# Position and Pose Estimation of the occluded Artificial Knee Joint in X-ray Fluoroscopy Images Based on Fuzzy Image Processing 

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#### Abstract

X-ray fluoroscopy images are widely used for evaluating kinematics of the artificial knee joint in vivo after the total knee Arthroplasty, TKA in short. Although some researchers have studied on computer-aided systems to analyze X-ray fluoroscopy images, there are no discussions about occluded images that artificial knee joint have overlapped on the images and they have separated from the field of view. This paper proposes a novel method for evaluating kinematics of artificial knee joint that is applicable to occluded images. Our method automatically estimates the position and pose of the artificial knee joint in vivo using fuzzy image matching of the given X-ray fluoroscopy image and the three-dimensional (3-D) geometric models of the artificial knee joint. To quantitatively evaluate our method, it was applied to simulation images artificially generated by a computer and X-ray fluoroscopy images that took the artificial knee joint in vitro fixed with arbitrary position and pose by a jig. The experimental results showed that the method could estimate the pose of the within the error of 2.8 degree in the occluded images. Also, the proposed system was applied to two patients after TKA.


## I. Introduction

The osteoarthritis of the knee is a disease that the joint damages by wearing the joint cartilage out, and becomes inconvenient for the knee bending with an ache, modification of the knee, and inflammation. As these disease cures, total knee Arthroplasty, TKA in short, has attracted attention in recent years. TKA is an operation which carries out the bone end of the surface of the damaged knee joint in due form, replaces with the artificial knee joint, and rebuilds the stable joint without an ache. The artificial knee joint mainly consists of femoral component, tibia insert, and tibia tray. After TKA, the design of the artificial knee joint determines the motion of the knee. So, in order to find the optimum design of the artificial knee joint, it is necessary to quantitatively evaluate of the motion of the artificial knee
joint in vivo.
There are some conventional methods for estimating the position and pose of artificial knee joint in vivo using X-ray fluoroscopy images [1-4]. The method proposed by Yamazaki et al. constructed a cone-like solid which connects a X-ray source and the projection contour of the artificial knee joint in virtual space, and presumed the position and pose of artificial knee joint by minimizing the total distance between the surface of the cone-like solid, and the 3-D geometric model surface of the artificial knee joint [1]. Also, Ref. [2] and [3] estimated the position and pose of artificial knee joint by matching the two-dimensional (2-D) projection of the 3-D geometric model of the artificial knee joint, and the X-ray fluoroscopy image. In these conventional studies, there are few discussions on occluded images because the methods estimate the position and pose of femoral component and tibia tray separately with no consideration of occlusion. The occluded images are generated when the tibia tray overlaps with femoral component, or a part of artificial knee joint separates from the field of view (FOV) of X-ray fluoroscopy equipment. However, when we acquire X-ray fluoroscopy images of patients, occlusion occurs frequently, rather it is difficult to obtain non-occluded images. In addition, when images are taken so that occlusion does not occur, it will force a different movement from natural knee motion.

This paper proposes a computer-aided system that quantifies the motion of the artificial knee joint and that is applicable to occluded images. The system estimates the position and pose of artificial knee joint by fuzzy image matching of X-ray fluoroscopy images and the 3-D geometric model. Fuzzy image matching can take into account the occlusion of objects by assigning fuzzy degrees to the occluded region. In order to evaluate the accuracy of our
system, it was applied to computer simulated images, and X-ray fluoroscopy images that took the artificial knee joint in vitro that were fixed with a jig. Also, we show the analysis results of two patients after TKA using the proposed system.

## II. Materials

In this study, we use static images captured from moving images that were acquired from a X-ray fluoroscopy equipment (SX-VA30, Hitachi Medical Corp., Tokyo). The static images were resolution of $640 \times 480$ pixel and stored in 8 bits. In our system, the value stored in a pixel is treated as intensity. An example of the X-ray fluoroscopy image and our coordinate system are shown in Fig. 1, Fig.2, respectively. The artificial knee joint mainly consists of femoral component, tibia insert, and tibia tray. Femoral component and tibia tray are made of metal, and tibia insert is made of polyethylene. So, in X-ray fluoroscopy images, femoral component and tibia tray appeared with lower intensity than the surrounding soft tissues, and tibia insert appeared with equivalent intensity to soft tissues. In preliminary, the static images were inverted, and the intensity outside the FOV (areas other than circle in Fig. 1) was set to zero for easy manipulation and to reduce computation time. An example of the preprocessed image is shown in Fig. 3(a). And, we use 3-D geometric models (stereolithography format, mesh size of 0.01 mm ) of the artificial knee joint. The projection image is rendered by fluoroscopy projection to the 2-D plane of this 3-D geometric model. An example of the projection image is shown in Fig. 3(b).


Fig. 1. An example of X-ray fluoroscopy image.


Fig. 2 Our coordinate system.


Fig. 3 (a) Preprocessed image and (b) projection image used in our system.

## III. Proposed Method

## A. Overview

The proposed computer-aided system consists of two methods; (1) fuzzy image matching and (2) quantification of knee motion. Fuzzy image matching estimates the position and pose of the artificial knee joint by evaluating a matching score with respect to fuzzy degrees belonging to occlusion. Using the estimated pose and position parameters of femoral component and tibia tray, we quantify the flexion angle and others according to a joint coordinate system proposed by Grood et al., [5].

## B. Fuzzy image matching

Recognizing position and pose of the artificial knee joint in the X-ray fluoroscopy image is to find rotation and translation parameters where the projection image of the 3-D geometric model of the artificial knee joint using the parameters is most similar to the X-ray fluoroscopy image. It is performed with fuzzy image matching. Fuzzy image matching gives a degree of similarity (i.e., matching score) between the given X-ray fluoroscopy image and a projection image, and finds the rotation and translation parameters that give the highest degree of similarity.

For given parameters $\left(\theta_{x}, \theta_{y}, \theta_{z}, t_{x}, t_{y}, t_{z}\right)$, degree of similarity, $\mu$, is calculated by

$$
\begin{equation*}
\mu=\kappa_{I} \mu_{I}+\kappa_{D} \mu_{D} \tag{1}
\end{equation*}
$$

where $\kappa_{I}$ and $\kappa_{D}$ are weighting parameters determined experimentally ( $\kappa_{I}=0.5$ and $\kappa_{D}=0.5$ were used in our experiment). $\mu_{I}$ is the degree of similarity between the given fluoroscopy image and the projection image. The degree, $\mu_{I}$, is defined as

$$
\begin{equation*}
\mu_{I}=\frac{\sum G(x, y) H(x, y)\left\{1.0-\mu_{B}(x, y)\right\} \mu_{F}(x, y)}{\sum H(x, y)\left\{1.0-\mu_{B}(x, y)\right\} \mu_{F}(x, y)} \tag{2}
\end{equation*}
$$

where $G(x, y)$ and $H(x, y)$ are the intensity values at the pixel $(x, y)$ in the given fluoroscopy image and the projection image, respectively. This degree evaluates the similarity of them with respect to the spatial distribution of intensity. Then, $\mu_{D}$ is the degree of similarity between the differential images of the given fluoroscopy image and projection image. This degree is defined as

$$
\begin{equation*}
\mu_{D}=\frac{\sum J(x, y) K(x, y)\left\{1.0-\mu_{B}(x, y)\right\} \mu_{F}(x, y)}{\sum K(x, y)\left\{1.0-\mu_{B}(x, y)\right\} \mu_{F}(x, y)} \tag{3}
\end{equation*}
$$

where $J(x, y)$ and $K(x, y)$ are the intensity values at the pixel $(x, y)$ of the differential images of the given fluoroscopy image and projection image, respectively. This degree evaluates the similarity with respect to the contour of the artificial knee joint. $\quad G(x, y), H(x, y), J(x, y)$ and $K(x, y)$ are normalized from 0 to 1 .
$\mu_{B}(x, y)$ is a fuzzy degree which means whether femoral component and tibia tray overlap each other at the pixel $(x, y)$ in the given fluoroscopy image. This fuzzy degree is calculated using a fuzzy membership function defined in Fig. 4. In this figure, $d(x, y)$ is the Euclidian distance between a


Fig. 4 The membership function about a occluded region.


Fig. 5 Fuzzy degree of occluded region.
boundary line and the pixel of interest. The boundary line is manually given by a user along with the overlapped region of femoral component and tibia tray. $T h_{d}$ is also arbitrarily given according to the overlap ratio of femoral component and tibia tray. For example, consider an image that a circle model has overlapped with a triangle model as shown in Fig. 5(a). By drawing a boundary line in this image, the fuzzy degree map is calculated as shown in Fig. 5(b). In this image, $T h_{d}=100$ was used. In addition, $\mu_{F}(x, y)$ is a fuzzy degree which means whether the artificial knee joint separates from FOV of X-ray fluoroscopy equipment. This fuzzy degree is calculated using a fuzzy membership function defined in Fig. 6. In this figure, $r(x, y)$ is the Euclidian distance between the center pixel of FOV and the pixel of interest. $\quad R_{F O V}$ is fixed and $R$ is arbitrarily given according to the size of FOV as shown in Fig. 7(a). The fuzzy degree map is calculated as shown in Fig. 7(b). When we calculate the matching score of this image, the matching score mostly depends on non-occluded region because the matching scores around the occluded region are ignored by the low fuzzy degree of $\mu_{B}(x, y)$ and $\mu_{F}(x, y)$. This will accelerate finding the true position and pose of the occluded objects.

The degree of similarity, $\mu$, takes a value from 0 to 1 , and takes the higher value when the X-ray fluoroscopy image and the projection image matches better. Then, the position and pose parameters of the 3-D geometric model with the highest degree of similarity are obtained as the position and pose of the artificial knee joint in the X-ray fluoroscopy image.

The parameters of rotation and translation of the 3-D geometric model are determined in order of the following steps.

1. Rotation on x-axis $\left(\theta_{x}-\Delta \theta \leqq \theta_{x}^{\prime} \leqq \theta_{x}+\Delta \theta\right)$
2. Rotation on y-axis $\left(\theta_{y}-\Delta \theta \leq \theta_{y}{ }^{\prime} \leq \theta_{y}+\Delta \theta\right)$
3. Rotation on z-axis $\left(\theta_{z}-\Delta \theta \leqq \theta_{z}{ }^{\prime} \leqq \theta_{z}+\Delta \theta\right)$
4. Translation on x-axis $\left(t_{x}-\Delta t \leqq t_{x}{ }^{\prime} \leqq t_{x}+\Delta t\right)$
5. Translation on y-axis $\left(t_{y}-\Delta t \leqq t_{y} \leqq t_{y}+\Delta t\right)$
6. Translation on z-axis $\left(t_{z}-\Delta t_{z} \leqq t_{z} \leqq t_{z}+\Delta t_{z}\right)$
where $\Delta \theta=3 \mathrm{deg}, \Delta t=5 \mathrm{~mm}$, and $\Delta t_{z}=50 \mathrm{~mm}$. For each step, a value with the highest degree of similarity is selected,


Fig. 6 The membership function about a FOV.

and parameters except the searching parameter are fixed. The parameters $\theta_{x}{ }^{\prime}, \theta_{y}^{\prime}$ and $\theta_{z}^{\prime}$ change every 1 deg , the parameters $t_{x}{ }^{\prime}$ and $t_{y}{ }^{\prime}$ change every 1 mm , and the parameter $t_{z}$ ' changes every 10 mm . After searching the all parameters, the six parameters are searched again with $\Delta \theta=0.3 \mathrm{deg}, \Delta t=$ 0.5 mm , and $\Delta t_{z}=5 \mathrm{~mm}$, and parameters change every 0.1 $\mathrm{deg}, 0.1 \mathrm{~mm}, 1 \mathrm{~mm}$. In the result of searching parameters, we obtain the six parameters ( $\left.\theta_{x}^{\prime}, \theta_{y}^{\prime}, \theta_{z}^{\prime}, t_{x}^{\prime}, t_{y}^{\prime}, t_{z}^{\prime}\right)$ with the highest degree of similarity among the all searching processes as the position and pose of the artificial knee joint in the X-ray fluoroscopy image. In this method, the initial position and rotation angles are manually determined by a user for the given X-ray fluoroscopy image.

## C. Quantification of knee motion

The position and pose estimation described above are applied to femoral component and tibia tray separately with considering the occlusion. Consequently, we obtained the six parameters for each of the both femoral component and tibia tray. Using the parameters, we quantify the rotation angles (flexion/extension, internal/external rotation, varus/valgus) that are used for clinical application. The rotation angles are calculated according to a joint coordinate system defined by Grood et al. [5] that has been widely used to analyze kinematics of the knee motion. The Grood's coordinate system defines the rotation angle based on the axis of the bone presumed from the bone shape of a femur and a tibia. However, in case of X-ray fluoroscopy image, it is difficult to presume the axis of the bone because only a part of femur and tibia are appeared within the FOV of X-ray fluoroscopy image. So, to apply the Grood's coordinate system to X-ray fluoroscopy images, we calculate the rotation angles based on the local coordinate system of femoral component and tibia tray. Each local coordinate system is defined in the 3-D geometric models. And they are rotated and translated in the world coordinate system according to the estimated parameters from the X-ray fluoroscopy images.

The calculation method of the rotation angles is illustrated in Fig. 8. First, unit vectors, $\vec{e}_{u f}, \vec{e}_{v f}$ and $\vec{e}_{w f}$, are defined in the local coordinate system (ut $\mathrm{u}_{\mathrm{f}}$-axis,
$\mathrm{v}_{\mathrm{f}}$-axis, $\mathrm{w}_{\mathrm{f}}$-axis) of femoral component. Similarly, unit vectors, $\vec{e}_{u t}, \vec{e}_{v t}$ and $\vec{e}_{w t}$, are defined in the local coordinate system of tibia tray. $\mathrm{w}_{\mathrm{f}}$-axis of femoral component is defined as a flexion axis, and $u_{t}$-axis of tibia tray is defined as a rotation axis. And a varus axis that intersects perpendicularly with these axes mutually is defined. The unit vector of the direction of the varus axis is set to $\vec{e}$, and it is given by

$$
\begin{equation*}
\vec{e}=\frac{\overrightarrow{e_{u t}} \times \overrightarrow{e_{w f}}}{\left|\overrightarrow{e_{u t}} \times \overrightarrow{e_{w f}}\right|} \tag{4}
\end{equation*}
$$

The flexion angle, $D_{\text {flexion }}$, is defined as the angle formed between the unit vector $\vec{e}$ and the unit vector $\vec{e}_{v f}$, the rotation angle, $D_{\text {rotation }}$, is defined as the angle formed between the unit vector $\vec{e}$ and the unit vector $\vec{e}_{v t}$. The varus angle, $D_{\text {varus }}$, is set to 0 deg when the flexion axis and the rotation axis intersect perpendicularly mutually. These angles are calculated by

$$
\begin{align*}
& D_{\text {flexion }}=\cos ^{-1}\left(\frac{\vec{e} \cdot \overrightarrow{e_{v f}}}{|\vec{e}|\left|\overrightarrow{e_{v f}}\right|},\right.  \tag{5}\\
& D_{\text {rotation }}=\cos ^{-1}\left(\frac{\vec{e} \cdot \overrightarrow{e_{v t}}}{|\vec{e}|\left|\overrightarrow{e_{v t}}\right|}\right. \text {, and }  \tag{6}\\
& D_{\text {rotation }}=\frac{\pi}{2}-\cos ^{-1}\left(\frac{\overrightarrow{e_{w f}}}{\left|\overrightarrow{e_{w f}}\right| \mid \overrightarrow{e_{u t}}}\right) . \tag{7}
\end{align*}
$$

## IV. EXPERIMENTAL RESULTS

## A. Comparison with the conventional matching method

First, fuzzy image matching was compared with the conventional matching method [3] (i.e., $\mu_{B}=0.0$ for all pixels) in order to test the validity of fuzzy image matching. Both of the method was applied to an image shown in Fig. 5(a). Fig. 9 shows the transition of degree of similarity when the position of the circle model was shifted horizontally between +1 mm and -1 mm from the true position. Comparison of transition of degree of similarity


Rotational axis
Fig. 8 The calculation method of the angle based on Grood's coordinate system.
between the fuzzy image matching and the conventional crisp image matching shows that the degree of similarity significantly decreased by leaving the true position. This suggests that the fuzzy image matching will have better performance of finding the true position in comparison with the conventional crisp method when the object of interest is occluded.

## B. Evaluation in overlapped images

Second, the proposed computer-aided system was applied to computer simulated images that the 3-D geometric model was projected on a plane so that occlusion occurs by inclining to projection axis. The images were convoluted by a Gaussian function (FWHM; full width half maximum = 3 pixel). For example, Fig. 10 (a) shows a computer simulated image when all of the flexion, rotation and varus angles were set to zero. In the results of applying the proposed system, the estimated rotation angles were 0.0 deg of the flexion angle, 0.9 deg of the rotation angle, and 0.0 deg of the varus angle. The rendering image of the 3-D geometric model with the estimated rotation angles is shown in Fig. 10(b). Also, we applied to images that were projected into another planes so that the overlap ratio of femoral component and tibia tray changes and the position and pose were not changed. The estimation results and errors are tabulated in Table I. As shown in this table, although it is in the tendency that the errors increase as the overlap ratio becomes large, we could estimate the rotation angles with the small error.

Next, we applied the proposed system to phantom data, which were X-ray fluoroscopy images taken the artificial knee joint fixed by a jig. For example, Fig. 11 (a) shows the X-ray fluoroscopy image taken when the artificial knee joint were fixed at zero flexion, zero rotation, and zero varus angles by the jig. The result of applying our system is shown in Fig. 11(b). In the similar way to the computer simulation experiments, Table II tabulates the estimation results and errors. The estimation errors included the human error of manual setting of the jig. And the quality of the X-ray fluoroscopy image will increase the estimation errors. Therefore, we are considering that the proposed system could be applied to the occluded images with small errors.


Fig. 9 Transition of degree of similarity near the true position.

## C. Evaluation in images that a part of knee joint out of FOV

Third, we evaluated the estimation performance of the proposed system when a part of artificial knee joint was out of the FOV of image through computer simulation and phantom experiments. Examples of computer simulation image and phantom image are shown in Fig. 12 (a) and (c), respectively. The images were generated by setting the all of the rotation angles to zero and by shifting the joint in the direction of the $y$-axis of -50 mm . The estimated rotation angles are tabulated in Table III. Also, the rendering images of the artificial knee joint with the estimated pose and position are shown in Fig. 12 (b) and (d). These experimental results showed that the proposed method could be applied successfully to images where a part of artificial knee joint had separated from the FOV.


(a) Phantom image.

(b) Estimation result of (a).

Fig. 11 Experiments in phantom data.
TABLE I The Analysis Result and Error of Simulation ImAGES.

| Overlap ratio |  | $D_{\text {flexion }}[\mathrm{deg}]$ | $D_{\text {rotation }}[\mathrm{deg}]$ | $D_{\text {varus }}[\mathrm{deg}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $0 \%$ | Result | 0.0 | 0.9 | 0.0 |
|  | Error | 0.0 | 0.9 | 0.0 |
| $15 \%$ | Result | 0.1 | 0.2 | 0.0 |
|  | Error | 0.1 | 0.2 | 0.0 |
| $25 \%$ | Result | 0.2 | 0.3 | 0.7 |
|  | Error | 0.2 | 0.3 | 0.7 |

TABLE II THE ANALYsis Result and Error of Phantom Images.

| Overlap ratio |  | $D_{\text {flexion }}[\mathrm{deg}]$ | $D_{\text {rotation }}[\mathrm{deg}]$ | $D_{\text {varus }}[\mathrm{deg}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $0 \%$ | Result | 1.4 | 1.0 | 0.6 |
|  | Error | 1.4 | 1.0 | 0.6 |
| $15 \%$ | Result | 0.5 | 1.2 | 1.4 |
|  | Error | 0.5 | 1.2 | 1.4 |
| $25 \%$ | Result | 0.9 | 0.5 | 2.0 |
|  | Error | 0.9 | 0.5 | 2.0 |

## D. Clinical results for TKA patients

Finally, to demonstrate the performance of the proposed system for analyzing the motion of the artificial knee joint in vivo, the system was applied to two female patients after TKA. The patients were subject 1 who was 63 years old and whose left knee was replaced by the artificial knee joint, and subject 2 who was 59 years old and whose right knee was replaced by the artificial knee joint. For each subject, X-ray fluoroscopy images were taken after 10 months and 3 months, respectively. The static images to analyze by our system were captured by one frame for one second from the moving images acquired by the X-ray fluoroscopy equipment. The representative images of each subject are shown in Fig. 13 (a) and Fig. 14 (a), respectively. The analysis results are given by Fig. 13 (b) and Fig. 14(b). Also, the time-transition of estimated rotation angles of each subject is shown in Fig. 13(c) and Fig. 14(c). In these images, the frames where the artificial knee joints overlapped each other are denoted by double arrows. And, the small flexion/extension angle means that the subject extended own knee, and the large angle does that the subject flexed own knee. As shown in these images, we have confirmed that our system could estimate the rotation angles of the artificial knee joint using X-ray fluoroscopy images even if the artificial knee joint overlapped.

## V. CONCLUSION

In this paper, we have proposed a novel computer-aided system for quantifying the motion of the artificial knee joint using the X-ray fluoroscopy images and the 3-D geometric


(a) Computer simulated image.

(c) Phantom image.

(b) Estimation results of (a)

(d) Estimation result of (c).

Fig. 12 Evaluation for images that a part of the artificial knee joint has separated from the FOV.

TABLE III The Analysis Result of the Image out of FOV.

|  |  | $D_{\text {flexion }}[\mathrm{deg}]$ | $D_{\text {rotation }}[\mathrm{deg}]$ | $D_{\text {varus }}[\mathrm{deg}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Simulation <br> image | Result | 0.2 | 0.8 | 0.0 |
|  | Error | 0.2 | 0.8 | 0.0 |
| Phantom <br> image | Result | 0.1 | 2.8 | 0.6 |
|  | Error | 0.1 | 2.8 | 0.6 |



(c) Time transition of estimated rotation angles.

Fig. 13 Motion analysis of Subject 1.
model of the artificial knee joint. In order to apply the system to occluded images, we have introduced fuzzy degree belonging to the boundary of the two components into image matching procedure that is called fuzzy image matching. To quantitatively evaluate the performance of estimating pose and position of the artificial knee joint, we applied the proposed system to computer simulated images and X-ray fluoroscopy images that taken the artificial knee joint fixed at the arbitrary pose and position by the jig. The experimental results showed that the system could estimate the pose of the artificial knee joint within the error of 2.8 deg even if the components of the artificial knee joint overlapped each other and a part of them was separated from the FOV. Therefore, we can analyze the natural kinematics of the knee without any constraint of knee motion although the conventional method has been limiting the knee motion so that the whole artificial knee joint is included in the FOV. In the future, we will quantify the other parameters of the knee motion such as the nearest point between the components. Also, we will apply the proposed system to other subjects in order to investigate the analysis performance and to study the knee motion under the free condition.

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(c) Time transition of estimated rotation angles.

Fig. 14 Motion analysis of Subject 2.
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