Adaptive Fuzzy Control of Helicopter

Zonghua Jin, Wonchang Lee, Geuntaek Kang
Department of Electronic Engineering, Pukyong National University, KOREA
email: gtkang@pknu.ac.kr

Abstract - **This paper presents an adaptive fuzzy control scheme for nonlinear helicopter system which has uncertainty or unknown variations in parameters. The proposed adaptive fuzzy controller is a model reference adaptive controller. The parameters of fuzzy controller are adjusted so that the plant output tracks the reference model output. It is shown that the adaptive law guarantees the stability of the closed-loop system by using Lyapunov function. Several experiments with a small model helicopter having parameter variations are performed to show the usefulness of the proposed adaptive fuzzy controller.**

Ⅰ INTRODUCTION

This paper presents an adaptive control scheme for nonlinear system by using TSK(Takagi-Sugeno-Kang) fuzzy system. The TSK fuzzy model consists of TSK fuzzy rules and the consequents are linear equations instead of fuzzy sets. The TSK fuzzy rule is represented as a linear input-output relation in the fuzzy subspace specified at the premise. The TSK fuzzy model can be identified using input-output data and represent a nonlinear system very well with a few fuzzy rules.[1-3]

In recently, many adaptive fuzzy control methods are proposed. [4-7] This paper proposes an adaptive control algorithm for continuous nonlinear systems using TSK fuzzy system. The proposed adaptive fuzzy controller is a model reference adaptive controller. The parameters of fuzzy controller are adjusted so that the output of plant follows the output of reference model. It is shown that the adaptive rule guarantees the stability of the closed-loop system by using Lyapunov function.

Several experiments with a small model helicopter having parameter variations are performed to show the usefulness of the proposed adaptive fuzzy controller.

Ⅱ ADAPTIVE FUZZY CONTROL SYSTEM

In order to apply the adaptive fuzzy control scheme to be shown in this paper, at first a TSK fuzzy model of the plant to be controlled is made by using input-output data., and a TSK fuzzy controller is designed from the TSK fuzzy model. The parameters of the fuzzy controller are adjusted to the change of the plant by using the adaptive scheme. The adaptive rule is made so that the output of plans follows the output of reference model. The structure of the model reference adaptive fuzzy control system is shown in Fig. 1.

A. TSK Fuzzy Model

The *i*-th rule of a TSK fuzzy input-output model for continous time systems is as follows.

$$\begin{aligned} M^i:\ & \text{if } z_1 \text{ is } F_1^i,\ z_2 \text{ is } F_2^i,\ \cdots,\ z_l \text{ is } F_l^i \\ & \text{then } \frac{d^n y(t)^i}{dt^n} = a_0^i + a_1^i y(t) + a_2^i \frac{dy(t)}{dt} \\ & \qquad + \cdots + a_n^i \frac{d^{n-1} y(t)}{dt^{n-1}} + b_1^i u(t) \end{aligned} \tag{1}$$

where, z_j is a premise variable, F_j^i is a fuzzy set on z_j. Let $x_1 = y$, $x_2 = dy/dt$, $\cdots$ $x_n = dy^{n-1}/dt^{n-1}$, then the state variable representation of the equation (1) is as follows.

$$\begin{aligned} M^i:\ & \text{if } z_1 \text{ is } F_1^i,\ z_2 \text{ is } F_2^i,\ \cdots,\ z_l \text{ is } F_l^i \\ & \text{then } \frac{dx^i(t)}{dt} = A^i x(t) + b^i u(t) + d^i \\ & \qquad y(t) = c\, x(t) \end{aligned} \tag{2}$$

where

$$A^i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1^i & a_2^i & a_3^i & \cdots & a_n^i \end{bmatrix},\quad b^i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_1^i \end{bmatrix},\quad d^i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_0^i \end{bmatrix},$$

$$c = [\,1 \ \ 0 \ \ \cdots \ \ 0$$

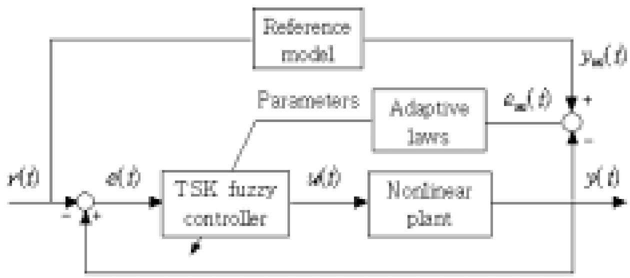


Fig. 1. The structure of model reference adaptive fuzzy control system

B. TSK Fuzzy Controller

The error state is defined as $\tilde{x}(t) = x(t) - x_d$, then the equation (2) is expressed as

$$\begin{aligned} M^i:\ & \text{if } z_1 \text{ is } F_1^i,\ z_2 \text{ is } F_2^i,\ \cdots,\ z_l \text{ is } F_l^i \\ & \text{then } \frac{d\tilde{x}^i(t)}{dt} = A^i \tilde{x}(t) + b^i u(t) + (d^i + A^i x_d) \\ & \qquad y(t) = c\,(\tilde{x}(t) + x_d) \end{aligned} \tag{3}$$

where x_d is the desired value of $x(t)$. The TSK fuzzy state controller is based on the TSK fuzzy state model of equation (3). The *i*-th rule of TSK fuzzy controller designed from the fuzzy model rule of the equation (3) is as follows.

$$C^l: \text{ if } z_1 \text{ is } F_1^l, \ z_2 \text{ is } F_2^l, \ \cdots, \ z_l \text{ is } F_l^l \quad \text{then } u^l(t) = -g^l \tilde{x}(t) + g_0^l \tag{4}$$

where the premise part of the rule C^l is the same with the premise part of the rule M^l, $g^l = (g_1^l \ g_2^l \cdots g_n^l)$ is a state feedback vector $(1 \times n)$, $g_0^l(t)$ is a scalar. g^l and g_0^l are obtained from the following equations.

$$\Phi = A^l - b^l g^l \tag{5}$$

$$g_0^l(t) = -(d^l(t) + A^l x_d) / b_1^l \tag{6}$$

where Φ is a desired state transition matrix. If the state transition matrix Φ is the form of equation (7) and stable, then the behaviour of the fuzzy model equation (3) controlled by the fuzzy controller equation (4) is the same as the linear system of which state transition matrix is the desired one Φ.

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 \\ \phi_1 & \phi_2 & \phi_3 & \cdot & \phi_n \end{bmatrix} \tag{7}$$

The control input u is obtained as follows.

$$u = \sum_{l=1}^{r} w^l(z) \ b_1^l u^l \ / \ \sum_{l=1}^{r} w^l(z) \ b_1^l \tag{8}$$

where r is the number of rules and $w^l(z)$ is as follows.

$$w^l(z) = \prod_{j=1}^{m} F_j^l(z_j) \tag{9}$$

where $F_j^l(z_j)$ is the membership value of z_j in the fuzzy set F_j^l.

C. Design of Adaptive Fuzzy Controller

Let the plant to be controlled be an *n*-th order nonlinear system as follows.

$$\begin{aligned} x^{(n)} &= f(x, \ \dot{x}, \ \ddot{x}, \cdots, x^{(n-1)}) \\ &\quad + g(x, \ \dot{x}, \ \ddot{x}, \cdots, \ x^{(n-1)}) u \\ y &= x \end{aligned} \tag{10}$$

where f and g are unknown nonlinear functions, u is the plant input, y is the plant output. Let the state variable $x = (x_1, \ x_2, \cdots, \ x_n)^T = (x, \ \dot{x}, \cdots, \ x^{(n-1)})^T$ can be observable. The ideal control input u^* for the system of the equation (10) is as follows.

$$u^* = \frac{1}{g(x)} [-f(x) + y_m^{(n)} + k^T e] \tag{11}$$

where y_m is the output of the reference model, $e = y_m - y = y_m - x$, $e = (e, \ \dot{e}, \ \ddot{e}, \cdots, \ e^{(n-1)})^T$. The elements of vector $k = (k_n, \cdots, \ k_1)^T$ are the roots of the polynomial equation $s^n + k_1 s^{n-1} + \cdots + k_n = 0$ and have negative real parts.

The control input obtained from the fuzzy controller of equation (8) is expressed as follows.

$$\begin{aligned} u &= \sum_{l=1}^{r} w^l b_1^l u^l \ / \ \sum_{l=1}^{r} w^l b_1^l \\ &= \frac{\sum_{l=1}^{r} w^l b_1^l (g_1^l \tilde{x}_1(t) + \cdots + g_n^l \tilde{x}_n(t) + g_0^l)}{\sum_{l=1}^{r} w^l \ b_1^l} \\ &= \frac{w^1 b_1^1 \tilde{x}_1(t) \ g_1^1}{\sum_{l=1}^{r} w^l b_1^l} + \frac{w^1 b_1^1 \tilde{x}_2(t) \ g_2^1}{\sum_{l=1}^{r} w^l b_1^l} + \cdots \\ &\quad + \frac{w^r b_1^r \tilde{x}_n(t) \ g_n^r}{\sum_{l=1}^{r} w^l b_1^l} + \frac{w^r b_1^r \ g_0^r}{\sum_{l=1}^{r} w^l b_1^l} \end{aligned} \tag{12}$$

The parameters of the controller $[g_1^1 \ g_2^1 \cdots g_n^r \ g_0^r]^T$ are adjusted so that the plant output follows the reference model output and the adaptive controller is adapted to the change of the plant. The control input u can be expressed as follows.

$$u = u_n(x \mid \theta) = \theta^T \phi(x) \tag{13}$$

where θ is to be adjusted and $\phi(x)$ can be measured.

$$\theta = [g_1^1 \ \ g_2^1 \cdots g_n^r \ \ g_0^r]^T \in R^{(n+1)r} \tag{14}$$

$$\phi(x) = \left(\frac{w^1 b_1^1 \tilde{x}_1(t)}{\sum_{l=1}^{r} w^l b_1^l} \ \ \frac{w^1 b_1^1 \tilde{x}_2(t)}{\sum_{l=1}^{r} w^l b_1^l} \cdots \ \frac{w^r b_1^r \tilde{x}_n(t)}{\sum_{l=1}^{r} w^l b_1^l} \ \ \frac{w^r b_1^r}{\sum_{l=1}^{r} w^l b_1^l} \right)^T \in R^{(n+1)r} \tag{15}$$

If the equations (11) and (13) are substituted to the equation (10), then the following error equation is obtained.

$$e^{(n)} = -k^T e + g(x)[u^* - u_n(x \mid \theta)] \tag{16}$$

The vector form of the error equation is as follows.

$$\frac{de}{dt} = Ae + b[u^* - u_n(x \mid \theta)] \tag{17}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & \cdots & \cdots & -k_1 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ g(x) \end{bmatrix} \tag{18}$$

Assume that the optimal parameters is as follows.

$$\theta^* = \arg \min_{\theta} \left[\sup_{x} |u_n(x \mid \theta) - u^*| \right] \tag{19}$$

Then the minimal error of fuzzy controller β is as follows.

$$\beta = u_n(x \mid \theta^*) - u^* \tag{20}$$

Using equations (13) and (20), the error equation (17) is

$$\frac{de}{dt} = Ae + b(\theta^* - \theta)^T \phi(x) - b\beta \tag{21}$$

The following Lyapunov function is used

$$V = \frac{1}{2} \ e^T Pe + \frac{g(x)}{2\gamma} (\theta^* - \theta)^T (\theta^* - \theta) \tag{22}$$

where γ is a positive constant, P is a positive definite matrix satisfying the Lyapunov equation (23), and. $g(x)>0$.

$$A^T P + PA = -Q \tag{23}$$

where Q is an $n \times n$ positive definite matrix and the matrix A is shown in equation (18). Using equations (20) and (22), the time derivative of V, $\frac{dV}{dt}$, is as follows.

$$\begin{aligned}\frac{dV}{dt} = &-\frac{1}{2}\, e^T Q e + e^T P b[(\theta^* - \theta)^T \phi(x) \\ &- \beta] - \frac{g(x)}{\gamma}(\theta^* - \theta)^T \frac{d\theta}{dt}\end{aligned} \tag{24}$$

If the last column of the positive definite matrix P is expressed as p_n, then $e^T P b = e^T p_n g(x)$ from $b = (0, \cdot\cdot\cdot, g(x))^T$. And the equation (24) is

$$\begin{aligned}\frac{dV}{dt} = &-\frac{1}{2}\, e^T Q e + \frac{g(x)}{\gamma}(\theta^* \\ &- \theta)^T[\gamma\, e^T p_n \phi(x) - \frac{d\theta}{dt}] - e^T p_n g(x)\beta\end{aligned} \tag{25}$$

Using the following adaptive rule,

$$\frac{d\theta}{dt} = \gamma\, e^T p_n \phi(x) \tag{26}$$

, the time derivative of Lyapunov function, dV/dt is

$$\frac{dV}{dt} = -\frac{1}{2}\, e^T Q e - e^T p_n g(x)\beta \tag{27}$$

The fuzzy controller is designed from the fuzzy model which represents the system very well, and the adaptive rule of equation (26) is used to adjust the parameters of the fuzzy controller to the change of the system. Hence, the β is very small, so that $|e^T p_n g(x)\beta| < \frac{1}{2}\, e^T Q e$ and the controlled system is stable.

Ⅲ ADAPTIVE FUZZY CONTROL OF HELICOPTER

A. Helicopter system

The proposed adaptive fuzzy controller is tested to a model small helicopter in laboratory. The helicopter consists of a base upon which an arm is mounted. the arm carries the helicopter body on one end and a counterweight on the other. The arm can pitch about an elevation axis as well as swivel about a vertical (travel) axis. Encoders mounted on these axis allows for measuring the elevation and travel of the arm. The helicopter mounted at the end of the arm is free to swivel about a pitch axis. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to the motors.

The purpose of the experiment in this paper is to design a adaptive fuzzy controller that makes the helicopter body have a desired travel rate.

The only way to apply a force in the travel direction is to pitch body of helicopter. Assume the body has pitched up by an angle p as shown in Fig. 2 The horizontal component of force F_g will cause a torque about that the travel axis which results in an acceleration about the travel axis.

$$J_t \dot{r} = -F_g \sin(p) L_a \tag{28}$$

where r is the travel rate

The pitch axis is controlled by the difference of the forces generated by the propellers

$$\begin{aligned} J_p \ddot{p} &= F_f L_h - F_b L_h \\ &= K_f L_h (V_f - V_b) \end{aligned} \tag{29}$$

where J_p is the moment of inertia of the helicopter body about the pitch axis, V_f and V_b are are the voltage applied to the front and back motors, K_f is the force constant of the motor/propeller combination., and L_h is the distance from the pitch axis to either moor.

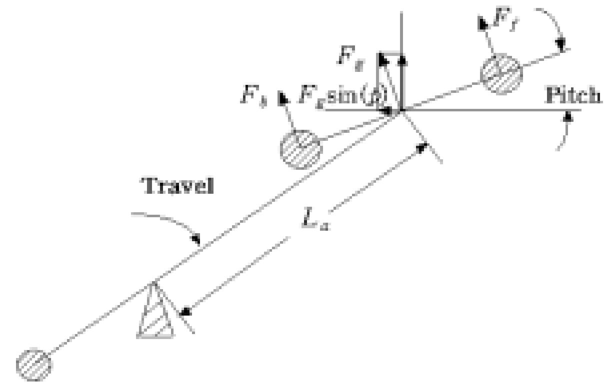


Fig. 2. The structure of the helicoper system

B. Travel rate control

Because the travel axis system is nonlinear as shown in equation (28), a fuzzy controller is need to control the travel rate. A TSK fuzzy model of the travel axis system is identified and the TSK fuzzy controller is designed from the TSK fuzzy model by using the algorithm explained above. The TSK fuzzy controller is as follows.

$$\begin{aligned} C^1 &: \textit{if } p \textit{ is } F_1 \textit{ then } u^1(t) = 5.843\tilde{x}(t) + 4.294 \\ C^2 &: \textit{if } p \textit{ is } F_2 \textit{ then } u^2(t) = 1.961\tilde{x}(t) \\ C^3 &: \textit{if } p \textit{ is } F_3 \textit{ then } u^3(t) = 5.843\tilde{x}(t) - 4.294 \end{aligned} \tag{30}$$

The fuzzy sets in equation (30) are shown in Fig. 3.

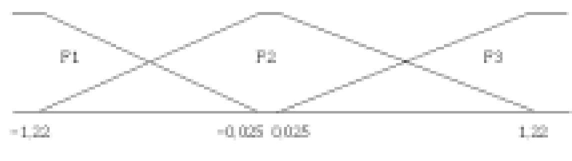


Fig. 3. The fuzzy sets in the fuzzy model (30)

The pitch axis system is a linear system of equation (29). So a linear controller is designed from the linear system, and the linear controller is used for the helicopter body to keep the desired pitch angle obtained from the TSK fuzzy controller of equation (30).

The parameters of the helicopter system may be changed and the adaptive rule of equation (26) is used for the fuzzy controller of equation (30). The structure of the travel rate adaptive fuzzy control system is shown in Fig. 4.

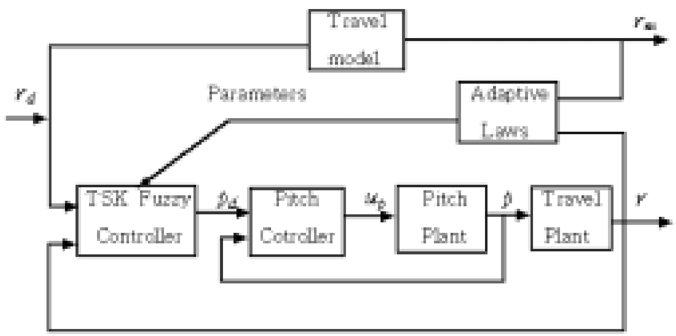


Fig. 4.. Travel rate adaptive fuzzy control system

C. Simulation and experiment results

Computer simulation results of the adaptive fuzzy control are shown in Fig. 4. At the simulation, a disturbance of size 0.15 is added to the output at the time 15 sec. and removed at the time 35 sec. It can be shown that the output controlled by the adaptive fuzzy controller follows the desired value more rapidly than the output controlled by the nonadaptive fuzzy controller.

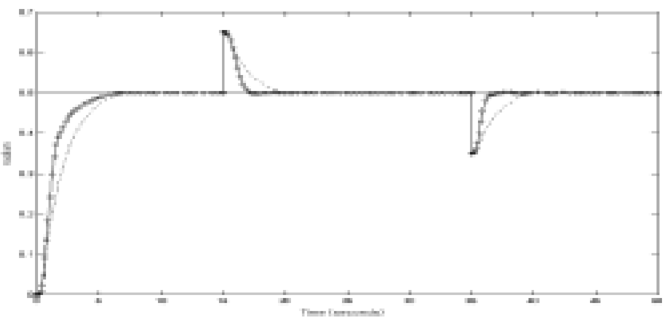
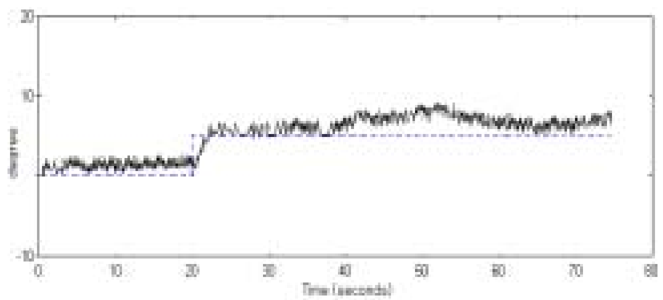


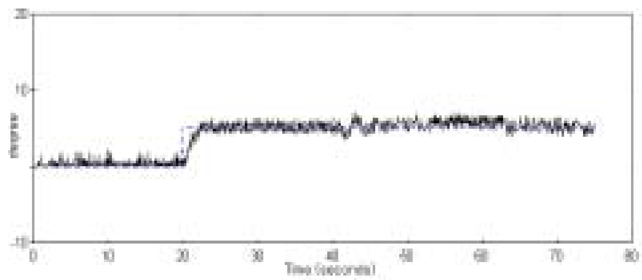
Fig. 4. Simulation results of Travel axis system (—— the desired output, —⊖— responses of the adaptive fuzzy controller, ·········· responses of the fuzzy controller)

The model helicopter experiment results are shown in Fig. 5. At the experiment, a disturbance weight of 0.1kg is put at the crossing point of the pitch axis and the travel axis at the time about 40 sec.

In the Fig. 5(a), it can be shown that the output controlled non adaptive fuzzy controller has large steady state error and does not follow the desired value after the disturbance weight is put. However, as shwon in Fig. 5 (b), if the helicopter is controlled by the adaptive fuzzy controller, the output follows the desired value very well.



(a) Results controlled by nonadaptive fuzzy controller



(b) Results controlled by adaptive fuzzy controlled

Fig. 5. Experiment results of travel axis system (—— : the plant output, ·········· : the desired output)

Ⅳ CONCLUSION

This paper proposed a model reference adaptive controller. The parameters of fuzzy controller are adjusted so that the output of plant tracks the reference model. It is shown that the adaptive law guarantees the stability of the closed-loop system by using Lyapunov function.

Several experiments with a small model helicopter having parameter variations are performed, the results reveal that suggested methods are practically feasible.

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