# Nonlinear Response Analysis of Multi-Story and Base Isolated Building Subjected to 2 Dimensional Ground Motion in Chaotic Dynamical System

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*Abstract*—— **This paper treats a numerical procedure computing nonlinear response of multi-story and base isolated building subjected to 2 dimensional ground motions in chaotic dynamical system. Idealized nonlinear stiffness of seismic isolators is presented and a nonlinear vibratory equation of the system is derived and solved directly by Runge-Kutta method. Nonlinear responses on 4 structural models enveloping eccentricity are shown and torsional behaviors and influence of initial condition in phase space to the response are discussed.**

## I . INTRODUCTION

Chaos theory has been studied in the many fields of science and the complex and nonperiodic behaviors in various vibration problems were discussed. For instance Ueda showed the famous chaos attractor called Japanese Attractor in the dynamical system governed by Duffing's equation in the late 1970's[1]. Sub-harmonic resonance and chaotic motion of offshore structures were summarized and published by Thompson and Stewart [2]. There essentially exists possibility of occurring these phenomena in the system governed by nonlinear equation. However the past dynamic analyses in the field of building structure never faced the real nonlinear phenomenon. Under these circumstances authors reviewed the past experiments and modelings on seismic isolator with hardening characteristics and discussed the nonlinear responses of one mass system idealized by Duffing's equation under sinusoidal and earthquake motions with large amplitudes, comparing other numerical solutions [3][4].

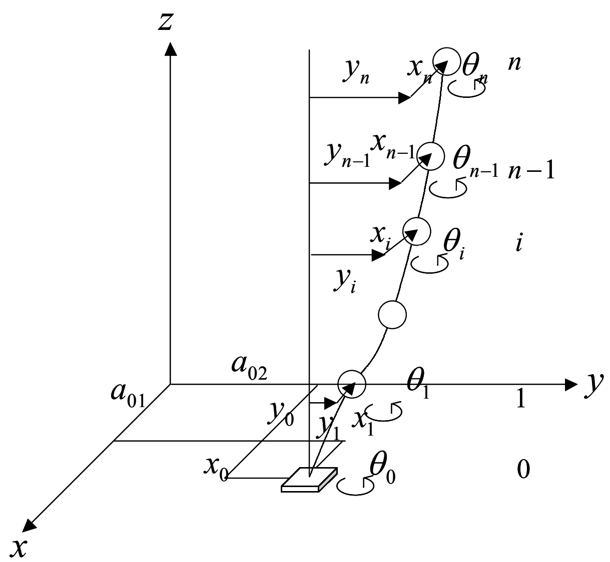



Figure 1. Multi-mass system subjected to 2D ground motion.

This paper presents a numerical method computing directly nonlinear responses of building subjected to 2 dimensional ground motions with large amplitudes and discuss nonlinear behaviors of multi-story and base isolated structure.

## II . IDEALIZED NONLINEAR RESTORNG FORCE OF SESMIC ISOLATOR

A building structure with eccentricity is assumed to be replaced into multi-mass system with 3 degrees of freedom in horizontal plane, $x_i$, $y_i$ and $\theta_i$ as shown Figure 1. Here the super structure is elastic and a hysteretic curve of seismic isolator [5][6] sustaining it is idealized by a cubic function shown in Figure 2. Since the isolators have circular cross sections and the same stiffnesses in arbitrary direction in plane, restoring force F of rigid base is expressed as

$$F = \sum_{i=1}^{n} ({}_i k_{H1} x + {}_i k_{H2} x^3) = (\sum_{i=1}^{n} {}_i k_{H1}) x + (\sum_{i=1}^{n} {}_i k_{H2}) x^3 = K_{H1} x + K_{H2} x^3 \tag{1}$$

,where ${}_i k_{H1}$ and ${}_i k_{H2}$ denote an elastic spring constant of i-th isolator and one on nonlinear term respectively. Here n is number of isolators. Restoring moment M is also expressed as

$$M = \sum_{i=1}^{n} \{ {}_i k_{H1} (l_{yi} \theta_0) + {}_i k_{H2} (l_{yi} \theta_0)^3 \} l_{yi} + \sum_{i=1}^{n} \{ {}_i k_{H1} (l_{xi} \theta_0) + {}_i k_{H2} (l_{xi} \theta_0)^3 \} l_{xi} \tag{2}$$

,where lxi and lyi denote distances in x and y directions from center of rotation to i-th isolator respectively. Here $\theta_0$ is a rotational angle of the rigid base. Equation (2) can be arranged into the following equation (3)

$$M = \{ \sum_{i=1}^{n} ({}_i k_{H1} l_{yi}^2 + k_{H1} l_{xi}^2) \} \theta_0 + \sum_{i=1}^{n} ({}_i k_{H2} l_{yi}^4 + k_{H2} l_{xi}^4) \theta_0^3 \} = K_{R1} \theta_0 + K_{R2} \theta_0^3 . \tag{3}$$

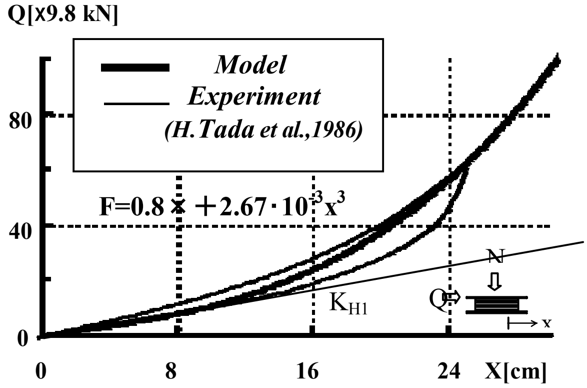



Figure 2. Idealized nonlinear hysteretic curve.

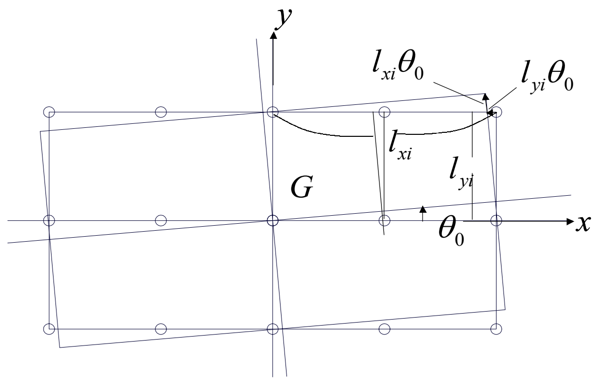



Figure 3. Rotation of a rigid base and horizontal displacements of isolators.

## III. NONLINEAR VIBRATORY EQUATION

Nonlinear vibratory equation with 3 degrees of freedom in horizontal plane is derived under the following assumptions.

1) The supper structure with eccentricity is elastic and a center of gravity of the rigid base sustained by the isolators is in accordance with a center of rigidity.
2) Nonlinear hysteretic curve of the isolator is idealized by a cubic function.

Nonlinear vibratory equation of the multi-mass system subjected to 2 dimensional ground motions shown Figure 1 can be expressed as

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} + \{\theta^3\} = -[m]\{\ddot{a}_0\} \tag{3}$$

,where [m] and [c] denote mass and damping matrices respectively. The stiffness matrix [k] can be written as

$$[k] = \begin{bmatrix}
\begin{bmatrix} K_{H1} + \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{xx}K_{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{xy}K_{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{x\theta}K_{ij} \\ \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{yx}K_{ij} & K_{H1} + \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{yx}K_{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{y\theta}K_{ij} \\ \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{\theta x}K_{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{\theta y}K_{ij} & K_{R1} + \sum_{i=1}^{n}\sum_{j=1}^{n} {}_{\theta\theta}K_{ij} \end{bmatrix} &
\begin{bmatrix} -\sum_{i=1}^{n} {}_{xx}K_{i1} & -\sum_{i=1}^{n} {}_{xy}K_{i1} & -\sum_{i=1}^{n} {}_{x\theta}K_{i1} \\ -\sum_{i=1}^{n} {}_{yx}K_{i1} & -\sum_{i=1}^{n} {}_{yy}K_{i1} & -\sum_{i=1}^{n} {}_{y\theta}K_{i1} \\ -\sum_{i=1}^{n} {}_{\theta x}K_{i1} & -\sum_{i=1}^{n} {}_{\theta y}K_{i1} & -\sum_{i=1}^{n} {}_{\theta\theta}K_{i1} \end{bmatrix} & \cdots &
\begin{bmatrix} -\sum_{i=1}^{n} {}_{xx}K_{in} & -\sum_{i=1}^{n} {}_{xy}K_{in} & -\sum_{i=1}^{n} {}_{x\theta}K_{in} \\ -\sum_{i=1}^{n} {}_{yx}K_{in} & -\sum_{i=1}^{n} {}_{yy}K_{in} & -\sum_{i=1}^{n} {}_{y\theta}K_{in} \\ -\sum_{i=1}^{n} {}_{\theta x}K_{in} & -\sum_{i=1}^{n} {}_{\theta y}K_{in} & -\sum_{i=1}^{n} {}_{\theta\theta}K_{in} \end{bmatrix} \\
\begin{bmatrix} -\sum_{i=1}^{n} {}_{xx}K_{1j} & -\sum_{i=1}^{n} {}_{xy}K_{1j} & -\sum_{i=1}^{n} {}_{x\theta}K_{1j} \\ -\sum_{i=1}^{n} {}_{yx}K_{1j} & -\sum_{i=1}^{n} {}_{yx}K_{1j} & -\sum_{i=1}^{n} {}_{yx}K_{1j} \\ -\sum_{i=1}^{n} {}_{\theta x}K_{1j} & -\sum_{i=1}^{n} {}_{\theta y}K_{1j} & -\sum_{i=1}^{n} {}_{\theta\theta}K_{1j} \end{bmatrix} &
\begin{bmatrix} {}_{xx}K_{11} & {}_{xy}K_{11} & {}_{x\theta}K_{11} \\ {}_{yx}K_{11} & {}_{yy}K_{11} & {}_{\theta\theta}K_{11} \\ {}_{xx}K_{11} & {}_{xx}K_{11} & {}_{xx}K_{11} \end{bmatrix} & \cdots &
\begin{bmatrix} {}_{xx}K_{1n} & {}_{xy}K_{1n} & {}_{x\theta}K_{1n} \\ {}_{yx}K_{1n} & {}_{yy}K_{1n} & {}_{y\theta}K_{1n} \\ {}_{\theta y}K_{1n} & {}_{\theta y}K_{1n} & {}_{\theta\theta}K_{1n} \end{bmatrix} \\
\vdots & \vdots & \ddots & \vdots \\
\begin{bmatrix} -\sum_{i=1}^{n} {}_{xx}K_{nj} & -\sum_{i=1}^{n} {}_{xy}K_{nj} & -\sum_{i=1}^{n} {}_{x\theta}K_{nj} \\ -\sum_{i=1}^{n} {}_{xy}K_{nj} & -\sum_{i=1}^{n} {}_{yy}K_{nj} & -\sum_{i=1}^{n} {}_{y\theta}K_{nj} \\ -\sum_{i=1}^{n} {}_{\theta x}K_{nj} & -\sum_{i=1}^{n} {}_{y\theta}K_{nj} & -\sum_{i=1}^{n} {}_{\theta\theta}K_{nj} \end{bmatrix} &
\begin{bmatrix} {}_{xx}K_{n1} & {}_{xy}K_{n1} & {}_{x\theta}K_{n1} \\ {}_{xy}K_{n1} & {}_{yy}K_{n1} & {}_{y\theta}K_{n1} \\ {}_{\theta x}K_{n1} & {}_{\theta y}K_{n1} & {}_{\theta\theta}K_{n1} \end{bmatrix} & \cdots &
\begin{bmatrix} {}_{xx}K_{nn} & {}_{xy}K_{nn} & {}_{x\theta}K_{nn} \\ {}_{yx}K_{nn} & {}_{yy}K_{nn} & {}_{y\theta}K_{nn} \\ {}_{\theta x}K_{nn} & {}_{\theta y}K_{nn} & {}_{\theta\theta}K_{nn} \end{bmatrix}
\end{bmatrix} . \tag{4}$$

Here the nonlinear term $\{\theta^3\}$ can be expressed as

$$\{\theta^3\} = \{K_{H2}x_0{}^3, K_{H2}y_0{}^3, K_{R2}\theta_0{}^3, 0, \cdot \cdot \cdot 0\}^T \qquad (5)$$

Other notations are as follows:

$$\{u\} = \{\{x_0, y_0, \theta_0\}, \{x_1, y_1, \theta_1\}, \cdot \cdot \cdot \{x_n, y_n, \theta_n\}\}^T \qquad (6)$$

$$[c] = \gamma[k], \quad \gamma = \frac{2h}{\varpi} \qquad (7)$$

h = damping coefficient, $\omega$ = natural circular frequency,
$\{a_0\}$ = 2 dimensional ground motions.

The nonlinear equation (3) is arranged into

$$\{\ddot{u}\} = -[m]^{-1}[c]\{\dot{u}\} - [m]^{-1}[k]\{u\} - [m]^{-1}\{\theta^3\} - \{\ddot{a}_0\} \qquad (8)$$

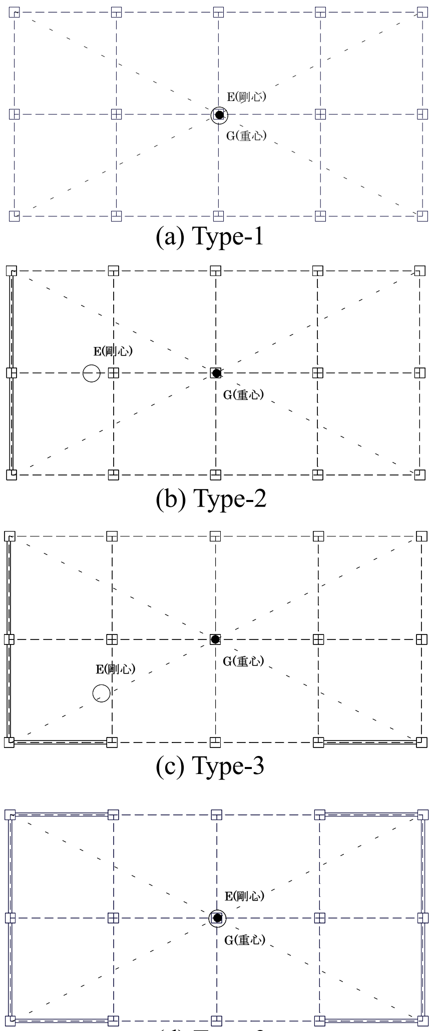
and solved directly by Runge-Kutta Method.

## IV. STRUCTURAL MODELS OF ANALYSIS

Four types of plans enveloping eccentric models are shown in Figure 4. Model Type-1 is a simple framework of reinforced concrete with no seismic wall and Type-2 has it one-sidedly in y direction. Type-3 has eccentricity in x and y directions. Type-4 with seismic walls in both directions has no eccentricity. Figure 5 shows a dimension of the framework and its story height is 3 meters and the span is set 6 meters. Mass, Young's modulus and other values are shown in Table1.

Table 1 List of parameters of response analysis.

| story | mass (kg) | monent of inertia (kg·cm$^3$) | story height (m) | E (kN/mm$^2$) | G (kN/mm$^2$) |
|---|---|---|---|---|---|
| 5 | $288\times10^3$ | $17277.4\times10^3$ | 3 | 20.58 | 7.84 |
| 4 | $288\times10^3$ | $17277.4\times10^3$ | 3 | 20.58 | 7.84 |
| 3 | $288\times10^3$ | $17277.4\times10^3$ | 3 | 20.58 | 7.84 |
| 2 | $288\times10^3$ | $17277.4\times10^3$ | 3 | 20.58 | 7.84 |
| 1 | $288\times10^3$ | $17277.4\times10^3$ | 3 | 20.58 | 7.84 |
| B | $576\times10^3$ | $34554.8\times10^3$ | 0.22 | 20.58 | 7.84 |
| column : 600×600, beam : 300×600 | | | | | |


(a) Type-1
(b) Type-2
(c) Type-3
(d) Type-3

● G : center of gravity
○ E : center of rigidity

Figure 4. Plan of structural models enveloping eccentricity.

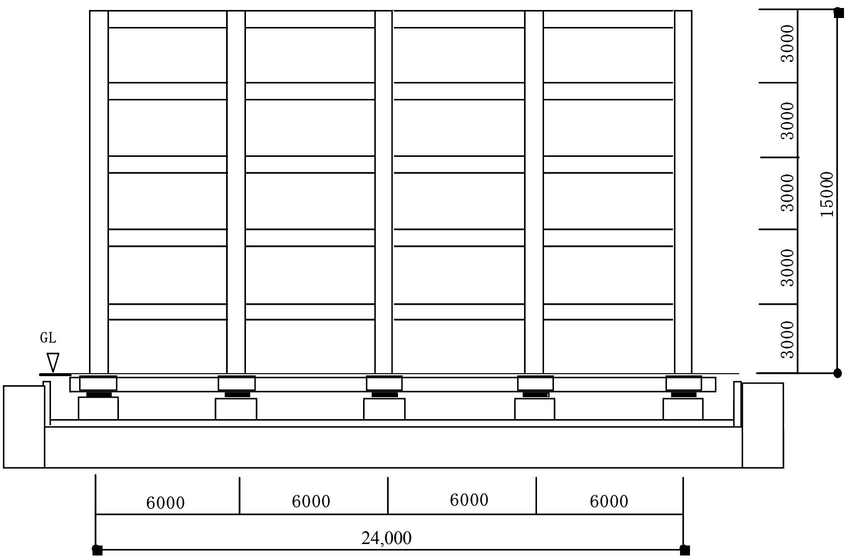


Figure 5. Reinforced concrete framework sustained by seismic isolators.

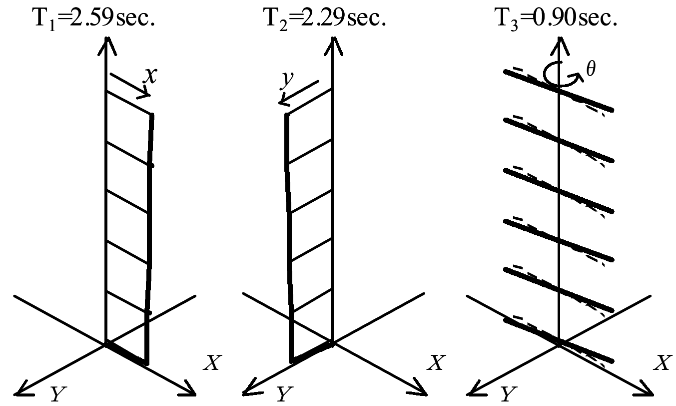
## V. ANALYTICAL RESULTS AND CONSIDERATION

### *A. Natural period and torsional mode*

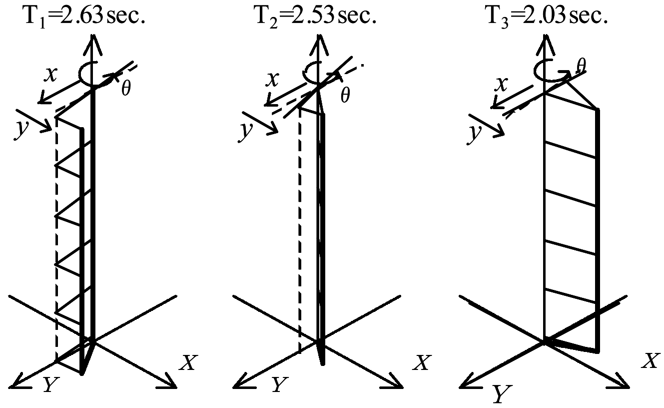
Vibratory modes and natural periods of the models of the structures should be considered in order to grasp the fundamental dynamic characteristics of them although it sometimes is difficult to anticipate nonlinear phenomenon by extending linear system.

Table 2 shows comparison of natural periods of the base isolated models with those under the condition of fixed base. The natural periods of the former are from 2.66 to 2.14 seconds while those of the latter are from 0.74 to 0.16 seconds. As for the 1st and 3rd modes in the fixed cases, eccentric seismic walls only shorten the natural periods lees than 21% since they cause torsional displacements. The seismic walls arrayed symmetrically do them about 78%. The eccentric walls attached to the base isolated models make the periods of the 1st and 2nd modes longer a little. However it seems almost negligible except Type-4 because almost of them is affected by the stiffness of isolators strongly.

Torsional behavior of the base isolated model was observed in the 2nd mode of Type-3 with eccentricity in x and y directions. The vibratory modes are shown in Figure 6 (b) with those of Type-1 in Figure 6 (a). It can be thought that the torsion has significant effect to response because the 2nd natural period of 2.53 seconds is extremely close to the first vibratory mode.




(a) Type-1, frame with no eccentricity




(b) Type-3, frame with eccentric wall

Figure 6. Natural period and base condition of structure.

### *B. Nonlinear response with torsion*

Figure 7 shows nonlinear responses of the base floor sustained by the isolators subjected simultaneously to El Centro 1940 NS and EW modified so that the maximum acceleration may reach 150 cm/sec (kine). Dampings are set 0% and 5% in each model and time increment is 0.001 seconds. Here the initial natural period is set 3 seconds by adjusting mass of the roof. Waves with nonperiodic and high frequency components can be observed in case of 0% damping. This trend is remarkable in Type-2 and Type-3 with eccentricity shown in Figure 7 (c), (d), (e) and (f). However the responses in 5% damping are stable. The maximum displacements of the isolators are about 50 centimeters. Additional devices such as an oil damper and equipments to reduce responses by hysteretic energy absorption are recommended if the super structure has to be kept elastic.

### *C. Effect of initial small displacement to response*

It is well known that the orbits of nonlinear responses are affected by initial condition in phase plane, which is normally given as a pair of displacement and velocity [2].

Figure 8 shows those of Type-2 and 3 models to the same

Table 2 Comparison of natural periods with base isolated building with those of fixed case.

| Type | base condition | 1st mode (sec) | 2nd mode (sec) | 3rd mode (sec) |
|---|---|---|---|---|
| Type-1 | fixed | 0.736, y-direction | 0.654, x-direction | 0.216, y-direction |
| | base-Isolated | 2.585, x-direction | 2.286, y-direction | 0.901, θ-direction |
| Type-2 | fixed | 0.647, x-direction | 0.568, y-direction | 0.204, x-direction |
| | base-Isolated | 2.655, y-direction | 2.627, x-direction | 0.421, x-direction |
| Type-3 | fixed | 0.578, y-direction | 0.245, x-direction | 0.173, y-direction |
| | base-Isolated | 2.632, y-direction | 2.531, θ-direction | 2.034, x-direction |
| Type-4 | fixed | 0.164, x-direction | 0.129, y-direction | 0.048, x-direction |
| | base-Isolated | 2.141, x-direction | 0.867, y-direction | 0.374, y-direction |

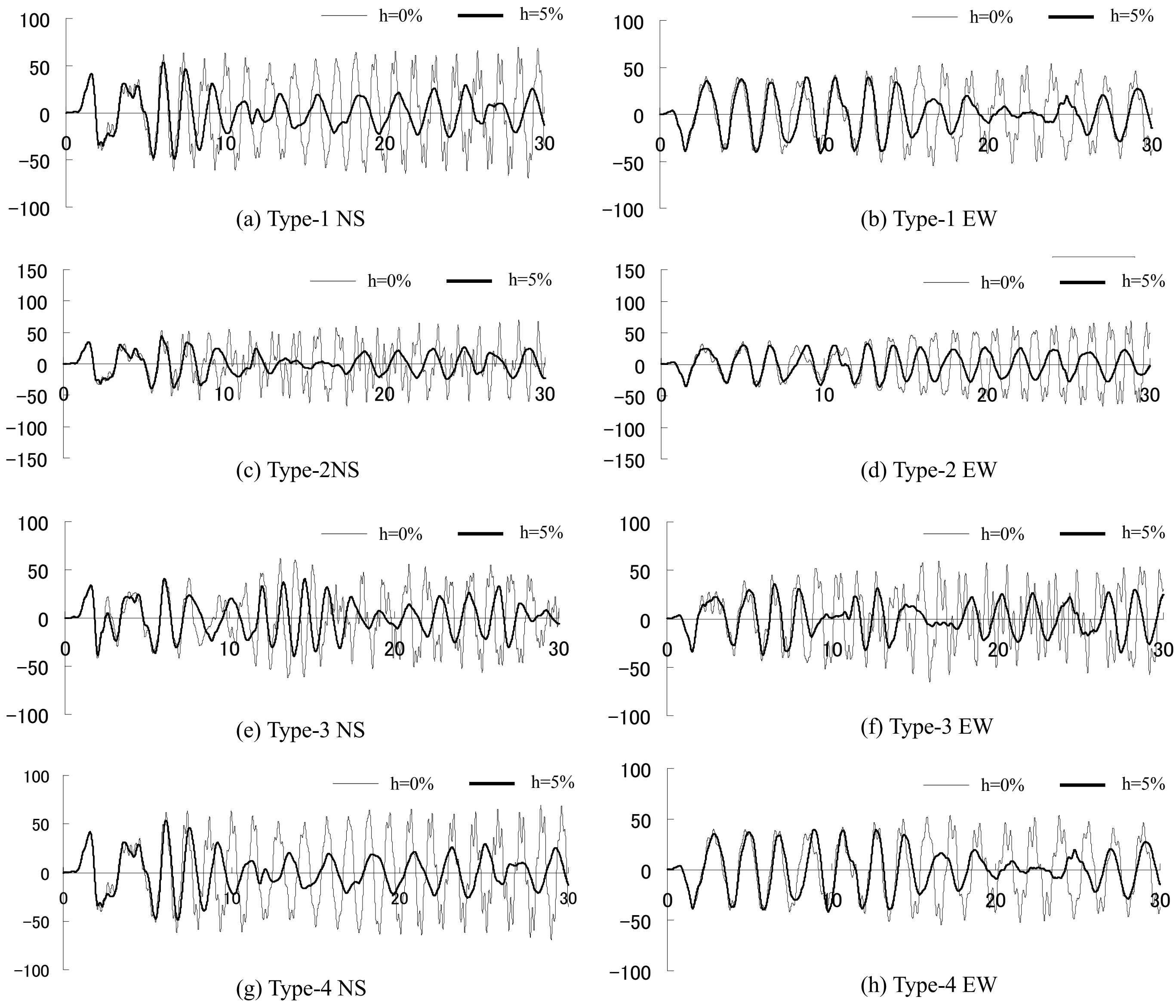



Figure 7. Influence of damping to nonlinear response with nonperiodic and high frequency component.

earthquake under the condition that they are given initial displacements of 2 centimeters in x and y directions. Thick line shows the above conditions and thin line means no initial displacement. The influence of them to response of Type-2 appears after 20 seconds in the time history of NS component (Figure 8 (a)) if 0% damping is given to the system. The same phenomenon can be observed in NS and EW components of response of Type-3 as shown in Figure 8 (e) and (f). Figure 8 (c), (d), (g) and (h) show no difference between thick and thin lines, which means the above nonlinear phenomena disappear if 5% damping is given. They also can't be observed in the structural model with no eccentricity such as Type-1 and 4 although the illustrations are omitted. Thus the above result suggests that array of seismic walls with eccentricity causes torsional vibration of structure and that sensitive nonlinear phenomenon appears if the damping of the system is extremely small and negligible as 0%.

## VI. CONCLUDING REMARKS

The following conclusions are summarized.

1) A numerical procedure to acquire mathematically nonlinear response of a base isolated building with eccentricity in supper structure was developed.
2) Nonperiodic and high frequency component can be observed in the models with eccentricity if the damping of the system is extremely small and negligible as 0%.
3) Influence of initial displacement to nonlinear response becomes larger in the structural models with eccentricity.

The above procedure adopts a hysteretic curve idealized by cubic function. Therefore it will be necessary to treat more

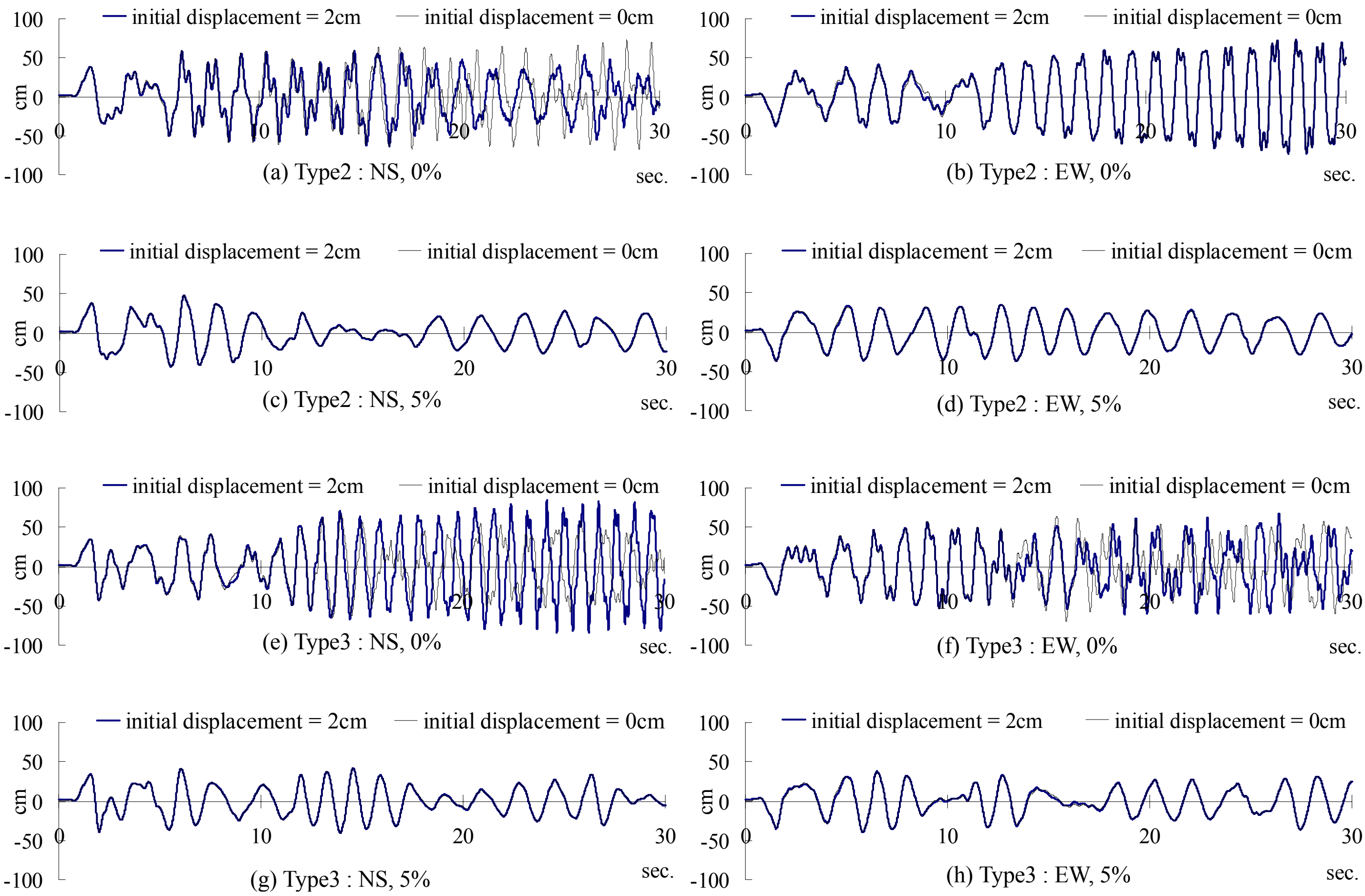



Figure 8. Influence of initial condition to nonlinear response of base isolated building.

realistic curve when applying it to design of a real structure.

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