H_∞ Control for Fuzzy Sampled-Data Systems

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Abstract

In this paper the design problem of output feedback H_∞ controllers for Takagi-Sugeno fuzzy sampled-data systems is considered. We take a jump system approach to design controllers. We first introduce the H_∞ performance (norm) for a stable fuzzy jump system and give a sufficient condition for the norm being less than a given number. We then consider the H_∞ problem with output feedback controllers. Since a fuzzy sampled-data system can be written in the form of a fuzzy jump system, H_∞ controller for fuzzy sampled-data systems can be readily derived from the corresponding results of fuzzy jump systems.

Keywords: H_∞ Control, Takagi-Sugeno Fuzzy Systems, Jump Systems, Sampled-Data Systems

1 Introduction

The Takagi-Sugeno fuzzy dynamic model is a system described by a set of fuzzy if-then rules which gives local linear representations of the underlying nonlinear system ([9], [10]). It is well-known in [11] that such a model can describe or approximate a wide class of nonlinear systems. Since the work by [10] on stability analysis and state feedback stabilization there has been much effort developing system theory for such systems. Robust stability is studied in [11] in terms of linear matrix inequalities (LMI) and in [5] using new stability conditions. Parallel to state feedback design, observer problems have also been studied. Now it is known in [4], [7] and [13] that the separation property of designing state feedback controls and observers hold. This property has been extended to sampled-data fuzzy systems in [8]. H_∞-control was also considered by many authors. The so-called bounded real lemma was formulated in terms of LMI's in [11] and the design of state feedback H_∞-controllers was considered in [1] and [2]. The design of output feedback H_∞-controllers, though more important and practical, was studied in [14] for continuous-time case and in [6] for discrete-time case.

When we consider practical systems, it is very natural that the state of the system is continuous-time, but the observations are taken only at each sampling instant. This class of system is called a sampled-data system. Sampled-data systems often appear in many practical systems and mathematical formulations, and describe the real situations of the systems. Recently, a jump system approach has been taken to analyze sampled-data systems. In fact, a jump system, whose state variable has jumps at certain time instants, has been proved useful to treat a wide class of systems, including continuous-time system, discrete-time system and sampled-data system. H_2 and H_∞ control problems for such jump system and sampled-data system were considered in [3]. Moreover, the stochastic control theory of jump systems was also explored. LQG control problem and risk-sensitive optimal control problem were solved in [15] and [12], respectively. In these paper, the results for jump systems were applied to sampled-data systems and the same control problems for sampled-data systems were solved.

In this paper we consider an H_∞ control problem for the Takagi-Sugeno fuzzy sampled-data systems via jump system approach. Since it has been shown that a sampled-data system can be written as a special case of a jump system, we first consider an H_∞ control problem for the Takagi-Sugeno jump systems. We give a design method of output feedback H_∞ controllers which is based on the two LMI's and give sufficient conditions for the proposed controller to be suboptimal. Then we apply the result to fuzzy sampled-data systems.

2 H_∞ Control

This section considers an H_∞ control problem for Takagi-Sugeno fuzzy jump systems, and gives a design method of output feedback H_∞ controllers.

2.1 Preliminary Results

First we recall some important results that help us obtain a main result of the paper.

The following lemma is known as the Schur complement formula.

Lemma 2.1 *Let M be a symmetric matrix of the form*

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix},$$

where M_{11} and M_{22} are assumed to be invertible. Then the following statements are equivalent.
1)$M > 0$,
2)$M_{11} > 0$ *and* $M_{22} - M_{12}^T M_{11}^{-1} M_{12} > 0$,
3)$M_{22} > 0$ *and* $M_{11} - M_{12} M_{22}^{-1} M_{12}^T > 0$.

Now we review the H_∞ norm for a linear stable jump system. Consider

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bw(t), \quad kh < t < (k+1)h, \\ x(kh^+) & = & A_d x(kh) + B_d w_{dk}, \quad t = kh, \\ z(t) & = & Cx(t) + Dw(t), \\ z_{dk} & = & C_d x(kh) + D_d w_{dk} \end{array} \tag{1}$$

where $x(t) \in \Re^n$ is the state which is left continuous at $t = kh$ where h is a constant number, $w(t) \in \Re^{m_1}$, $w_{dk} \in \Re^{m_d}$ are the continuous-time and discrete-time disturbances, respectively, $z(t) \in \Re^{p_1}$, $z_{dk} \in \Re^{p_d}$ are the controlled outputs, respectively. The matrices $A, A_d, B, B_d, C, C_d, D$ and D_d are of appropriate dimensions.

Definition 2.1 *The system (1) is said to be input-output stable if $(z, z_d) \in L^2(0,\infty;\Re^q) \times l^2(0,\infty;\Re^{q_d})$ for any $(w, w_d) \in L^2(0,\infty;\Re^m) \times l^2(0,\infty;\Re^{m_d})$ where $L^2(0,\infty;\cdot)$ is the space of square integrable functions and $l^2(0,\infty;\cdot)$ is the space of square summable functions.*

H_∞ disturbance attenuation performance for the system (1) is defined in the following.

Definition 2.2 *Given a scalar $\gamma > 0$, the system (1) is said to be stable with H_∞ disturbance attenuation γ if it is exponentially stable and input-output stable with*

$$\| z \|_{L_2}^2 + \| z_d \|_{l_2}^2 \leq d^2 (\| w \|_{L_2}^2 + \| w_d \|_{l_2}^2) \tag{2}$$

for some $0 < d < \gamma$.

The following lemma gives necessary and sufficient conditions for (2) and is known as the Bounded Real Lemma.

Lemma 2.2 *([3]) Let $\gamma > 0$ be given, and consider the system*

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bw(t), \; kh < t < (k+1)h, \\ x(kh^+) & = & A_d x(kh) + B_d w_{dk}, \; t = kh, \\ z(t) & = & Cx(t) + Dw(t), \\ z_{dk} & = & C_d x(kh) + D_d w_{dk}. \end{array}$$

Then it is stable with disturbance attenuation γ if and only if there exists a matrix $X > 0$ such that

$$\begin{array}{c} \begin{bmatrix} \dot{X}(t) + A^T X(t) + X(t)A & X(t)B & C^T \\ B^T X(t) & -\gamma^2 I & D^T \\ C & D & -I \end{bmatrix} < 0, \\ kh < t < (k+1)h, \\ \begin{bmatrix} -X^{-1}(kh) & 0 & A_d & B_d \\ 0 & -I & C_d & D_d \\ A_d^T & C_d^T & -X(kh^-) & 0 \\ B_d^T & D_d^T & 0 & -\gamma^2 I \end{bmatrix} < 0, \\ t = kh. \end{array} \tag{3}$$

2.2 Fuzzy Jump Systems

Consider the Takagi-Sugeno fuzzy jump system described by the following fuzzy rules:

$$\begin{array}{ll} \textbf{IF} & \xi_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_q \text{ is } M_{iq}, \\ \textbf{THEN} & \dot{x}(t) = A_i x(t) + B_{1i} w(t), \\ & \qquad kh < t < (k+1)h, \\ & x(kh^+) = A_d x(kh) + B_2 u_k, \; t = kh, \\ & z(t) = C_{1i} x(t), \\ & z_{dk} = C_{di} x(kh) + D_{12} u_k, \\ & y_k = C_{2i} x(kh) + D_{21i} w_{dk} + D_{22i} u_k, \\ & \qquad i = 1, \cdots, r \end{array} \tag{4}$$

where $u_k \in \Re^{m_2}$ is the control input, $y_k \in \Re^{p_2}$ is the observation. All the matrices are of appropriate dimensions. r is the number of IF-THEN rules. M_{ij} are fuzzy sets and ξ_1, $\cdots$, ξ_q are premise variables. We set $\xi = \begin{bmatrix} \xi_1 & \cdots & \xi_q \end{bmatrix}^T$. Here we assume that the premise variables are given.

The state, controlled output and observation are defined as follows:

$$
\begin{aligned}
\dot{x}(t) = & \sum_{i=1}^{r} \lambda_i(\xi(t))\{A_i x(t) + B_{1i} w(t)\}, \\
& \qquad\qquad kh < t < (k+1)h, \\
x(kh^+) = & A_d x(kh) + B_2 u_k, \ t = kh, \\
z(t) = & \sum_{i=1}^{r} \lambda_i(\xi(t)) C_{1i} x(t), \\
z_{dk} = & \sum_{i=1}^{r} \lambda_i(\xi(kh))(C_{di} x(kh) + D_{12} u_k), \\
y(t) = & \sum_{i=1}^{r} \lambda_i(\xi(t))\{C_{2i} x(kh) + D_{21i} w_{dk} + D_{22i} u_k\}
\end{aligned}
\tag{5}
$$

where

$$
\lambda_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{i=1}^{r} \beta_i(\xi(t))}, \quad \beta_i(\xi(t)) = \prod_{j=1}^{q} M_{ij}(\xi_j(t))
$$

and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume

$$
\beta_i(\xi(t)) \geq 0, i = 1, \cdots, r, \ \sum_{i=1}^{r} \beta_i(\xi(t)) > 0
$$

for any $\xi(t)$. Hence $\lambda_i(\xi(t))$ satisfy

$$
\lambda_i(\xi(t)) \geq 0, i = 1, \cdots, r, \ \sum_{i=1}^{r} \lambda_i(\xi(t)) = 1
$$

for any $\xi(t)$.

Remark 2.1 *The state equations of (5) can be generalized as*

$$
\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t))\{A_i x(t) + B_{1i} w(t)\}, \\
& \qquad\qquad kh < t < (k+1)h, \\
x(kh^+) &= \sum_{i=1}^{r} \lambda_i(\xi(kh))\{A_{di} x(kh) + B_{2i} u_k\}, t = kh.
\end{aligned}
$$

However, we keep in mind that our goal is H_∞ *control of sampled-data fuzzy systems. As we see in Section 3, sampled-data fuzzy systems can be written in the form (5). Thus we proceed our argument with (5).*

Suppose that the following rules concerning H_∞ controllers for each subsystem (4) are given.

$$
\begin{array}{ll}
\textbf{IF} & \xi_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_q \text{ is } M_{iq}, \\
\textbf{THEN} & \dot{\hat{x}}(t) = \hat{A}_i \hat{x}(t), \ kh < t < (k+1)h, \\
& \hat{x}(kh^+) = \hat{A}_{di} \hat{x}(kh) + \hat{B}_i y_k, \ t = kh, \\
& u_k = \hat{C}_i \hat{x}(kh), \quad i = 1, \cdots, r
\end{array}
\tag{6}
$$

where $\hat{x}(t) \in \Re^{\hat{n}}$ and all matrices are of appropriate dimensions. Then an actual choice of a controller is

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t)) \hat{A}_i \hat{x}(t), \ kh < t < (k+1)h, \\
\hat{x}(kh^+) &= \sum_{i=1}^{r} \lambda_i(\xi(t))\{\hat{A}_{di} \hat{x}(kh) + \hat{B}_i y_k\}, \ t = kh, \\
u_k &= \sum_{i=1}^{r} \lambda_i(\xi(t)) \hat{C}_i \hat{x}(kh).
\end{aligned}
\tag{7}
$$

We use the same weights $\lambda_i(\xi(t))$ as those for the rules (6) of the fuzzy system.

Definition 2.3 *A controller (7) is said to be a* γ*-suboptimal controller if it makes the system (5) stable with* H_∞ *disturbance attenuation* γ*.*

Now we propose a method to design a γ-suboptimal controller based on the linear matrix inequalities(LMI's). To this end, for some matrices G_j, let us define common matrices $X > 0$ and $Z > 0$ satisfying

$$
\begin{aligned}
& \dot{X}(t) + A_i^T X(t) + X(t) A_i + C_{1i}^T C_{1i} \\
& \quad + \frac{1}{\gamma^2} X(t) B_{1i} B_{1i}^T X(t) < 0, \ kh < t < (k+1)h, \\
& X(kh^-) - A_d^T X(kh) A_d - C_{di}^T C_{di} + F_i^T V F_i > 0, t = kh, \\
& \dot{Z}(t) + (A_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T X(t))^T Z(t) \\
& \quad + Z(t)(A_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T X(t)) < 0, \ kh < t < (k+1)h, \\
& \begin{bmatrix}
\frac{1}{\gamma^2} Z^{-1}(kh) & -G_j D_{21i} & A_d - G_j C_{2i} & 0 \\
-D_{21i}^T G_j^T & \gamma^2 I & 0 & 0 \\
(A_d - G_j C_{2i})^T & 0 & \gamma^2 Z(kh^-) & F_i^T \\
0 & 0 & F_i & V^{-1}
\end{bmatrix} > 0, \\
& \qquad\qquad\qquad\qquad t = kh, \quad \forall \ i, j
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
V(kh) &= D_{12}^T D_{12} + B_2^T X(kh) B_2, \\
F_i(kh) &= -V^{-1}(kh)(B_2^T X(kh) A_d + D_{12}^T C_{di}).
\end{aligned}
$$

The following theorem gives a γ-suboptimal controller for fuzzy jump system (5).

Theorem 2.1 *Suppose that for some matrices* G_j *there exist common matrices* $X > 0$ *and* $Z > 0$ *satisfying (8).*

Then the controller

$$
\begin{aligned}
\dot{\hat{x}}(t) = &\quad \sum_{i=1}^{r} \lambda_i(\xi(t))(A_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T X(t))\hat{x}(t), \\
& \qquad\qquad\qquad kh < t < (k+1)h, \\
\hat{x}(kh^+) = &\quad \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(kh))\{A_d\hat{x}(kh) + B_2 u_k \\
& \quad + G_j(y_k - C_{2i}\hat{x}(kh) - D_{22i}u_k)\},\ t = kh, \\
u_k = &\quad \sum_{i=1}^{r} \lambda_i(\xi(t)) F_i \hat{x}(kh)
\end{aligned}
\tag{9}
$$

is a γ-suboptimal controller.

Proof: The closed-loop system (5) and (9) becomes

$$
\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix}(t) &= A_c \begin{bmatrix} x \\ e \end{bmatrix}(t) + B_c w(t), \\
& \qquad\qquad kh < t < (k+1)h, \\
\begin{bmatrix} x \\ e \end{bmatrix}(kh^+) &= A_{dc} \begin{bmatrix} x \\ e \end{bmatrix}(kh) + B_{dc} w_{dk},\ t = kh, \\
z(t) &= C_c \begin{bmatrix} x \\ e \end{bmatrix}(t), \\
z_{dk} &= C_{dc} \begin{bmatrix} x \\ e \end{bmatrix}(kh)
\end{aligned}
\tag{10}
$$

where $e = x - \hat{x}$ and

$$
A_c = \sum_{i=1}^{r} \lambda_i(\xi) \begin{bmatrix} A_i & 0 \\ -\frac{1}{\gamma^2} B_{1i} B_{1i} X(t) & A_i + \frac{1}{\gamma^2} B_{1i} B_{1i} X(t) \end{bmatrix},
$$

$$
B_c = \sum_{i=1}^{r} \lambda_i(\xi) \begin{bmatrix} B_{1i} \\ B_{1i} \end{bmatrix},\ C_c = \sum_{i=1}^{r} \lambda_i(\xi) \begin{bmatrix} C_{1i} & 0 \end{bmatrix},
$$

$$
A_{dc} = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi)\lambda_j(\xi) \begin{bmatrix} A_d + B_2 F_i & -B_2 F_i \\ 0 & A_d - G_j C_{2i} \end{bmatrix},
$$

$$
B_{dc} = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi)\lambda_j(\xi) \begin{bmatrix} 0 \\ -G_j D_{21i} \end{bmatrix},
$$

$$
C_{dc} = \sum_{i=1}^{r} \lambda_i(\xi) \begin{bmatrix} C_{di} + D_{12} F_i & -D_{12} F_i \end{bmatrix}.
$$

Now we shall show that for the closed-loop system (10) the positive definite matrix

$$
X_c = \begin{bmatrix} X & 0 \\ 0 & \gamma^2 Z \end{bmatrix}
$$

satisfies (3). In fact, for $kh < t < (k+1)h$

$$
\begin{aligned}
& \dot{X}_c(t) + A_c^T X_c(t) + X_c(t) A_c + C_c^T C_c \\
& \qquad\qquad + \frac{1}{\gamma^2} X_c(t) B_c B_c^T X_c(t) \\
= & \sum_{i=1}^{r} \lambda_i(\xi(t)) \begin{bmatrix} X_{11}(t) & 0 \\ 0 & \gamma^2 X_{22}(t) \end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
X_{11}(t) = &\quad X(t) + A_i^T X(t) + X(t) A_i + C_{1i}^T C_{1i} \\
& + \frac{1}{\gamma^2} X(t) B_{1i} B_{1i}^T X(t) < 0, \\
X_{22}(t) = &\quad \dot{Z}(t) + (A_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T X(t))^T Z(t) \\
& + Z(t)(A_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T X(t)) \\
& + Z(t) B_{1i} B_{1i}^T Z(t) < 0.
\end{aligned}
$$

For $t = kh$, define

$$
\begin{aligned}
\Phi &\triangleq \begin{bmatrix} X_c^{-1}(kh) & 0 & A_{dc} & B_{dc} \\ 0 & I & C_{dc} & 0 \\ A_{dc}^T & C_{dc}^T & X_c(kh^-) & 0 \\ B_{dc}^T & 0 & 0 & \gamma^2 I \end{bmatrix} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t)) \begin{bmatrix} \Phi_{11} & \Phi_{12ij} \\ \Phi_{12ij}^T & \Phi_{22} \end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_{11} &= \begin{bmatrix} X^{-1}(kh) & 0 & 0 \\ 0 & \frac{1}{\gamma^2} Z^{-1}(kh) & 0 \\ 0 & 0 & I \end{bmatrix}, \\
\Phi_{12ij} &= \begin{bmatrix} A_d + B_2 F_i & -B_2 F_i & 0 \\ 0 & A_d - G_j C_{2i} & -G_j D_{21i} \\ C_{di} + D_{12} F_i & -D_{12} F_i & 0 \end{bmatrix}, \\
\Phi_{22} &= \begin{bmatrix} X(kh^-) & 0 & 0 \\ 0 & \gamma^2 Z(kh^-) & 0 \\ 0 & 0 & \gamma^2 I \end{bmatrix},
\end{aligned}
$$

and we need to show $\Phi > 0$. We calculate

$$
\begin{aligned}
& \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \Phi \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \\
& \triangleq \hat{\Phi} = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t)) \begin{bmatrix} \hat{\Phi}_{11i} & \hat{\Phi}_{12i} \\ \hat{\Phi}_{12i}^T & \hat{\Phi}_{22ij} \end{bmatrix}
\end{aligned}
$$

where

$$
\hat{\Phi}_{11i} = \begin{bmatrix} X(kh^-) & (A_d + B_2 F_i)^T & (C_{di} + D_{12} F_i)^T \\ A_d + B_2 F_i & X^{-1}(kh) & 0 \\ C_{di} + D_{12} F_i & 0 & I \end{bmatrix},
$$

$$
\hat{\Phi}_{12i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_2 F_i \\ 0 & 0 & -D_{12} F_i \end{bmatrix},
$$

$$
\hat{\Phi}_{22ij} = \begin{bmatrix} \frac{1}{\gamma^2} Z^{-1}(kh) & -G_j D_{21i} & A_d - G_j C_{2i} \\ -D_{21i}^T G_j^T & \gamma^2 I & 0 \\ (A_d - G_j C_{2i})^T & 0 & \gamma^2 Z(kh^-) \end{bmatrix}.
$$

Clearly, $\Phi > 0$ if and only if $\hat{\Phi} > 0$. In order to show $\hat{\Phi} > 0$, we refer to Lemma 2.1 and need to check if

$$
\hat{\Phi}_{11i} > 0,\ \hat{\Phi}_{22ij} - \hat{\Phi}_{12i}^T \hat{\Phi}_{11i}^{-1} \hat{\Phi}_{12i} > 0,\ \ \forall\, i, j.
$$

Since we have

$$\begin{aligned}&X(kh^-)\\&\quad-\begin{bmatrix}(A_d+B_2F_i)^T & (C_{di}+D_{12}F_i)^T\end{bmatrix}\\&\qquad\times\begin{bmatrix}X(kh) & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_d+B_2F_i\\ C_{di}+D_{12}F_i\end{bmatrix}\\&=X(kh^-)-A_d^TX(kh)A_d-C_{di}^TC_{di}+F_i^TVF_i>0,\end{aligned}$$

we can show $\hat{\Phi}_{11i}>0$ by Lemma 2.1. Next we calculate

$$\bar{\Phi}_{ij}\triangleq\hat{\Phi}_{22ij}-\hat{\Phi}_{12i}^T\hat{\Phi}_{11i}^{-1}\hat{\Phi}_{12i}=\begin{bmatrix}\frac{1}{\gamma^2}Z^{-1}(kh) & -G_jD_{21i} & A_d-G_jC_{2i}\\ -D_{21i}^TG_j^T & \gamma^2I & 0\\ (A_d-G_jC_{2i})^T & 0 & \gamma^2Z(kh^-)-F_i^TVF_i\end{bmatrix}.$$

$\bar{\Phi}_{ij}>0$ for all i,j if and only if

$$\begin{bmatrix}\frac{1}{\gamma^2}Z^{-1}(kh) & -G_jD_{21i} & A_d-G_jC_{2i} & 0\\ -D_{21i}^TG_j^T & \gamma^2I & 0 & 0\\ (A_d-G_jC_{2i})^T & 0 & \gamma^2Z(kh^-) & F_i^T\\ 0 & 0 & F_i & V^{-1}\end{bmatrix}>0$$

for all i,j. This completes the proof.

3 Application to Sampled-Data Systems

In this section, we shall give a method of designing a γ-suboptimal controller for fuzzy sampled-data systems, which is our main result in the paper. As will be noted in this section, a fuzzy sampled-data system is a special case of a fuzzy jump system. Thus we can apply the results in the previous section to sampled-data systems. First we shall show that fuzzy sampled-data systems can be written in the form of the fuzzy jump system (5). Consider the Takagi-Sugeno fuzzy system described by the following fuzzy rules:

IF ξ_1 is M_{i1} and $\cdots$ and ξ_q is M_{iq},
THEN $\dot{x}(t)=A_ix(t)+B_{1i}w(t)+B_{2i}\tilde{u}(t)$,
$z(t)=C_{1i}x(t)+D_{12}\tilde{u}(t)$,
$y_k=C_{2i}x(kh)+D_{21i}w_{dk}, i=1,\cdots,r$

where $\tilde{u}(t)\in\Re^m$ is the zero-order hold control input and all the matrices are of appropriate dimensions. Then the state, the controlled output and observation are defined as follows;

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^r\lambda_i(\xi(t))\{A_ix(t)+B_{1i}w(t)+B_{2i}\tilde{u}(t)\},\\ z(t)&=\sum_{i=1}^r\lambda_i(\xi(t))\{C_{1i}x(t)+D_{12}\tilde{u}(t)\},\\ y_k&=\sum_{i=1}^r\lambda_i(\xi(t))\{C_{2i}x(kh)+D_{21i}w_{dk}\}\end{aligned}\tag{11}$$

Since $\tilde{u}(t)$ is the zero-order hold input, it implies that $\tilde{u}(t)=u_k,\ kh<t<(k+1)h$ where h is a sampling time. That is, since the input $\tilde{u}(t)$ is constant between two sampling periods, we can take the following state space representation:

$$\dot{\bar{x}}=0,\ \bar{x}(kh^+)=u_k,\ kh<t<(k+1)h.$$

Clearly $\tilde{u}(t)=\bar{x}(t)$. If we define $x_e(t)=\begin{bmatrix}x^T & \bar{x}^T\end{bmatrix}^T$, then the fuzzy sampled-data system (11) becomes the following fuzzy jump system:

$$\begin{aligned}\dot{x}_e(t)&=\sum_{i=1}^r\lambda_i(\xi(t))\{\mathcal{A}_ix_e(t)+\mathcal{B}_{1i}w(t)\},\\&\qquad\qquad kh<t<(k+1)h,\\ x_e(kh^+)&=\mathcal{A}_dx_e(kh)+\mathcal{B}_2u_k,\ t=kh,\\ z(t)&=\sum_{i=1}^r\lambda_i(\xi(t))\mathcal{C}_{1i}x_e(t),\\ z_{dk}&=\sqrt{h}\mathcal{D}_{12}u_k,\\ y_k&=\sum_{i=1}^r\lambda_i(\xi(t))\{\mathcal{C}_{2i}x_e(kh)+\mathcal{D}_{21i}w_{dk}\}\end{aligned}$$

where

$$\mathcal{A}_i=\begin{bmatrix}A_i & B_{2i}\\ 0 & 0\end{bmatrix},\ \mathcal{A}_d=\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix},\ \mathcal{B}_{1i}=\begin{bmatrix}B_{1i}\\ 0\end{bmatrix},$$
$$\mathcal{B}_2=\begin{bmatrix}0\\ I\end{bmatrix},\ \mathcal{C}_{1i}=\begin{bmatrix}C_{1i} & 0\end{bmatrix},\ \mathcal{C}_{2i}=\begin{bmatrix}C_{2i} & 0\end{bmatrix},$$
$$\mathcal{D}_{12}=D_{12},\ \mathcal{D}_{21i}=D_{21i},$$

and $\sqrt{h}$ comes from

$$\begin{aligned}\int_0^\infty\tilde{u}^T(t)D_{12}^TD_{12}\tilde{u}(t)dt&=\sum_{k=0}^\infty\int_0^h u_k^T\mathcal{D}_{12}^T\mathcal{D}_{12}u_kdt\\&=\sum_{k=0}^\infty u_k^T(h\mathcal{D}_{12}^T\mathcal{D}_{12})u_k.\end{aligned}$$

Thus we can apply the jump system result in the previous section to obtain a γ-suboptimal controller for fuzzy sampled-data systems.

Theorem 3.1 *Suppose that there exist common matrices $X>0$, $Z>0$ and matrices $\mathcal{G}_j$ such that*

$$\begin{aligned}&\dot{X}(t)+\mathcal{A}_i^TX(t)+X(t)\mathcal{A}_i+\mathcal{C}_{1i}^T\mathcal{C}_{1i}\\&\quad+\frac{1}{\gamma^2}X(t)\mathcal{B}_{1i}\mathcal{B}_{1i}^TX(t)<0,\ kh<t<(k+1)h,\\&X(kh^-)-\mathcal{A}_d^TX(kh)\mathcal{A}_d+\mathcal{F}^T\mathcal{V}\mathcal{F}>0,\ t=kh,\\&\dot{Z}(t)+(\mathcal{A}_i+\frac{1}{\gamma^2}\mathcal{B}_{1i}\mathcal{B}_{1i}^TX(t))^TZ(t)\\&\quad+Z(t)(\mathcal{A}_i+\frac{1}{\gamma^2}\mathcal{B}_{1i}\mathcal{B}_{1i}^TX(t))<0,\ kh<t<(k+1)h,\\&\begin{bmatrix}\frac{1}{\gamma^2}Z^{-1}(kh) & -\mathcal{G}_j\mathcal{D}_{21i} & \mathcal{A}_d-\mathcal{G}_j\mathcal{C}_{2i} & 0\\ -\mathcal{D}_{21i}^T\mathcal{G}_j^T & \gamma^2I & 0 & 0\\ (\mathcal{A}_d-\mathcal{G}_j\mathcal{C}_{2i})^T & 0 & \gamma^2Z(kh^-) & \mathcal{F}^T\\ 0 & 0 & \mathcal{F} & \mathcal{V}^{-1}\end{bmatrix}>0,\\&\qquad\qquad t=kh,\quad\forall\ i,j\end{aligned}$$

where

$$\begin{aligned}\mathcal{V}(kh) &= h\mathcal{D}_{12}^T\mathcal{D}_{12}+\mathcal{B}_2^TX(kh)\mathcal{B}_2,\\ \mathcal{F}(kh) &= -\mathcal{V}^{-1}(kh)\mathcal{B}_2^TX(kh)\mathcal{A}_d.\end{aligned}$$

Then the controller

$$\begin{aligned}\dot{\hat{x}}(t) = &\sum_{i=1}^r \lambda_i(\xi(t))(\mathcal{A}_i+\frac{1}{\gamma^2}\mathcal{B}_{1i}\mathcal{B}_{1i}^TX(t))\hat{x}(t),\\ &\qquad\qquad kh<t<(k+1)h,\\ \hat{x}(kh^+) = &\sum_{i=1}^r\sum_{j=1}^r \lambda_i(\xi(t))\lambda_j(\xi(kh))\{\mathcal{A}_d\hat{x}(kh)+\mathcal{B}_2u_k\\ &+\mathcal{G}_j(y_k-\mathcal{C}_{2i}\hat{x}(kh))\},\ t=kh,\\ u_k = &\mathcal{F}\hat{x}(kh)\end{aligned}$$

is a γ-suboptimal controller.

4 Conclusion

We have considered the output feedback H_∞ control problem for the Takagi-Sugeno fuzzy sampled-data systems, and have given a design method of an H_∞ controller based on LMI's. First, a fuzzy jump system has been investigated. Then the result for a jump system has been applied to a fuzzy sampled-data system.

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