An Avoidance Obstacle in the Chaotic Robot

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***Abstract*— In this paper, we propose a method to avoid obstacles that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When a chaos robot meets an obstacle in a Lorenz equation, Hamilton and Hyper-chaos equation trajectory, the obstacle reflects the robot.**

We also show computer simulation results of Lorenz equation, Hamilton and Hyper-chaos equation trajectories with one or more Van der Pol as an obstacles. We proposed and verified the results of the method to make the embedding chaotic mobile robot to avoid with the chaotic trajectory in any plane. It avoids the obstacle when it meets or closes to the obstacle with dangerous degree.

***Index Terms*— chaos, mobile robot, Lorenz, Hamilton, Hyper-choas equation, obstacle avoidance.**

1.INTRODUCTION

CHAOS theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a method to avoid obstacles using unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaos robots meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Lorenz equation, Hamilton and hyper-chaos equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided with as a Lorenz, Hamilton equation and hyper-chaos equation. We proposed and verified the results of the method to make the embedding chaotic mobile robot to avoid with the chaotic trajectory in any plane. It avoids the obstacle when it meets or closes to the obstacle.

2. CHAOTIC MOBILE ROBOT S EQUATION

A. Mobile Robot

As the mathematical model of mobile robots, we assume a two- wheeled mobile robot as shown in Fig. 1.

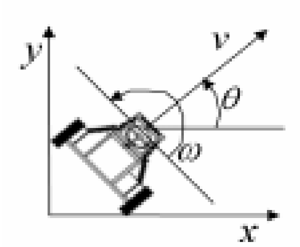


Fig. 1 two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity ω[rad/s] be the inputs in the system. The state equation of the two-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot.

B. Chaos equations

In order to generate chaotic motions for the mobile robot, we apply chaos equations such as a Lorenz, Hamilton, Hyper-choas equation.

1) Lorenz equation

We define the Lorenz equation as follows:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad (2)$$

where $\sigma = 10, r = 28, b = 8/3$. The Lorenz equation describes the famous chaotic phenomenon.

2) Hamilton equation

Hamilton equation is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods. We can derive the state equation of

Hamilton equation as follows.

$$\dot{x}_1 = x_1(13 - x_1^2 - y_1^2)$$
$$\dot{x}_2 = 12 - x_1(13 - x_1^2 - y_1^2) \tag{3}$$

3) Hyper-chaos Equation

Hyper-chaos equation is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods for complex chaotic dynamic. We can easily make hyper-chaotic equation by using some of connected N-double scroll. We can derive the state equation of N-double scroll equation as followings.

$$\begin{aligned} \dot{x} &= a[y - h(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{4}$$

Where,

$$h(x) = m_{2n-1}x + 1/2\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|$$

In order to make a hyper-chaos, we have compose to 1 dimensional CNN(Cellular Neural Network) which are identical two N-double scroll circuits and then we have to connected each cell by using unidirectional coupling or diffusive coupling. In this paper, we used to diffusive coupling method. We represent the state equation of x-diffusive coupling and y-diffusive coupling as follows.

x-diffusive coupling

$$\begin{aligned} \dot{x}^{(j)} &= a[y^{(j)} - h(x)^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{z}^{(j)} &= -\beta y^{(j)},\ j = 1,2...L \end{aligned} \tag{5}$$

y-diffusive coupling

$$\begin{aligned} \dot{x}^{(j)} &= a[y^{(j)} - h(x)^{(j)}] \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)} \\ \dot{z}^{(j)} &= -\beta y^{(j)},\ j = 1,2....L \end{aligned} \tag{6}$$

where, L is number of cell.

C. Embedding of chaos circuit in the robot

In order to embed the chaos equation into the mobile robot, we define and use the Lorenz equation, Hamilton equation, Hyper-chaos equation as follows.

1) Lorenz equation

Combination of equation (1) and (2), we define and use the following state variables:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sigma(y - x) \\ \gamma x - y - xz \\ xy - bz \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \tag{7}$$

Eq. (7) is including Lorenz equation. The behavior of Lorenz equation is chaos. We can get chaotic mobile robot trajectory such as Fig. 2 by using Eq. (7) with coefficient and initial conditions as follows:

Coefficients: v= 1[m/s]

Initial conditions:

$$x_1 = 0.10,\ x_2 = 0.265,\ x_3 = 0.27,\ \ y = 0.5$$

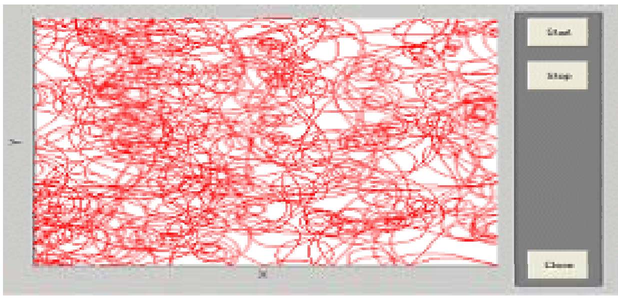


Fig. 2 Trajectory of mobile robot of Lorenz equation

2) Hamilton Equation

Combination of equation (1) and (3), we define and use the following state variables (8)

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x_1(13 - x_1^2 - y_1^2) \\ 12 - x_1(13 - x_1^2 - y_1^2) \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \tag{8}$$

Using equation (8), we obtain the embedding chaos robot trajectories with Hamilton equation. Fig. 3 shows the phase plane of the Hamilton equation with coefficient and initial conditions as follows:

Coefficients: v= 1[m/s]

Initial conditions:

$$x_1 = random,\ x_2 = random,\ \ x = 0.1,\ y = 0.1$$

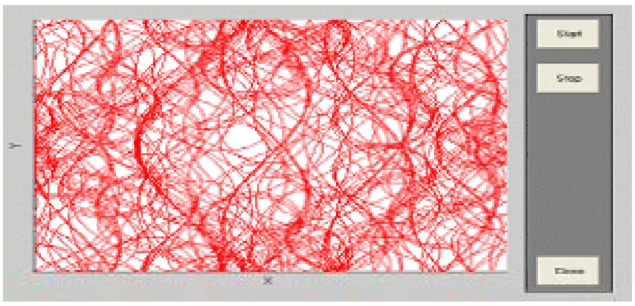


Fig. 3 Trajectory of mobile robot of Hamilton equation

3) Hyper-chaos equation

Combination of equation (1) and (5) or (6), we define and use the following state variables (9) or (10)

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (9)$$

3.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)}] \\ x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (10)$$

Using equation (8) and (9), we obtain the embedding chaos robot trajectories with Hyper-chaos equation. Fig.43 shows the phase plane of the Hyper-chaos equation with coefficient and initial conditions as follows:

Coefficients:

$\alpha = 9,\ \beta = 12.787$, v= 1[m/s], $D_y = 0.01$

Initial conditions:

$x_1 = 0.1,\ x_2 = -0.1\ \ x_3 = 0.1,\ x = 0,\ y = 0$

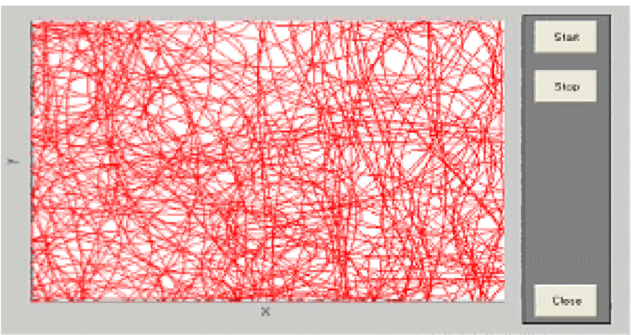


Fig. 4 Trajectory of mobile robot of Hyper-chaos equation

D. Mirror Mapping.

Equation (7) - (10) assume that the mobile robot moves in a smooth state space without boundaries. However, real robots move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the robots approach walls or obstacles using Eq. (11) and (12). Whenever the robots approach a wall or obstacle, we calculate the robots' new position by using Eq. (11) or (12).

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (11)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (12)$$

We can use equation (11) when the slope is infinity, such as $\theta = 90$, and use equation (12) when the slope is not infinity.

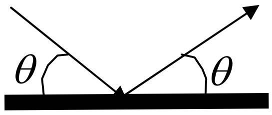


Fig. 5 Mirror mapping

3. The Chaotic Behavior of Chaos Robot with Mirror Mapping and Obstacle

In this section, we will study avoidance behavior of a chaos trajectory with obstacle mapping, relying on the Lorenz equation, Hamilton equation and Hyper-chaos equation respectively.

Fig. 6 through 8 shows that a chaos robot trajectories to which mirror mapping was applied in the outer wall and in the inner obstacles as well using Eq. (11) and (12), relying on Lorenz equation (7), Hamilton equation(8) and Hyper-chaos equation (9) or (10). The chaos robot has two fixed obstacles, and we can confirm that the robot adequately avoided the fixed obstacles in the Lorenz, Hamilton and Hyper-chaos robot trajectories.

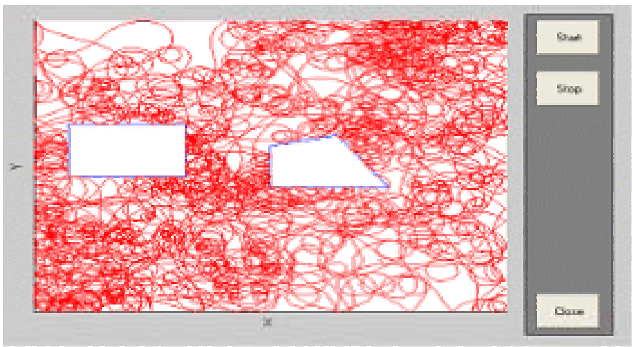


Fig. 6 Lorenz equation trajectories of chaos robot with obstacle

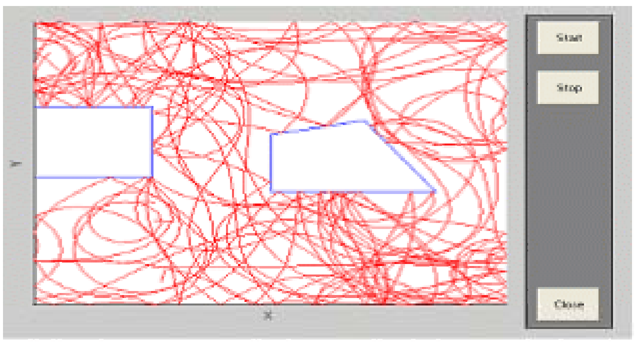


Fig. 7 Hamilton equation trajectories of chaos robot with obstacles

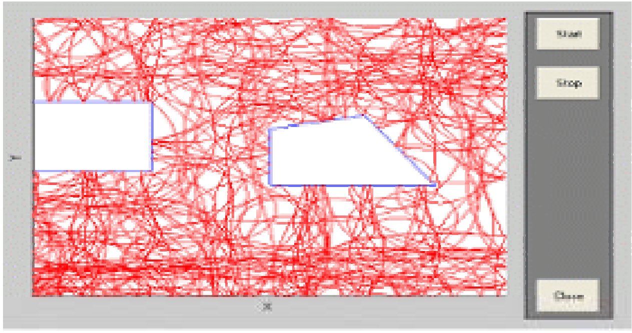


Fig.8 Hyper-chaos equation trajectories of chaos robot with obstacles

4. THE MOBILE ROBOT WITH VAN DER POL EQUATION OBSTACLE.

In this section, we will discuss the mobile robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot can not move close to the obstacle and the obstacle is avoided.

A.. VDP equation as a hidden obstacle

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2)y - x \end{aligned} \tag{13}$$

From equation (13), we can get the following limit cycle as shown in Fig. 9.

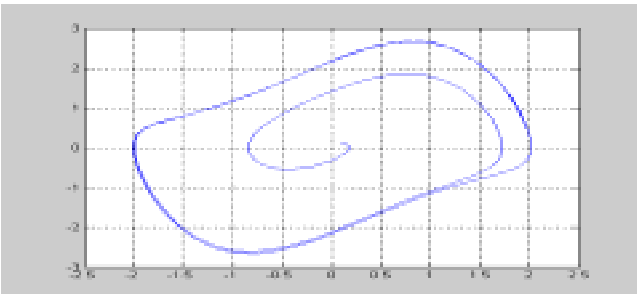


Fig. 9 Limit cycle of VDP

B. Magnitude of Distracting force from the obstacle

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \tag{14}$$

where D_k is the distance between each effective obstacle and the mobile robot.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \vec{x}_k \\ \vec{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1-(y_o - y)^2)(y_o - y) - (x_o - x) \end{bmatrix} \tag{15}$$

where (x_o, y_o) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual robot (I) and the enlarged coordinates (I/2L) of the magnitude of the virtual robot in VDP(x_k', y_k') as follows:

$$\begin{aligned} L &= \sqrt[2]{(\vec{x}_{vdp}^{\,2} + \vec{y}_{vdp}^{\,2})} \\ I &= \sqrt{(x_r^{\,2} + y_r^{\,2})} \\ x_k' &= \frac{\vec{x}_k}{L}\frac{I}{2}, \quad y_k' = \frac{y_k}{L}\frac{I}{2} \end{aligned} \tag{16}$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{bmatrix} \dfrac{\sum_k^n \left(\left(1 - \frac{D_k}{D_0}\right)\vec{x} + \frac{D_k}{D_0}\vec{x}_k\right)}{n} \\ \dfrac{\sum_k^n \left(\left(1 - \frac{D_k}{D_0}\right)\vec{y} + \frac{D_k}{D_0}\vec{y}_k\right)}{n} \end{bmatrix} \tag{17}$$

Using equations (14)-(17), we can calculate the avoidance method of the obstacle in the Lorenz equation, Hamilton and Hyper-chaos equation trajectories with one or more VDP obstacles.

In Fig. 10, the computer simulation result shows that the chaos robot has two robots and a total of 5 VDP obstacles, including two VDP obstacles at the origin in the Lorenz equation trajectories. We can see that the robot sufficiently avoided the obstacles in the Lorenz equation trajectories.

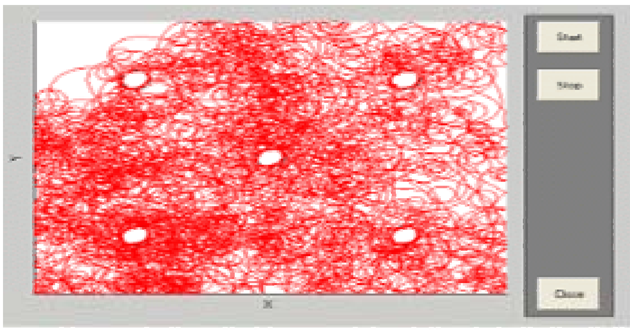


Fig. 10 Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Lorenz equation trajectories.

In Fig. 11, the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Hamilton equation trajectory. We can see that the robot sufficiently avoided the obstacles in the Hamilton equation trajectory.

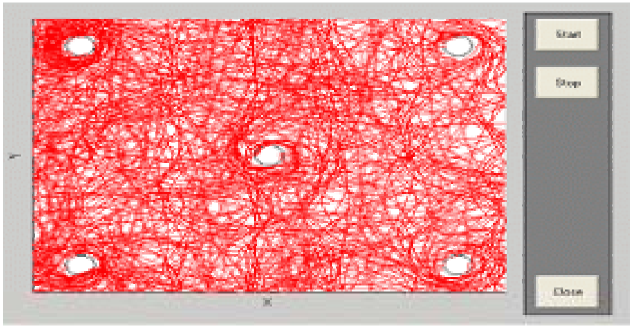


Fig. 11 Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Hamilton equation trajectory.

In Fig. 12, the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Hyper-chaos equation trajectory. We can see that the robot sufficiently avoided the obstacles in the Hyper-chaos equation trajectory.

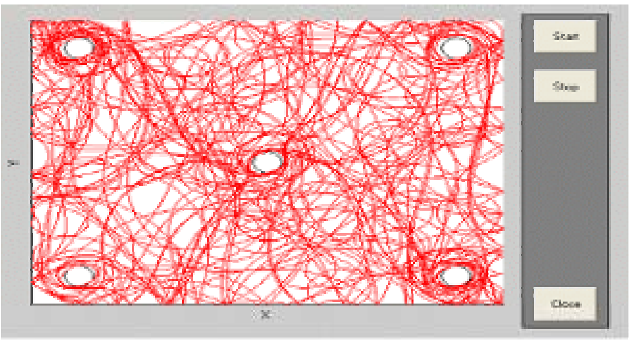


Fig. 12 Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Hyper-chaos equation trajectory

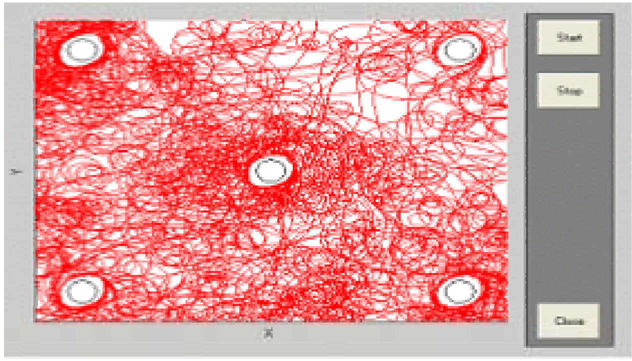
5. According to Dangerous degree obstacle avoidance method

In this section, we proposed a new obstacle avoidance method which is according to dangerous degree with Lorenz equation, Hamilton equation, hyper-chaos equation.

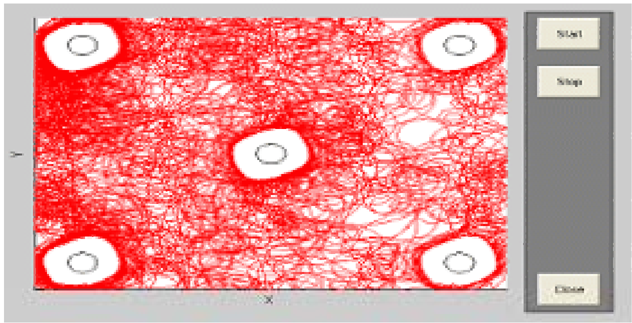
We have to ensure against robot risk with distance limit, if there is a dangerous situation when robots are avoid to obstacle. To do this, we constrained approach obstacle distance for the degree of robots.

A. Lorenz equation

In Fig. 13, we can see the robot trajectories of obstacle avoidance results, dangerous degree are low situation (a), high situation (b) in the Lorenz chaos robot respectively.



(a)

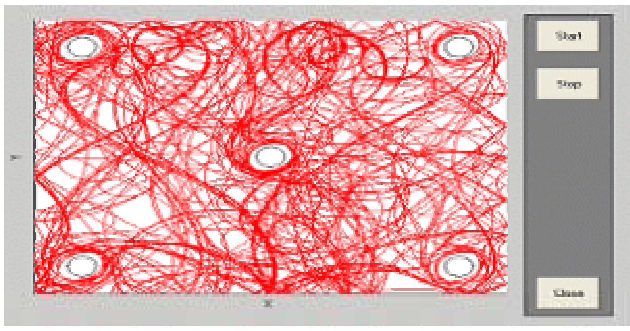


(b)

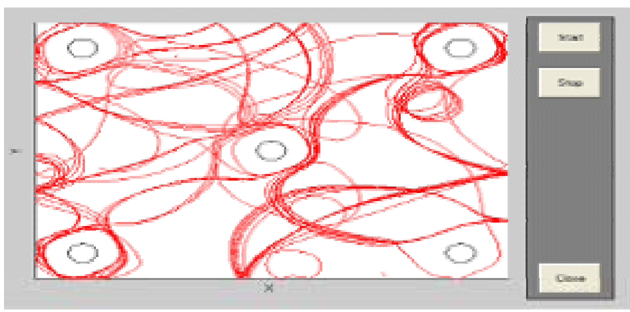
Fig. 13 An obstacle avoidance result for dangerous degree in the Lorenz chaos robot, dangerous degree low (a), high (b)

B. Hamilton equation

In Fig. 14, we can see the robot trajectories of obstacle avoidance results, dangerous degree are low situation (a), high situation (b) in the Hamilton chaos robot respectively.



(a)

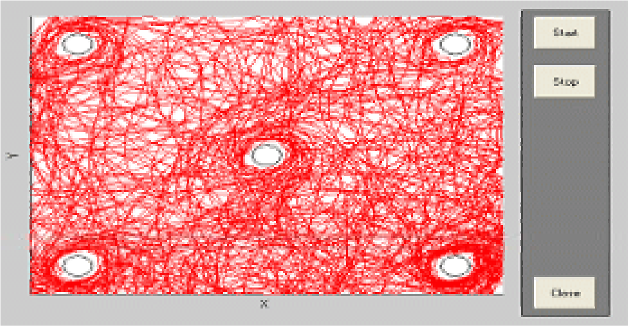


(b)

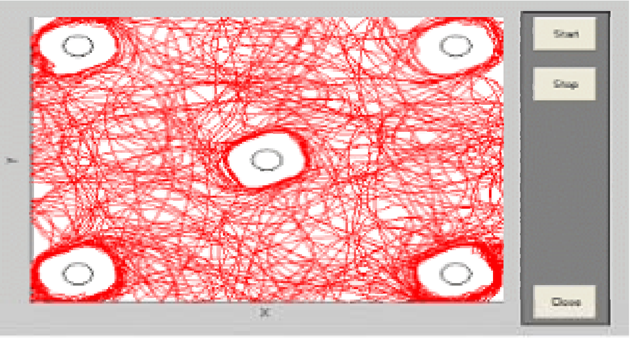
Fig. 14 An obstacle avoidance result for dangerous degree in the Hamilton equation, dangerous degree low (a), high (b)

C. Hyper-chaos equation

In Fig. 15, we can see the robot trajectories of obstacle avoidance results, dangerous degree are low situation (a), high situation (b) in the Hyper-chaos robot respectively.



(a)



(b)

Fig. 15 An obstacle avoidance result for dangerous degree in the Hyper-chaos equation, dangerous degree low (a), high (b)

From Fig. 13, 14, 15, we can compare between the low and high dangerous degree for an obstacle avoidance results.

6. CONCLUSION

In this paper, we proposed a chaotic mobile robot, which employs a mobile robot with Lorenz equation, Hamilton equation and Hyper-chaos equation trajectories, and also proposed an obstacle avoidance method in which we assume that the obstacle has a Van der Pol equation with an unstable limit cycle.

We designed robot trajectories such that the total dynamics of the mobile robots was characterized by an Lorenz equation, Hamilton and hyper-chaos equation and we also designed the robot trajectories to include an obstacle avoidance method with dangerous degree. By the numerical analysis, it was illustrated that obstacle avoidance methods with a Van der Pol equation that has an unstable limit cycle gave the best performance.

In order to make an obstacle avoidance result in the robot system, we applied the Lorenz equation, Hamilton equation and hyper-chaos equation with dangerous degree. As a result, we realized that there are satisfy to obstacle avoidance method with dangerous degree..

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REFERENCE

[1] E. Ott, C.Grebogi, and J.A York," Controlling Chaos", Phys. Rev.Lett., vol. 64, no.1196-1199, 1990.

[2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, " Using small perturbations to control chaos", Nature, vol. 363, pp. 411-417, 1993.

[3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundmentals. vol.E77-A, no. , pp.1000-1005, 1994.

[4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.

[5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, " Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.

[6] K. M. Short, " Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.

[7] L. Kocarev, " Chaos-based cryptography: A brief overview," IEEE, Vol. pp. 7-21. 2001.

[8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", Phys. Rev. E 67, pp. 036208, 2003.

[9] K. Krantz, f. H. Yousse, R.W , Newcomb, "Medical usage of an expert system for recognizing chaos", Engineering in Medicine and Biology Society, 1988. Proceedings of the Annual International Conference of the IEEE, 4-7, pp. 1303 -1304,1988 .

[10] Nakamura, A. , Sekiguchi. " The chaotic mobile robot", , IEEE Transactions on Robotics and Automation , Volume: 17 Issue: 6 , pp. 898 -904, 2001.

[11] J.A.K.Suykens, "N-Double Scroll Hypercubes in 1-D CNNs" Int. J. Bifurcation and Chaos, vol. 7, no. 8, pp. 1873-1885, 1997.

[12] Y. Bae, J. Kim, Y.Kim, " The obstacle collision avoidance methods in the chaotic mobile robot", 2003 ISIS, pp.591-594, 2003.

[13] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Chua's equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp.5-8, 2003.

[14] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003.

[15] Y. C. Bae, J.W. Kim, Y.G. Kim , " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003

[16] Y. C. Bae, J.W. Kim, N.S. Choi , The Collision Avoidance Method in the Chaotic Robot with Hyperchaos Path", KIMICS Conference 2003 Fall , vol. 7, no. 2, pp.584-588, 2003.

[17] Y. C. Bae, J.W. Kim, N.S. Choi ," The Analysis of Chaotic Behaviour in the Chaotic Robot with Hyperchaos Path ov Van der Pol(VDP) Obstacle", KIMICS Conference 2003 Fall , vol. 7, no. 2, pp.589-593, 2003.

[18] Y. C. Bae, J.W. Kim, Y.I, Kim, Chaotic Behaviour Analysis in the Mobile of Embedding some Chaotic Equation with Obstacle", J.ournal of Fuzzy Logic and Intelligent Systems, vol. 13, no.6호, pp.729-736, 2003.

[19] Y. C. Bae, J. W. Kim, Y.I, Kim, Obstacle Avoidance Methods in the Chaotic Mobile Robot with Integrated some Chaotic Equation", International Journal of Fuzzy Logic and Intelligent System, vol. 3, no. 2. pp. 206-214, 2003.

[20] Y. C. Bae, J.W. Kim, Y.I, Kim," The Obstacle Collision Avoidance Methods in the Chaotic Mobile Robots", ISIS 2003 Proceeding of the 4th International symposium on Advanced Intelligent System, pp. 591-594, 2003.