On the Design of Adaptive Fuzzy Logic Controllers

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Abstract – A simple fuzzy logic controller (FLC) is introduced to simplify the design of the conventional FLC. It uses a single fuzzy input variable to represent the rule antecedent. The FLC has an adaptation capability in itself. However additive adaptation algorithm can more or less improve the performance of a conventional fuzzy logic controller. We here discuss the design of some single-input adaptive fuzzy logic controllers based on the concept of the single input FLC.

I. INTRODUCTION

Most FLC's use the error and the change-of-error as antecedent variables of if-then rules regardless of the complexity of the controlled plants. Either control input or incremental control input is typically used as a consequent variable [1]. Such FLC's are suitable for simple lower order plants. All process states are typically required for a good performance of complex higher order plants. Choi *et al.* proposed a single-input FLC that uses a sole input variable consists in all state variables [2]. It gave a useful method to simplify the design process for a proper FLC.

A fuzzy logic system is a nonlinear approximator. Furthermore it has an adaptation capability in itself. The FLC has emerged as one of the most active areas in the application of the fuzzy set theory. They are useful in situations where 1) there is no acceptable mathematical model for the plant to be controlled and 2) there are experienced human operators who can adequately control the plant by some qualitative control rules.

Although the FLC has a kind of adaptability in itself, it still lacks in the case of some complicated situations, where the operating conditions can be subject to change. Wang developed an adaptive fuzzy logic control method that ensures the stability of the overall system using a Lyapunov-based learning law [3]. He presented here a fuzzy basis function that provides a natural framework for combining numerical and linguistic information in a uniform fashion. Despite its advantages it has some drawbacks: it can occur a kind of high control action due to its supervisory control input, and it must adapt itself to every change of the reference signal. Su and Stepanenko introduced a modified version of this approach that incorporates a variable structure controller to keep the system state within defined boundaries [4]. Fischle and Schroder [5] presented some improved stable adaptive fuzzy control methods for resolving some drawbacks existed in [3]. Besides these direct adaptive FLC's, many indirect or hybrid adaptive FLC's also published in the related fields [6-8]. Park *et al* developed an indirect adaptive control algorithm robust against the reconstruction errors using fuzzy systems for single input single output nonlinear systems with unknown nonlinearities[9].

In this paper we discuss the design of some adaptive fuzzy logic controllers equipped with adaptation algorithms based on the stability analysis. Some linguistic fuzzy information from experienced human operators is incorporated into the closed-loop control

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system through a fuzzy basis function. This is especially useful to the complex systems with high nonlinearities and uncertainties and improves the control performance.

We first explain a single input FLC that uses a sole variable in the antecedent part of the fuzzy control rule, and then propose some stable adaptive FLC's that automatically adjust some parameters of the single input FLC. This paper is organized as follows. We simply describe the design strategy for the single input FLC in Section Ⅱ. Section Ⅲ is a core and we here propose some adaptive fuzzy logic controllers designed by the following schemes: 1) using a Hurwitz error dynamics, 2) utilizing a switching function of the sliding mode control (SMC), 3) using a gradient decent method, and 4) using a recursive least square algorithm. In Section Ⅳ we represent general discussions, respectively.

Ⅱ. SINGLE INPUT FLC

We first consider a 2-nd order system, and then extend to a n-th order case.

Let the controlled process be a system with n-th order (linear or nonlinear) state equation:

$$\begin{aligned} x^{(n)} &= f(\mathrm{x}, t) + b(\mathrm{x}, t)u(t) + d(t), \\ y &= x, \end{aligned} \tag{1}$$

with

$$\begin{aligned} \mathrm{x} &= [x_1, x_2, \cdots, x_n]^T \\ &= [x, \dot{x}, \cdots, x^{(n-1)}]^T, \end{aligned} \tag{2}$$

where $f(\mathrm{x}, t)$ and $b(\mathrm{x}, t)$ are partially known continuous functions, $d(t)$ is the unknown external disturbance, and $u(t) \in R$ and $y(t) \in R$ are the input and output of the system, respectively. $\mathrm{x}(t) \in R^n$ is the process state vector.

The control problem is to force $y(t)$ to follow a given bounded reference input signal $x_d(t)$. Let $e(t)$ be the tracking error vector as follows

$$\begin{aligned} \mathrm{e}(t) &= \mathrm{x}(t) - \mathrm{x_d}(t) \\ &= [e, \dot{e}, \cdots, e^{(n-1)}]^T. \end{aligned} \tag{3}$$

The rule form for the conventional FLC using two fuzzy input variables of the error and the change-of-error is as follows:

$$R^{ij}_{old}: \text{ If } e \text{ is } LE_i \text{ and } \dot{e} \text{ is } LDE_j, \text{ then } u \text{ is } LU_{ij}$$

where $i = 1, 2, \cdots, M$, $j = 1, 2, \cdots, N$, and LE, LDE, and LU are the linguistic values taken by the process state variables e, $\dot{e}$, and u, respectively.

Table 1. Rule table for the conventional FLC.

$\dot{e}$ \ e	LE_{-2}	LE_{-1}	LE_0	LE_1	LE_2
LDE_2	LU_0	LU_{-1}	LU_{-1}	LU_{-2}	LU_{-2}
LDE_1	LU_1	LU_0	LU_{-1}	LU_{-1}	LU_{-2}
LDE_0	LU_1	LU_1	LU_0	LU_{-1}	LU_{-1}
LDE_{-1}	LU_2	LU_1	LU_1	LU_0	LU_{-1}
LDE_{-2}	LU_2	LU_2	LU_1	LU_1	LU_0

In Table 1, subscripts −2, −1, 0, 1, and 2 denote fuzzy linguistic values of Negative Big (NB), Negative Small (NS), ZeRo (ZR), Positive Small (PS), and Positive Big (PB), respectively.

Conventional FLC's for minimum-phase systems have skew-symmetric rule tables similar to Table 1. Then Table 1 can be reduced by Table 2.

Table 2. Rule table for the single input FLC.

d_s	LDL_{-2}	LDL_{-1}	LDL_0	LDL_1	LDL_2
u	LU_2	LU_1	LU_0	LU_{-1}	LU_{-2}

That is, a single input FLC can be designed from the conventional FLC with the skew-symmetric rule table. In Table 2, d_s is a variable defined as follows:

$$d_s = \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}}, \tag{4}$$

where $\lambda > 0$ is a design parameter. And the rule for the single input FLC has the following form:

$$R^k_{new}: \text{ If } d_s \text{ is } LDL_k \text{ then } u \text{ is } LU_k,$$

where LDL_k is the linguistic value of d_s in the k-th rule.

Consider an n-input FLC with the following rule form:

$$R_{GO}^{k}:$$

$$\text{If } e_1 \text{ is } LE_k^1,\ e_2 \text{ is } LE_k^2,\ \cdots,\ \text{and } e_n \text{ is } LE_k^n \text{ then } u \text{ is } LU_k,$$

where $k = 1, 2, \cdots, m^n$, m is the number of fuzzy sets for each fuzzy input variable and LE_k^i ($i = 1, 2, \cdots, n$) is the linguistic value taken by the process state variable $e_i\ (= x^{(i-1)} - x_d^{(i-1)})$ in the k-th rule. In this case, the rule table is established on n-dimensional space of e_1, e_2, $\cdots$, and e_n.

Even this case an n-dimensional rule table can be reduced as Table 2 if the controlled plant has the minimum-phase property. Here d_s is replaced by D_s defined as follows:

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots \lambda_2\dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \cdots \lambda_2^2 + \lambda_1^2}}\ . \qquad (5)$$

Ⅲ. DESIGN OF ADAPTIVE FLC's

A fuzzy IF-THEN rule can directly be expressed by a rigorous mathematical equation. Let LDL^k be a fuzzy set in U, then the fuzzy logic system with the singleton fuzzifier, product inference, and the height defuzzifier is of the following form:

$$u(D_s) = \frac{\sum_{k=1}^{K} \overline{u}^k (\mu_{LDL^k}(D_s))}{\sum_{k=1}^{K} (\mu_{LDL^k}(D_s))}, \qquad (6)$$

where $\overline{u}^k$ is the point in R at which μ_{LU^k} achieves its maximum value (assume that $\mu_{LU^k}(\overline{u}^k) = 1$), and K is the number of one-dimensional control rules. And the fuzzy basis function (FBF) is summarized as:

$$\xi^k(D_s) = \frac{\mu_{LDL^k}(D_s)}{\sum_{k=1}^{K} (\mu_{LDL^k}(D_s))}. \qquad (7)$$

Therefore an one-dimensional fuzzy control rule R_{new}^k can be expressed as a rigorous mathematical formula:

$$u(D_s) = \Theta_u^T \Xi_u(D_s), \qquad (8)$$

where $\Theta_u = [\ \overline{u}^1,\ \overline{u}^2, \cdots,\ \overline{u}^K]^T$ is an adjustable parameter vector, and $\Xi_u(D_s) = [\xi^1(D_s), \xi^2(D_s), \cdots, \xi^K(D_s)]^T$ is a regressive vector.

The control purpose is to determine a feedback control input

$$u = u_f(D_s|\Theta_u) + u_a \qquad (9)$$

such that the tracking error should be as small as possible under some constraints, where u_f is a control law by the single input FLC and u_a is an auxiliary control input to ensure the closed-loop stability.

A. Design by Hurwitz Error Dynamics :

$$u_1 = u_{f1}(D_s|\Theta_{u1}) + u_{a1}$$

Let $c = [c_n, c_{n-1}, \cdots, c_1]^T \in R^n$ be a real valued vector such that all roots of the polynomial $h(s) = s^n + c_1 s^{n-1} + \cdots + c_n$ are in the open left-half plane, where s is the Laplace variable.

If the functions f, b, and d are known in the controlled plant (1), then the control law is as follows:

$$u_1^* = b^{-1}(-f - d + x_d^{(n)} - c^T e), \qquad (10)$$

where e is the tracking error vector that is given by Eq. (3). Substituting Eq. (10) into Eq. (1), the following error dynamics is obtained.

$$e^{(n)} + c_1 e^{(n-1)} + \cdots + c_n e = 0. \qquad (11)$$

Since Eq. (11) is a Hurwitz from the definition of the constant parameter vector c, $\lim_{t\to\infty} e(t) = 0$.

However, we don't know exact information about the functions f, b, and d, except for the sign of b(x,t). Substituting Eq. (9) into Eq. (1) and adding and subtracting bu_1^* in the right hand side, Eq. (1) is summarized as follows:

$$\begin{aligned} x^{(n)} &= f + b(u_{f1} + u_{a1}) + d + bu_1^* - bu_1^* \\ &= b(u_{f1} - u_1^*) + bu_{a1} + x_d^{(n)} - c^T e. \end{aligned} \qquad (12)$$

It can also be rewritten as Eq. (13).

$$\dot{\mathrm{e}} = \mathrm{Ce} + \mathrm{B}(u_{f1} - u_1^*) + \mathrm{B}u_{a1}, \tag{13}$$

where

$$\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ -c_n & -c_{n-1} & -c_{n-2} & \cdots & -c_1 \end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ b \end{bmatrix}. \tag{14}$$

Now we define the optimal parameter Θ_{u1}^* and the minimum approximation error related to the control input ε_{u1} as follows:

$$\Theta_{u1}^* = \arg\min_{\Theta_{u1}}[\sup_{x}|u_{f1}(D_s|\Theta_{u1}) - u_1^*|] \tag{15}$$

and

$$\varepsilon_{u1} = u_{f1}^* - u_1^*, \tag{16}$$

where $u_{f1}^* = u_{f1}(D_s|\Theta_{u1}^*)$. Furthermore ε_{u1} will maintain a very small value due to the universal approximating property of the fuzzy logic system [9]. That is,

$$|\varepsilon_{u1}| = |u_{f1}^* - u_1^*| \le \varepsilon, \tag{17}$$

where $\varepsilon > 0$ is a small value. Then the error equation (12) can be rewritten as

$$\begin{aligned} \dot{\mathrm{e}} &= \mathrm{Ce} + \mathrm{B}(u_{f1} - u_{f1}^*) + \mathrm{B}\varepsilon_{u1} + \mathrm{B}u_{a1} \\ &= \mathrm{Ce} + \mathrm{B}\Phi_{u1}^T\Xi_{u1} + \mathrm{B}\varepsilon_{u1} + \mathrm{B}u_{a1}. \end{aligned} \tag{18}$$

where $\Phi_{u1} = \Theta_{u1} - \Theta_{u1}^*$ and Ξ_{u1} is the FBF.

Now we replace the $u_{f1}(D_s|\Theta_{u1})$ by a fuzzy logic system (8) and develop an adaptive law to update the parameter vector Θ_{u1}. It is obtained by the followings:

Consider a control law (10) and a stable error dynamics (11). If we choose the auxiliary control input u_{a1} as

$$u_{a1} \le -sgn(\mathrm{e^T PB})|\varepsilon_{u1}|, \tag{19}$$

then the proposed system is stable in the sense of the Lyapunov and the parameter adaptation law is given as

$$\dot{\Theta}_{u1} = -sgn(b)\gamma_1 \mathrm{e^T P_n}\Xi_{u1}, \tag{20}$$

where P is a positive definite symmetric n×n matrix that satisfies the Lyapunov equation

$$\mathrm{C^T P + PC} = -\mathrm{Q}. \tag{21}$$

Here, Q is an arbitrary positive definite matrix. $\gamma_1 > 0$ is a constant that determines a kind of learning rate, and $\mathrm{P_n}$ is the last column of P.

The proposition is proved by as follows:

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}\mathrm{e^T Pe} + \frac{|b|}{2\gamma_1}\Phi_{u1}^T\Phi_{u1}. \tag{22}$$

Then,

$$\begin{aligned} \dot{V}_1 &= -\frac{1}{2}\mathrm{e^T Qe} + \mathrm{e^T PB}(\Phi_{u1}^T\Xi_{u1} + \varepsilon_{u1} + u_{a1}) \\ &\quad + \frac{|b|}{\gamma_1}\Phi_{u1}^T\dot{\Phi}_{u1} \\ &= -\frac{1}{2}\mathrm{e^T Qe} + \frac{|b|}{\gamma_1}\Phi_{u1}^T(\dot{\Phi}_{u1} + sgn(b)\gamma_1\mathrm{e^T P_n}\Xi_{u1}) \\ &\quad + \mathrm{e^T PB}\varepsilon_{u1} + \mathrm{e^T PB}u_{a1}. \end{aligned} \tag{23}$$

From Eq. (23) we can get the following parameter adaptation law:

$$\dot{\Phi}_{u1} = -sgn(b)\gamma_1\mathrm{e^T P_n}\Xi_{u1}. \tag{24}$$

Since $\dot{\Phi}_{u1} = \dot{\Theta}_{u1}$, Eq. (24) is equivalent to the adaptation law (20). Also if we choose the auxiliary control input such that the given condition (19) is satisfied, then Eq. (23) is summarized as follows:

$$\dot{V}_1 \le -\frac{1}{2}\mathrm{e^T Qe}. \tag{25}$$

Thus, the proposed adaptive fuzzy logic controller is stable in the sense of the Lyapunov.

□

B. Design by Switching Function of SMC : $u_2 = u_{f2}(D_s|\Theta_{u2}) + u_{a2}$

A method designed above requires some tuning parameters such as c_i $(i=1, 2, \cdots, n)$, and it makes the design of an adaptive FLC somewhat difficult. So we propose another method. It uses a switching function of the sliding mode control. Then the number of tuning parameters is also reduced.

Consider the following switching function $S_l = 0$ that is used in SMC:

$$\begin{aligned} S_l &= 0 \\ &= e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots + \lambda_2\dot{e} + \lambda_1 e. \end{aligned} \tag{26}$$

We first determine the control law u_2^* when the functions f, b, and d of the controlled plant (1) are known. Here two cases must independently be considered: $S_l = 0$ and $S_l \neq 0$.

In the case of $S_l = 0$, the control law is easily determined by the following equation.

$$u_2^* = b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)}\right). \tag{27}$$

As $S_l \neq 0$, the control law can be derived from the concept of the SMC. That is, it can be determined by the following sliding condition:

$$S_l \dot{S}_l \leq -\eta|S_l|, \tag{28}$$

where η is a positive constant. From Eq. (26),

$$\begin{aligned} \dot{S}_l &= e^{(n)} + \lambda_{n-1}e^{(n-1)} + \cdots \lambda_1\dot{e} \\ &= f + bu_2 + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)}. \end{aligned} \tag{29}$$

Multiplying both sides of Eq. (29) by S_l,

$$\begin{aligned} S_l \dot{S}_l &= S_l\left(f + bu_2 + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)}\right) \\ &= -\eta|S_l|. \end{aligned} \tag{30}$$

From Eq. (30),

$$\begin{aligned} u_2^* &\leq b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} - \eta\right) \quad \text{for } S_l > 0, \\ u_2^* &\geq b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} + \eta\right) \quad \text{for } S_l < 0. \end{aligned} \tag{31}$$

Combining Eq. (27) and (31), we obtain the following closed form for the control law.

$$u_2^* = b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} - \rho\, sgn(S_l)\,\eta_m\right), \tag{32}$$

where $\rho = \begin{cases} 1 & \text{for } S_l \neq 0 \\ 0 & \text{for } S_l = 0 \end{cases}$ and $\eta_m \geq \eta$.

However we don't know exact information about the controlled plant (1) except for the sign of b(x,t). Adding and subtracting bu_2^* in the right side of Eq. (29),

$$\begin{aligned} \dot{S}_l &= f + b(u_{f2} + u_{d2}) + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)} \\ &\quad + bu_2^* - bu_2^* \\ &= b(u_{f2} - u_2^*) + bu_{d2} - \rho\, sgn(S_l)\,\eta_m. \end{aligned} \tag{33}$$

Consider another optimal parameter Θ_{u2}^* and minimum approximation error related to the control input ε_{u2}:

$$\Theta_{u2}^* = \arg\min_{\Theta_{u2}}[\sup_x |u_{f2}(D_s|\Theta_{u2}) - u_2^*|], \tag{34}$$

and

$$\varepsilon_{u2} = u_{f2}^* - u_2^*, \tag{35}$$

where $u_{f2}^* = u_{f2}(D_s|\Theta_{u2}^*)$. ε_{u2} also has a very small value due to the universal approximating property of the fuzzy logic system [9]. That is,

$$|\varepsilon_{u2}| = |u_{f2}^* - u_2^*| \leq \varepsilon, \tag{36}$$

where $\varepsilon > 0$ is a small value. And Eq. (33) can be rewritten as

$$\begin{aligned} \dot{S}_l &= b(u_{f2} - u_{f2}^*) + b\varepsilon_{u2} + bu_{d2} - \rho\, sgn(S_l)\,\eta_m \\ &= b\Phi_{u2}^T\Xi_{u2} + b\varepsilon_{u2} + bu_{d2} - \rho\, sgn(S_l)\,\eta_m, \end{aligned} \tag{37}$$

where $\Phi_{u2} = \Theta_{u2} - \Theta_{u2}^*$ and Ξ_{u2} is the FBF.

Now we replace the $u_{f2}(D_s|\Theta_{u2})$ by a fuzzy logic system (8) and develop an adaptive law to update the parameter vector Θ_{u2}. It is obtained by the followings:

Consider a control law (32) and a switching function (26). If we choose the auxiliary control input u_{d2} as

$$u_{d2} \leq -sgn(bS_l)\,|\varepsilon_{u2}|. \tag{38}$$

then the proposed system is stable in the sense of the Lyapunov and the parameter adaptation law is given as

$$\dot{\Theta}_{u2} = -sgn(b)\gamma_2 S_l \Xi_{u2}, \tag{39}$$

where γ_2 is a positive constant that determines a kind of learning rate.

This proposition is also proved by the similar process to the case of Section A.

C. Other Methods

In this Section we discuss other schemes briefly.

In Reference [10], Lu and Chen used the following gradient descent method as an learning rule for updating parameters.

$$\dot{\Theta}_u = -\Gamma \frac{\partial S_l \dot{S}_l}{\partial \Theta_u}, \tag{43}$$

where Γ is a learning gain.

In Reference [7], Feng used the following least square algorithm as an learning rule for updating parameters.

$$\dot{\Theta}_u = \frac{P\phi e}{1+\phi^T P\phi} \tag{44}$$

$$\dot{P} = \frac{P\phi\phi^T P}{1+\phi^T P\phi}$$

$$P(0) = k_0 I, \quad k_0 > 0,$$

where ϕ and I are a regressor vector and unit matrix, respectively.

Ⅳ. CONCLUDING REMARKS

We first explained the single input FLC and then proposed some adaptive FLC's. The single input FLC was simply derived based on the skew-symmetric property of the control rule table for conventional FLC's.

We discussed the design of some adaptive FLC's. We here introduced two methods based on Lyapunov stability using the concept of the single input FLC. One was designed based on a Hurwitz error dynamics and the other a switching function of the SMC. The closed-loop stability of both cases was ensured in the sense of the Lyapunov.

We also addressed another two methods, the gradient descent method and the least square algorithm.

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