set of all possible single feature point state.  $\mathbf{X}_k$  is the ‘‘state’’ that we will estimate. We denote the observed position of  $j$ -th feature point at discrete-time  $k$  by

$$\mathbf{z}_{k,j} = [X_{k,j}, Y_{k,j}]. \quad (3)$$

Let  $M(k)$  be the number of observed feature points, then we can from a set of observed feature point as

$$\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \dots, \mathbf{z}_{k,M(k)}\} \subseteq E_o, \quad (4)$$

where  $E_o$  is set of all possible feature points in observation. Appearance and occlusion of feature points are represented by birth and death of points in  $\mathbf{X}_k$ . The set of observed feature points may contain missing and false detection. Thus,  $N(k)$  and  $M(k)$  can vary depending on time step  $k$ .

It is suitable to use finite random sets (FRS) on  $E_s$  and  $E_o$  to represent this situation. State transition given the previous state  $\mathbf{X}_{k-1}$  is denoted by system equation of FRS  $\Xi_k$  on  $E_s$

$$\Xi_k = S_k(\mathbf{X}_{k-1}) \cup F_k \quad (5)$$

where  $S_k(\mathbf{X}_{k-1})$  is FRS of feature points survived from  $\mathbf{X}_{k-1}$ , and  $F_k$  is FRS of new birth feature points. Observation process given the current state  $\mathbf{X}_k$  is denoted by observation equation of FRS  $\Sigma_k$  on

$E_o$

$$\Sigma_k = E_k(\mathbf{X}_k) \cup C_k \quad (6)$$

where  $E_k(\mathbf{X}_k)$  is FRS of observed feature points coming from  $\mathbf{X}_k$ , and  $C_k$  is FRS of false detection. These two FRS,  $\Xi_k$  and  $\Sigma_k$ , are represented with conditional probability density  $f_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1})$  and  $g_k(\mathbf{Z}_k|\mathbf{X}_k)$  respectively

$$\Xi_k \sim f_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1}), \quad (7)$$

$$\Sigma_k \sim g_k(\mathbf{Z}_k|\mathbf{X}_k). \quad (8)$$

Backing to eq.(5),  $S_k(\mathbf{X}_{k-1})$  is

$$S_k(\mathbf{X}_{k-1}) = S(\mathbf{x}_{k-1,1}) \cup S(\mathbf{x}_{k-1,2}) \cup \dots \cup S(\mathbf{x}_{k-1,N(k-1)}), \quad (9)$$

$$S(\mathbf{x}_{k-1,j}) = \begin{cases} \emptyset, & \text{with prob. } e(\mathbf{x}_{k-1,j}) \\ \{\mathbf{x}_{k,j}\}, & \text{with prob. } 1-e(\mathbf{x}_{k-1,j}) \end{cases} \quad (10)$$

where  $\mathbf{x}_{k,j} \sim f_{k|k-1}(\mathbf{x}_k|\mathbf{x}_{k-1})$ , with  $f_{k|k-1}(\mathbf{x}_k|\mathbf{x}_{k-1})$  be single target density, and  $e(\mathbf{x}_{k-1})$  is probability such that feature point  $\mathbf{x}_{k-1}$  still existing at time  $k$ .  $F_k$  is

$$F_k = \{\mathbf{x}_{k-1,N(k-1)+1}, \mathbf{x}_{k-1,N(k-1)+2}, \dots, \mathbf{x}_{k-1,N(k-1)+N_B(k-1)}\}, \quad (11)$$

$$N_B(k-1) \sim \text{Poisson}(\mu_B V), \quad (12)$$

$$\mathbf{x}_{k,N(k-1)+j} \sim U(E_s), \quad (13)$$

where  $N_B(k-1)$  is the number of new birth feature points, and  $U(E)$  is uniform distribution over  $E$ . Then  $N(k)$  becomes  $N(k-1)+N_B(k-1)$ .

In eq.(6),  $E_k(\mathbf{X}_k)$  is

$$E_k(\mathbf{X}_k) = E(\mathbf{x}_{k,1}) \cup E(\mathbf{x}_{k,2}) \cup \dots \cup E(\mathbf{x}_{k,N(k)}), \quad (14)$$

$$E(\mathbf{x}_{k,j}) = \begin{cases} \emptyset, & \text{with prob. } 1-p_D(\mathbf{x}_{k,j}) \\ \{\mathbf{z}_{k,j}\}, & \text{with prob. } p_D(\mathbf{x}_{k,j}) \end{cases}, \quad (15)$$

where  $\mathbf{z}_{k,j} \sim g_k(\mathbf{z}_k|\mathbf{x}_{k,j})$  with  $g_k(\mathbf{z}_k|\mathbf{x}_{k,j})$  be single target observation density, and  $p_D(\mathbf{x}_{k,j})$  is probability of detection. Let  $M_D(k)$  be the number of detected target, i.e. the number of non-empty set at right hand side of eq.(14).  $C_k$  is

$$C_k = \{\mathbf{z}_{k,M_D(k)+1}, \mathbf{z}_{k,M_D(k)+2}, \dots, \mathbf{z}_{k,M_D(k)+M_c(k)}\}, \quad (16)$$

where  $M_c(k)$  is number of false detections such that

$$M_c(k) \sim \text{Poisson}(\mu_c V), \quad (17)$$

and

$$\mathbf{z}_{k,M_D(k)+j} \sim U(E_o). \quad (18)$$

Then  $M(k) = M_D(k) + M_c(k)$ .

## 3. State estimation

### 3.1 PHD filter

State estimation is a term which means to infer state  $\mathbf{X}_k$  given series of observations  $Z_{1:k}$ , i.e., obtaining conditional probability distribution  $p(\mathbf{X}_k|Z_{1:k})$ . State estimation by Probability Hypothesis Density (PHD) filter is done based on PHD. PHD is 1st order moment of FRS, so PHD filter corresponds to fixed covariance Kalman filter, which estimate mean vector only.

PHD is defined as follows. Let  $\delta_{\mathbf{x}}(\mathbf{y})$  be Dirac delta function with mass centered at  $\mathbf{x}$ , i.e.,

$$\int_S \delta_{\mathbf{x}}(\mathbf{y}) d\mathbf{y} = \begin{cases} 1 & \mathbf{x} \in S \\ 0 & \mathbf{x} \notin S \end{cases}. \quad (19)$$

We define distribution of finite set  $\mathbf{X}$  as

$$\delta_{\mathbf{x}}(\mathbf{y}) = \sum_{\mathbf{x} \in \mathbf{X}} \delta_{\mathbf{x}}(\mathbf{y}). \quad (20)$$

When we integrate  $\delta_{\mathbf{x}}(\mathbf{y})$  over a region  $S$ , we have the number of elements of  $\mathbf{X}$  contained in  $S$ .

PHD is defined as expectation of density of FRS  $\Delta$

$$D_{\Delta}(\mathbf{x}) \equiv E[\delta_{\Delta}(\mathbf{x})] = \int \delta_{\mathbf{x}}(\mathbf{x}) P_{\Delta}(d\mathbf{X}). \quad (21)$$

When we integrate  $D_{\Delta}(\mathbf{x})$  over a region  $S$ , then we have expected number of points of FRS  $\Delta$  in the region  $S$ .

PHD filter deals with PHD of conditional distribution  $p(\mathbf{X}_k|Z_{1:k})$  instead of the distribution itself. Algorithm of PHD filter consists of prediction and update procedures similarly to Kalman filter.

Prediction procedure is to calculate

$$D_{k|k-1}(\mathbf{x}|Z_{0:k-1}) = \gamma_k(\mathbf{x}) + \int \phi_{k|k-1}(\mathbf{x}, \xi) D_{k-1|k-1}(\xi|Z_{0:k-1}) d\xi \quad (22)$$

where

$$\phi_{k|k-1}(\mathbf{x}, \xi) = e_{k|k-1}(\xi) f_{k|k-1}(\mathbf{x}|\xi). \quad (23)$$

Symbols appeared here are as follows:

$e_{k|k-1}(\mathbf{x}_{k-1})$ : Probability of feature point  $\mathbf{x}_{k-1}$  still surviving to time  $k$ .

$\gamma_k(\mathbf{x})$ : PHD of FRS  $F_k$ , which represents new birth feature points to time  $k$ .

Update procedure is to evaluate

$$D_{k|k}(\mathbf{x}|Z_{0:k}) = \left[ v(\mathbf{x}) + \sum_{z \in \mathcal{Z}_k} \frac{\psi_{k,z}(\mathbf{x})}{\mu_k c_k(\mathbf{z}) + \left( \psi_{k,z} D_{k|k-1}(\cdot|Z_{0:k-1}) \right)} \right] D_{k|k-1}(\mathbf{x}|Z_{0:k-1}), \quad (24)$$

where

$$v(\mathbf{x}) = 1 - p_D(\mathbf{x}), \quad (25)$$

$$\psi_{k,z}(\mathbf{x}) = p_D(\mathbf{x}) g_k(\mathbf{z}|\mathbf{x}). \quad (26)$$

Symbols appeared here are as follows:

$p_D(\mathbf{x})$ : Probability of detection.

$g_k(\mathbf{z}|\mathbf{x})$ : Likelihood of single target.

$c_k(\mathbf{z})$ : Probability density of false detection on  $E_o$ .

$\mu_k(\mathbf{x})$ : Average number of false detection points.

$\langle \cdot \rangle$ : Convolution of two functions  $f(\mathbf{x})$  and  $g(\mathbf{x})$ .

### 3.2 SMC implementation of PHD filter

PHD filter is very difficult to perform its computation analytically because it contains a lot of integrals with no closed form. To circumvent this difficulty, [10] proposes a method using technique of particle filters to the computation of PHD filter. In this method,

PHD  $D(\mathbf{x})$  is approximated by weighted particles  $\left\{ \left( \mathbf{x}_{0:k}^{(i)}, w_k^{(i)} \right) \right\}_{i=1}^{L_k}$ ,

where  $\mathbf{x}_{0:k}^{(i)}$  is realization of  $\mathbf{x}$ , which is called ‘‘particle’’, and

$w_k^{(i)}$  is corresponding weight with  $w_k^{(i)} > 0$ . Notice that

$\hat{N}_{k|k} = \sum_{i=1}^{L_k} w_k^{(i)}$  is expected number of feature points at

discrete-time  $k$ . We let  $\rho$  be number of particles per one feature point. Then,  $L_k$ , number of particles, is variable depending on

$\hat{N}_{k|k}$  such that

$$L_k = \rho \hat{N}_{k|k}, \quad (27)$$

which effectively reduce the number of particles to reasonable one. For the particles and the weights, sequential operation of state estimation is conducted by three steps; prediction, update, and resampling.

In the prediction step, we first create particles of  $S_k(\mathbf{X}_{k-1})$  and calculate corresponding weights as

$$\tilde{\mathbf{x}}_k^{(i)} \sim q_k(\cdot|\mathbf{x}_{k-1}^{(i)}, \mathbf{Z}_k), \quad i = 1, \dots, L_{k-1}, \quad (28)$$

$$\tilde{w}_{k|k-1}^{(i)} = \frac{\phi_k(\tilde{\mathbf{x}}_k^{(i)}, \mathbf{x}_{k-1}^{(i)})}{q_k(\tilde{\mathbf{x}}_k^{(i)}|\mathbf{x}_{k-1}^{(i)}, \mathbf{Z}_k)} w_{k-1}^{(i)}, \quad i = 1, \dots, L_{k-1}, \quad (29)$$

where  $q_k(\cdot|\mathbf{x}_{k-1}^{(i)}, \mathbf{Z}_k)$  is called ‘‘proposal distribution’’. Next we

create particles of  $F_k$ , and calculate corresponding weights as

$$\tilde{\mathbf{x}}_k^{(i)} \sim p_k(\cdot|\mathbf{Z}_k), \quad i = L_{k-1} + 1, \dots, L_{k-1} + J_k, \quad (30)$$

$$\tilde{w}_{k|k-1}^{(i)} = \frac{\gamma_k(\tilde{\mathbf{x}}_k^{(i)})}{J_k p_k(\tilde{\mathbf{x}}_k^{(i)}|\mathbf{Z}_k)}, \quad i = L_{k-1} + 1, \dots, L_{k-1} + J_k, \quad (31)$$

where  $p_k(\cdot|\mathbf{Z}_k)$  denotes proposal distribution, which is different from

$q_k(\cdot|\mathbf{x}_{k-1}^{(i)}, \mathbf{Z}_k)$ , to create particles of new birth feature points.  $J_k$

corresponds to number of new birth feature points at discrete time  $k$ , and it is set as

$$J_k = \rho \int \gamma_k(\mathbf{x}) d\mathbf{x}.$$

In the update step, according to eq.(24), we update the weights as

$$\tilde{w}_k^{(i)} = \left[ v(\tilde{\mathbf{x}}_k^{(i)}) + \sum_{\mathbf{z} \in \mathbf{Z}_k} \frac{\psi_{k,\mathbf{z}}(\tilde{\mathbf{x}}_k^{(i)})}{\kappa_k(\mathbf{z}) + C_k(\mathbf{z})} \right] \tilde{w}_{k|k-1}^{(i)}. \quad (32)$$

Where  $C_k(\mathbf{z})$  is

$$C_k(\mathbf{z}) = \sum_{j=1}^{L_{k-1}+J_k} \psi_{k,\mathbf{z}}(\tilde{\mathbf{x}}_k^{(j)}) \tilde{w}_{k|k-1}^{(j)}. \quad (33)$$

In the resampling step, we calculate the total mass  $\hat{N}_{k|k}$  as an estimate of expected number of feature points

$$\hat{N}_{k|k} = \sum_{j=1}^{L_{k-1}+J_k} \tilde{w}_k^{(j)} \cong \int_{E_s} D_{k|k}(\mathbf{x}) \quad (34)$$

Then we set  $L_k$ , the number of particles, as in eq.(27). Now, resample from  $\{(\tilde{\mathbf{x}}_k^{(i)}, \tilde{w}_k^{(i)})\}_{i=1}^{L_{k-1}+J_k}$  as

$$\mathbf{x}_k^{(i)} = \begin{cases} \tilde{\mathbf{x}}_k^{(1)} & \text{with prob. } \alpha_k^{(1)} \\ \vdots & \\ \tilde{\mathbf{x}}_k^{(L_{k-1}+J_k)} & \text{with prob. } \alpha_k^{(L_{k-1}+J_k)} \end{cases} \quad (35)$$

and set  $w_k^{(i)} \propto \tilde{w}_k^{(i)}/\alpha_k^{(i)}$  subject to  $\sum_{i=1}^{L_k} w_k^{(i)} = \hat{N}_{k|k}$ , then we obtain  $\{(\mathbf{x}_k^{(i)}, w_k^{(i)})\}_{i=1}^{L_k}$ . Where  $\alpha_k^{(i)} > 0$  and  $\sum_{i=1}^{L_{k-1}+J_k} \alpha_k^{(i)} = 1$ , one typically choose  $\alpha_k^{(i)} = \tilde{w}_k^{(i)}/\hat{N}_{k|k}$ .

## 4. Experiment

We investigate the performance of proposed method by conducting simulation and real image experiments. These experiments use computer (CPU:2.20GHz, memory:512MB).

### 4.1 Simulation

Suppose a situation that multiple objects move independently with appearance and occlusion, and they are observed with missing and false detection. We synthesized simulation data on 2 dimension according to this situation. Number of feature points is initially 2. Birth and death occur according to Poisson distribution with intensity 0.5. As conditions for observation, missing occurs according to Poisson with intensity 0.5 and false detection Poisson with intensity 2.0. We use  $N(0,1)$  as observation noise. Plot of simulation data is shown in Figure 1. Figure 2 and 3 respectively show time evolution of x and y elements of the data.

Conditions for estimation are given in Table 1. We have used constant values for most of them, although we can chose variable values depending on  $\mathbf{x}_k$  or  $\mathbf{z}_k$ . Figure 4 and 5 show result of estimation of PHD mangnalized with respect to x and y elements of position. To plot these figures, we have used kernel density estimation slightly modified to have total mass greater than 1.

By looking at Figure 4 and 5, PHD has high values around

correctly detected feature points. We can also see that PHD becomes smaller value around missing and it immediately recover to the right level when the point is detected again. Additionally, PHD gently grows up at birth of feature points, and gently comes down at death of feature points with eventually it disappeared.

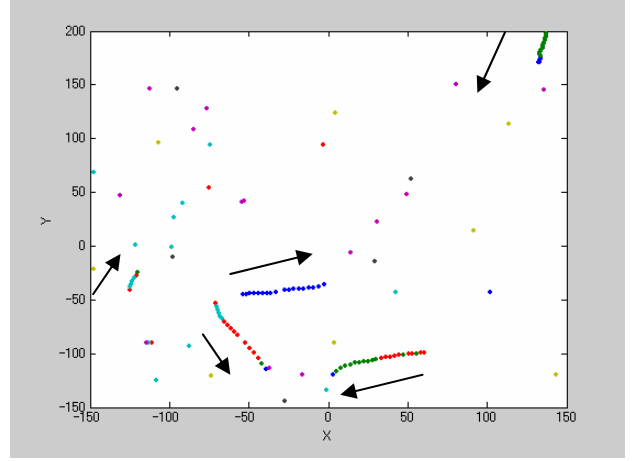


Figure 1 Simulation data.

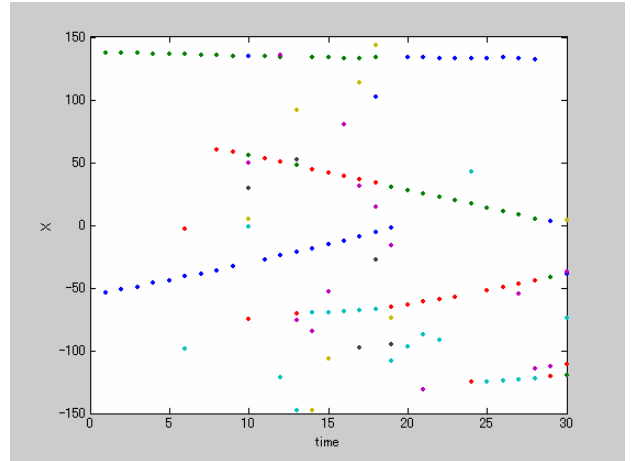


Figure 2 Time evolution of simulation data (x element).

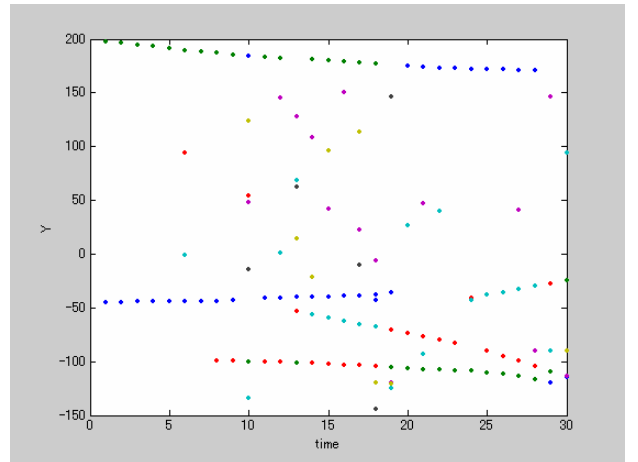


Figure 3 Time evolution of simulation data (y element).

Table 1 Conditions of estimation on simulation.

$\rho$ : Number of particles per feature point.	500
$\gamma_k$ : PHD of FRS $F_k$ which represents, new birth feature points.	$0.3V^{-1}$
$e_{k k-1}$ : Probability of feature point $\mathbf{x}_{k-1}$ still surviving to time $k$ .	0.9
$c_k$ : Probability density of false detection.	$V^{-1}$
$\mu_k$ : Average number of false detection points.	3.0
$p_D$ : Probability of detection.	0.8

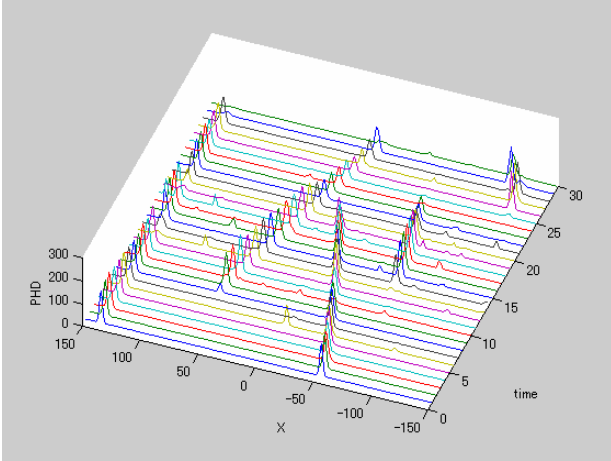


Figure 4 Marginal PHD on x element of position for simulation data.

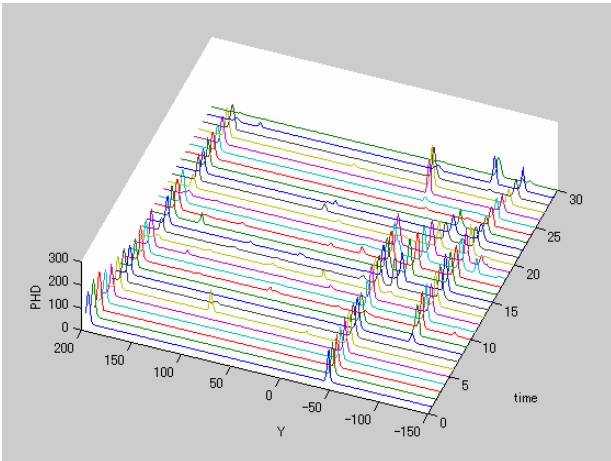


Figure 5 Marginal PHD on y element of position for simulation data.

## 4.2 Real image

Dynamic image of 30 frames (about 0.1 [sec/frame]) is taken with a scene where man's foot and car are moving independently. We extracted feature points independently for each frame. As the result, we obtained 12~21 feature points per frame. 1st, 10th, 20th and 30th frames of dynamic image are shown in Figure 6. Figure 7 shows the plot of extracted feature points. Figure 8 and 9 show time evolution of x and y elements of positions of the feature points.

Conditions for estimation are given in Table 2. Figure 10 and 11 show the result of estimation of PHD with respect to x and y elements of positions.

By looking at Figure 10 and 11, PHD has high values around at feature points on car, man's foot, back ground. When the car hides behind the front object, PHD around feature points on the car gradually decreases and eventually disappears, and when the car appear again, PHD for the car becomes high.

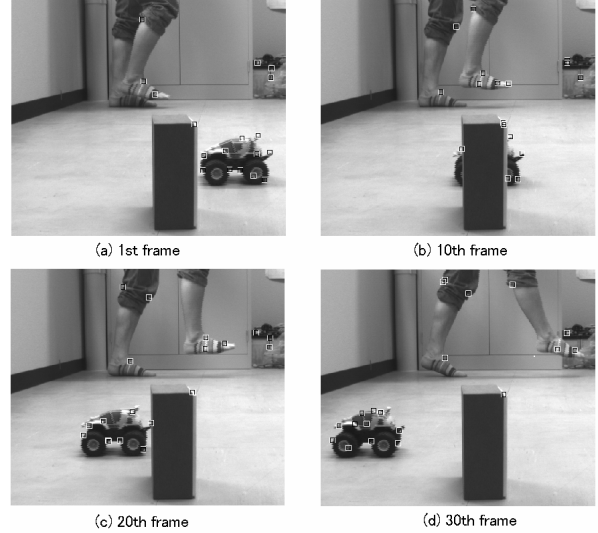


Figure 6 Dynamic image: a scene with man and car are moving independently.

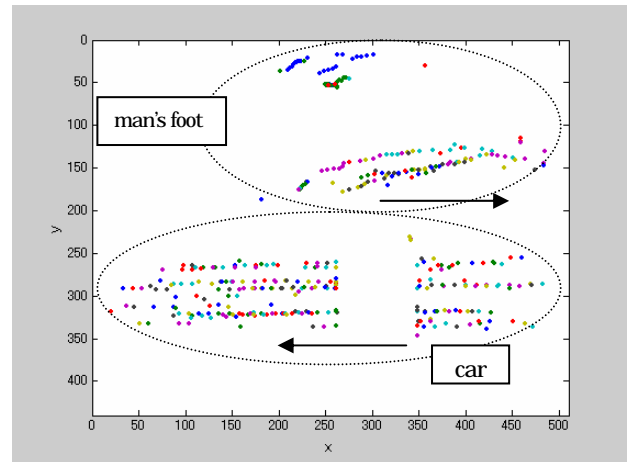


Figure 7 Plot of feature points of dynamic image.

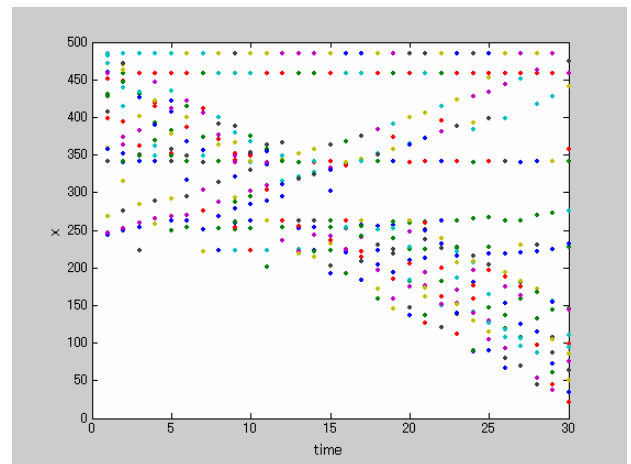


Figure 8 Time evolution of feature points (x element) of dynamic image.

dynamic image.

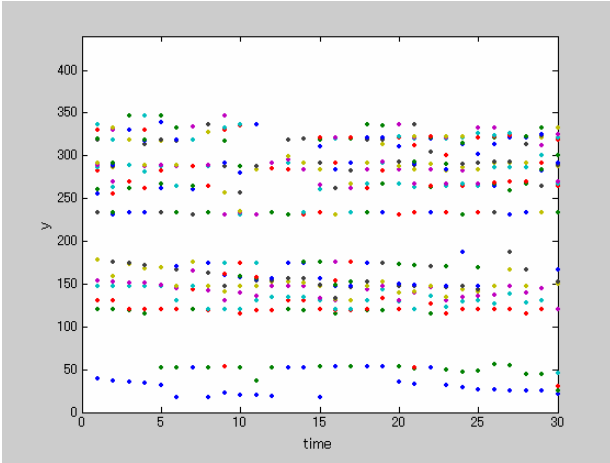


Figure 9 Time evolution of feature points (y element) of dynamic image.

Table 2 Condition of estimation on real image

$\rho$ : Number of particles per feature point.	300
$\gamma_k$ : PHD of FRS $F_k$ which represents, new birth feature points.	$0.5V^{-1}$
$e_{k k-1}$ : Probability of feature point $\mathbf{x}_{k-1}$ still surviving to time $k$ .	0.9
$c_k$ : Probability density of false detection.	$V^{-1}$
$\mu_k$ : Average number of false detection points.	5.0
$p_D$ : Probability of detection.	0.8

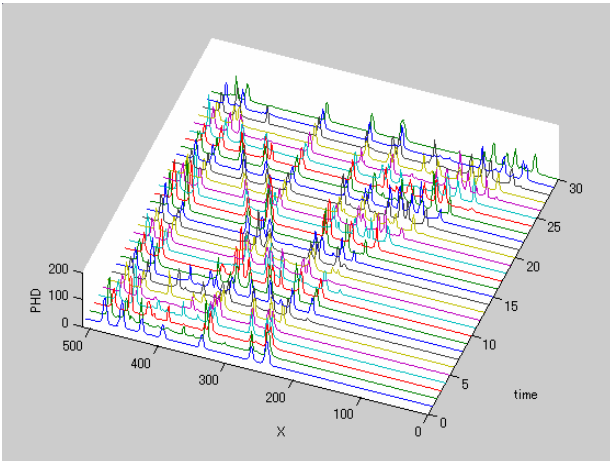


Figure 10 Marginal PHD on x element position of feature points for dynamic image.

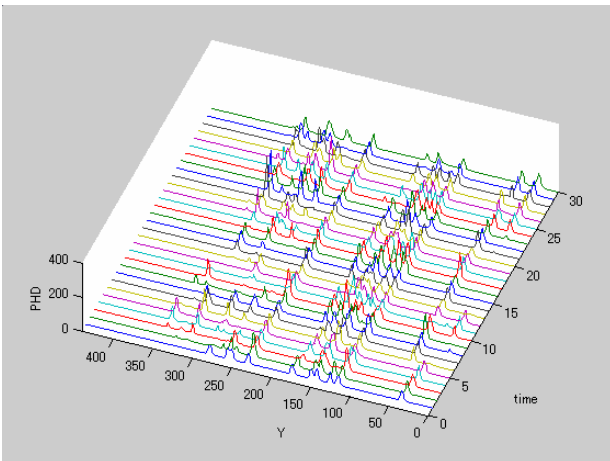


Figure 11 Marginal PHD on y element position of feature points for

## 5. Conclusion

For a scene with multiple objects moving independently and the objects having appearance and occlusion, we proposed a state space model based on finite random set, having birth, death, missing, and false detection of feature points. We estimate the state of the model by SMC implementation of PHD filter to track the feature points in the scene. We demonstrated the performance of proposed method by conducting simulation and real image experiments.

For future works, we are considering an extension of the model to deal with 3D reconstruction with camera projection model and multiple camera situation.

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