

Self-tuning Fuzzy Robust Control for Seismically Excited Buildings with Sliding Bearing Isolations

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Abstract - The purpose of this paper is to apply self-tuning fuzzy robust control for the structural control of buildings with sliding bearing isolations. Combining the fuzzy control and robust control such as sliding mode control one can reduce the complexity of fuzzy rule bases. It also ensures the stability and robustness. The Lyapunov theory is used to develop the self-tuning law. Finally, stiffness uncertainty and time delay is utilized to illustrate the robustness of this proposed algorithm. The effectiveness of this algorithm is demonstrated by simulation results for Taiwan Chi Chi earthquake occurred in 1999. The simulations show that self-tuning fuzzy robust control can achieve satisfactory results in the application of structural control for buildings with sliding bearing isolators.

I. INTRODUCTION

Recently, several hybrid systems have been shown to be quite effective for reducing the damage to the structures due to environmental disturbances (e.g. earthquake, etc.). The idea of hybrid control is to utilize the advantages of both active and passive control systems. The base-isolation system is used to reduce the ground motion transmitted to the building, whereas the active control devices are used to reduce the response of building. Base-isolation attempts to uncouple the structure from the seismic ground motion by means of replaceable devices, placed between the building and ground. The disadvantage of the isolators is large lateral displacement may induce damages. Since the dynamic behavior of base isolation systems, such as frictional-type sliding bearings, is either highly nonlinear or inelastic, it is necessary to found a nonlinear control method for such nonlinear systems. The concept of structural control in civil engineering applications was originated in the early 1970s[1]. Some of widely used structural control methods are LQR optimal control[2], pole assignment[3], instantaneous optimal control[4], Recent H₂[5], H-infinite[6] optimal control, sliding-mode control[7], LQG/LTR[8] and fuzzy control methodology[9] were introduced for structural control problem.

This paper applies self-tuning fuzzy robust control for structural control of buildings with sliding bearing isolations.

In industrial sector, systems with complex mechanism, non-linear, and/or ill-defined are difficult to model mathematically, but the operator can control and operate the system adequately. Operator's control strategy is based on intuition and experiences, and can be considered as a set of heuristic decision rules. The theory of fuzzy logic and algorithms can be used to evaluate and implement these imprecise linguistic statements directly and effectively. However, several difficulties still exist in fuzzy control design: (1) the huge amount of fuzzy rules for a high-order system makes the analysis complex; (2) the suitable parameters of membership functions should be given by time-consuming trial and error procedure; (3) no stability analysis tool s can be applied to fuzzy control system[10]. Generally, system parameters are difficult to be known exactly, but the bounds on the uncertainty can be known. For the unmodeled dynamics systems, the robust control such as sliding mode control is a useful control strategy [11]. It provides a systematic approach to solve the problem of maintaining stability and consistent performance. Yager and Filev[12] determined the fuzzy rules according to the sliding mode condition. The sliding surface can dominate the dynamic behaviors of the control system and reduce the rule numbers of fuzzy rule base. Adaptive fuzzy control[13] used a linear combination of fuzzy basic functions and tune the consequent parameters with an adaptive mechanism. The self-tuning law of control in this paper is derived by Lyapunov theory. It is used to tune the centers of membership functions consequence. Finally, the algorithm is illustrated by several examples applied in buildings with sliding bearing isolations.

II. EQUATION OF MOTION OF STRUCTURAL SYSTEM

Assume that the equation of motion for a base-isolated building controlled by actuators and subjected to ground excitation $\ddot{x}_g$ can be written as follows:

$$\begin{bmatrix} M & MI \\ I^T M & I^T MI + m_b \end{bmatrix} \begin{pmatrix} \ddot{\bar{x}}(t) \\ \ddot{x}_b(t) \end{pmatrix} + \begin{bmatrix} C & 0 \\ 0 & c_b \end{bmatrix} \begin{pmatrix} \dot{\bar{x}}(t) \\ \dot{x}_b(t) \end{pmatrix} + \begin{bmatrix} K & 0 \\ 0 & k_b \end{bmatrix} \begin{pmatrix} \bar{x}(t) \\ x_b(t) \end{pmatrix} = \begin{pmatrix} MI \\ I^T MI + m_b \end{pmatrix} \ddot{x}_g(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (U(t) - f(t)) \quad (1)$$

or (1) can be represented as

$$M^* \ddot{\bar{X}}(t) + C^* \dot{\bar{X}}(t) + K^* \bar{X}(t) = b(U(t) - f(t)) - \bar{M} \ddot{x}_g \quad (2)$$

where $\bar{x} = [\bar{x}_1, \bar{x}_2, \dots, \bar{x}_r]^T \in R^r$ = r-vector with $\bar{x}_i$ denoting the *i*th floor displacement relative to base; x_b = base displacement relative to the ground. Matrices M , C , and $K = r \times r$ mass, damping, and stiffness matrices, respectively, for the superstructure; $I = r$ -vector denoting the influence of the earthquake excitation; m_b , c_b , and k_b are base mass, damping, and stiffness matrices, respectively; $U(t)$ corresponds to the actuator forces (generated via active tendon system or an active mass damper, for example), $f(t)$ is the forces from the isolators; this is only a static model, neglecting the dynamic equations of actuators. The frictional force of the sliding bearings is given by

$$f(t) = \mu mgv(t) \quad (3)$$

in which mg = the weight of the structural system above the sliding bearing; and μ is the coefficient of friction. Generally, the coefficient of friction μ is velocity-dependent. A approximate model for the frictional coefficient μ of sliding bearings using Teflon/strainless-steel plates is obtained experimentally[14, 15]:

$$\mu = \mu_m - \mu_f e^{-a_\mu |\dot{x}_b|} \quad (4)$$

in which μ_m , μ_f and a_μ are constants to be obtained experimentally using curve-fitting procedures; and x_b is the relative displacement of the sliding system. The constants μ_m , μ_f and a_μ depend on the surface condition and the pressure of sliding bearings.

$v(t)$ is the hysteretic component of the sliding bearings governed by

$$\dot{v}(t) = D_y^{-1} (\alpha \dot{x}_b - \beta |\dot{x}_b|^{\eta-1} v - \gamma |\dot{x}_b|^\eta) \quad (5)$$

where D_y is the yield deformation; and α, β, γ and η are the parameters defining the characteristics of the hysteresis loop of the frictional force.

For controller design, the standard first-order state equation corresponding to (2) is

$$\dot{X}(t) = AX(t) + B(U(t) - f(t)) + E \ddot{x}_g \quad (6)$$

where $X^T = [\bar{X}^T \quad \dot{\bar{X}}^T]^T = 2(r+1)$ vector and

$$A = \begin{bmatrix} 0 & I \\ -(M^*)^{-1} K^* & -(M^*)^{-1} C^* \end{bmatrix}, B = \begin{bmatrix} 0 \\ (M^*)^{-1} b \end{bmatrix}, E = \begin{bmatrix} 0 \\ -(M^*)^{-1} \bar{M} \end{bmatrix} \quad (7)$$

III. SELF-TUNING FUZZY ROBUST CONTROL

For a complete account of robust control such as sliding

mode control theory, the reader can consult the references[16, 17]. The basic concept is that the controller changes its structures according to the position of the state trajectory with respect to a chosen sliding surface. The control is designed to force the state trajectory of the system onto the sliding surface and to maintain it there. This is accomplished by a high speed switching law. The design of a sliding mode controller consists of two steps: (1) The design of the sliding surface (2) The design of the control strategy to steer the state trajectory to the sliding surface.

The design of the sliding surface is described in the following. Consider the equation of system have the form

$$\dot{X} = AX + BU + BH + F + W \quad (8)$$

where $X(t)$ is a n state vector, $n=2(r+1)$, A is a $n \times n$ system matrix, B is a $n \times m$ matrix, H is a n vector which contains the uncertainty and nonlinear of system and satisfy matching condition, F is a n vector which contains the uncertainty and nonlinear of system, but F don't satisfy matching condition. W is a n excitation vector.

Suppose $\{x | S(X) = 0\}$ is the chosen sliding surface.

$$S(X) = PX \quad (9)$$

Consider the nominal system

$$\dot{X} = AX + BU \quad (10)$$

The optimal sliding modes method[7,17] is used to the determination of P . The sliding surface is obtained by minimizing the integral of the quadratic function of the state vector

$$I = \int_0^\infty X^T Q X dt \quad (11)$$

where Q is a $(n \times n)$ positive definite weighting matrix.

The second step is the design of the controller. The controllers are designed to drive the state trajectory into the sliding surface $S=0$.

Define a Lyapunov function V

$$V = 0.5 S^T S \quad (12)$$

In (8) F is a n vector which contains the uncertainty and nonlinear of system, W is a n excitation vector. Generally, system parameters are difficult to be known exactly, but the bounds on the uncertainty can be known.

$$\|F\| \leq \delta_f, \quad \|W\| \leq \delta_w \quad (13)$$

Let

$$U = U_{eq} - (\gamma + \eta) \text{sgn}(S^T PB)^T \quad (14)$$

where $\gamma = \frac{\delta}{\|B\|}$, $\delta = \delta_f + \delta_w$

$$\begin{aligned} \dot{V} &= S^T P (AX + BU + BH + F + W) \\ &= S^T P (AX - B(PB)^{-1} PAX - B(\gamma + \eta) \text{sgn}(S^T PB)^T + BH + F + W) \\ &= S^T P (-B(\gamma + \eta) \text{sgn}(S^T PB)^T + BH + F + W) \\ &= S^T PB (-(\gamma + \eta) \text{sgn}(S^T PB)^T + H) + S^T P (F + W) \\ &\leq -\eta \|S^T PB\| - \gamma \|S^T PB\| + \|S^T PB\| \|H\| + S^T P (F + W) \\ &= -\eta \|S^T PB\| + \|S^T PB\| \|H\| \\ &\quad - \gamma \|S^T PB\| (1 - (S^T P (F + W)) / \gamma \|S^T PB\|) \end{aligned}$$

$$\begin{aligned}
&= -\eta \|S^T P B\| \|H\| + \|S^T P B\| \|H\| \\
&- \gamma \|S^T P B\| (1 - (S^T P(F+W))/\frac{\delta}{\|B\|}) \|S^T P B\| \\
&\leq -(\eta - \|H\|) \|S^T P B\| < 0 \quad (\eta \geq \|H\|)
\end{aligned}$$

Let $K = \eta + \gamma$, $\bar{S} = (S^T P B)^T$, (14) control force $U = U_{eq} - K \text{sgn}(\bar{S})$, stability can be obtained when

$$K \geq \eta + \|P\| \delta \quad (15)$$

$\|\cdot\|$ denotes the Euclidean norm.

A drawback to the control law given in equation (14) that is discontinuous and tends to excite high frequency modes of the plant. The problem can be alleviated with the insertion of a boundary layer about the sliding surface. The characteristic $u_f = f(\bar{S})$ of the sliding mode controller with boundary layer is linear, but the one of fuzzy sliding mode controller is nonlinear.

A fuzzy sliding mode controller is proposed, in which a fuzzy inference mechanism is used to estimate the second part of (14), i.e. u_f . The range of u_f is $[-K, K]$. The fuzzy rule is as follows.

If $\bar{S}$ is PB and $\dot{\bar{S}}$ is PB then u_f is NB.

Fuzzy output u_f can be calculated by the center of area

defuzzification

$$u_f = \frac{\sum_{i=1}^l w_i c_i}{\sum_{i=1}^l w_i} = \frac{\begin{bmatrix} c_1 & \dots & c_l \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_l \end{bmatrix}}{\sum_{i=1}^l w_i} = v^T \Psi \quad (16)$$

where $v = [c_1, \dots, c_l]^T$ is a adjustable parameter vector; c_i is the center of membership function, $\Psi = \frac{[w_1, \dots, w_l]^T}{\sum_{i=1}^l w_i}$ is a firing

strength vector.

From above can see that the sliding mode controller requires the upper bound of uncertainty. While the uncertainty increases, the control cost increases as well. But the optimal value of uncertainty can not be obtained exactly owing to the unknown of structure or system complexity. Therefore, an adaptive fuzzy control is developed to deal with the problem and to estimate the minimum control cost.

Assume there exists a specific $\hat{u}_f$ which achieves minimum control cost. $\hat{u}_f$ satisfies the sliding mode condition.

From (16), $\hat{u}_f$ can be written as follows:

$$\hat{u}_f = \hat{v}^T \Psi \quad (17)$$

where $\hat{v}$ is the optimal vector which achieves the minimum control cost.

Define the parameter vector as

$$\tilde{v} = v - \hat{v} \quad (18)$$

Let the Lyapunov function for each controller as

$$V = \frac{1}{2} (s^2 + \frac{1}{\alpha} \tilde{v}^T \tilde{v}) \quad (19)$$

where α is a positive constant.

$$\dot{V} = s P b_i u_f + s P b_i H + s P(F+W) + \frac{1}{\alpha} \tilde{v}^T \dot{\tilde{v}}$$

$$= s P b_i (u_f - \hat{u}_f) + s P b_i \hat{u}_f + s P b_i H + s P(F+W) + \frac{1}{\alpha} \tilde{v}^T \dot{\tilde{v}}$$

$$= s P b_i (v - \hat{v}) \Psi + s P b_i \hat{u}_f + s P b_i H + s P(F+W) + \frac{1}{\alpha} \tilde{v}^T \dot{\tilde{v}}$$

$$= \frac{1}{\alpha} \tilde{v}^T (\dot{\tilde{v}} + \alpha s P b_i \Psi) + s P b_i \hat{u}_f + s P b_i H + s P(F+W)$$

$$\leq -(\eta_i - h_i) |s P b_i| < 0 \quad (\hat{u}_f \text{ satisfies the sliding mode condition and let } \dot{\tilde{v}} = -\alpha s P b_i \Psi)$$

Finally, the self-tuning law is obtained as

$$\dot{\tilde{v}} = -\alpha s P b_i \Psi \quad (20)$$

The self-tuning law adjusts the centers of the membership function. Next section some examples is used to illustrated the self-tuning fuzzy robust control in isolated buildings.

IV. NUMERICAL SIMULATION AND RESULTS

The self-tuning fuzzy robust controller is used to control building with sliding bearing isolations. Fig.1 shows a base-isolated six floor building. The nominal value of each floor mass is 345600kg, base mass is 450000kg, stiffness of each floor is 3.1e8 nt/m, damping ratio is 0.02. The coefficient of friction μ for Teflon /stainless-steel bearings is given by (3) with $\mu_m = 0.1$, $\mu_f = 0.05$, $a_\mu = 0.2$ s/cm. The parameter values for (4) are $\alpha = 1.0$, $\beta = 0.5$, $\eta = 2$, $\gamma = 0.5$, and $D_y = 0.012$ cm. Firstly, the 1999 Taiwan Chi Chi earthquake (ew direction) whose peak ground acceleration is over 1g is used as input excitation. Fig. 2 is the time history of Chi Chi earthquake.

The optimal sliding mode method is used to determine the sliding surface with a diagonal weighting matrix Q ; $Q_{77} = 1$, $Q_{ii} = 5e3$, for $i = 1, 2, \dots, 6$ and $Q_{ii} = 1$, for $i = 8, 9, \dots, 14$. The base displacement history of building with sliding bearing for no estimation error and time delay is shown in Fig.3. To examine the robustness of the self-tuning fuzzy robust control, we vary the stiffness of all the floor of the building by $\pm 40\%$ in design controller and suppose 30 ms time delay. All maximum response quantities of building with sliding bearing are shown in Table 1. x_i , $\ddot{x}_i$, and $U_{\max}$, are the interstory deformation of each floor or base, the absolute acceleration of each floor or base, and maximum control force, respectively. Table 1 shows that the self-tuning fuzzy robust control can not only reduces the deformation of base, but the

response of the superstructure and the amplitude of floor's acceleration are also be decreased. The maximum control forces of self-tuning fuzzy robust control are rather low. They are all small than 9% of the superstructure weight.

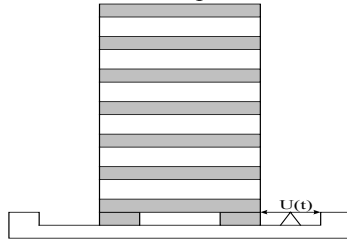


Fig. 1 A base-isolated six floor building

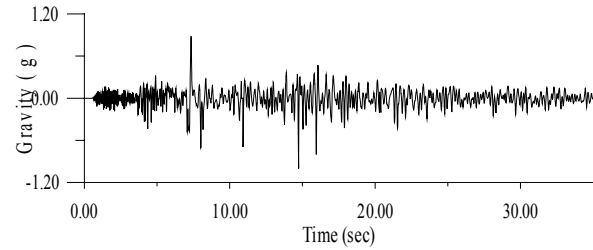


Fig. 2 The Chi Chi earthquake

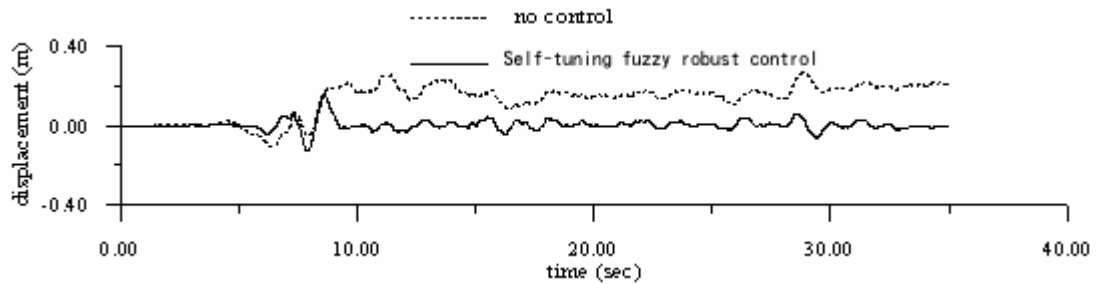


Fig. 3 The base displacement history of building with sliding bearing

Table 1. Maximum response quantities of building with sliding bearing

	No control $U_{\max} = 0 \text{ kN}$		Control no error $U_{\max} = 2174 \text{ kN}$		Stiffness +40% $U_{\max} = 2174 \text{ kN}$		Stiffness -40% $U_{\max} = 2174 \text{ kN}$		Time delay 30ms $U_{\max} = 2174 \text{ kN}$	
	$x_i \text{ (m)}$	$\ddot{x}_i \text{ (m/s}^2\text{)}$	$x_i \text{ (m)}$	$\ddot{x}_i \text{ (m/s}^2\text{)}$	$x_i \text{ (m)}$	$\ddot{x}_i \text{ (m/s}^2\text{)}$	$x_i \text{ (m)}$	$\ddot{x}_i \text{ (m/s}^2\text{)}$	$x_i \text{ (m)}$	$\ddot{x}_i \text{ (m/s}^2\text{)}$
B	2.68E-01	11.066	1.19E-01	8.7792	1.56E-01	8.7266	1.41E-01	7.9341	2.02E-01	8.8813
1	7.77E-03	5.0928	2.89E-03	4.6607	3.61E-03	4.747	3.27E-03	4.9528	4.54E-03	5.0426
2	7.98E-03	6.7986	2.53E-03	4.9707	3.54E-03	5.0731	2.76E-03	5.0102	4.73E-03	5.2476
3	7.29E-03	6.7512	2.09E-03	3.6555	3.33E-03	4.5918	3.06E-03	4.4755	3.89E-03	5.1132
4	6.83E-03	6.0155	2.38E-03	3.7513	3.25E-03	4.1997	4.07E-03	3.5907	3.50E-03	4.0271
5	6.07E-03	5.4851	1.80E-03	3.5419	2.42E-03	4.163	3.75E-03	3.6777	2.74E-03	4.3466
6	4.63E-03	6.7718	1.60E-03	3.4441	1.84E-03	3.7724	2.71E-03	4.1679	2.77E-03	4.4257

V. CONCLUSIONS

It is shown that buildings equipped with isolation can reduce the interfloor drift and floor absolute acceleration from the simulations. The proposed self-tuning fuzzy robust control not only reduce the base displacement, but all the above response quantities. The maximum control forces of self-tuning fuzzy robust control are rather low. They are all small than 9% of the superstructure weight. Table 1 demonstrates this control method can work well in estimation error and time delay. It is robust. It can be used to structural

control with nonlinear, uncertainty and time delay. It can be used in practical application.

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