

Mean-variance Approach to Project Management ¹

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Abstract

In this paper, we propose a project portfolio management in order to progress a project smoothly and quickly. In our method, we employ a mean-variance analysis based on the achievement evaluation of project members to select participating members, and employ a mean-variance analysis to decide the reallocation of funds in preceding term minimizing the difference from the allocation of funds in present term. Mean-variance approach is a method of investment over the securities or stocks proposed by H.Markowitz. The future result of project members(named expected evaluation) is to be maximized in executing a strategy under the condition that the project commits a risk in the minimum. It can be formulated as a two-objective non-linear programming problem. Furthermore, we introduce a concept of fuzzy sets to illustrate a decision maker's vague aspiration level for each of expected return rate and risk.

In the mean-variance model, we will not only evaluate the expected evaluation and risk, but also minimize the difference between the preceding and next allocations of funds which satisfies a decision maker's aspiration level. In this paper, we propose a fuzzy reallocation model in the mean-variance analysis. If we decide the fund distribution to each project member after a project was executed one term, the past evaluation of team member should be taken into consideration and the difference of the fund distribution of continuity selected members between the past and next terms should be minimized in order to guarantee their continuity of the plan of each team company as possible.

Keywords: Strategy of Group Management, Fuzzy Mean-Variance Analysis, Achievement Evaluation, Fund distribution

1.Introduction

In recent years, the processing speed in transferring, accumulating and selecting information accelerates agile management and also a drastic global change of corporation environment requires the quick decision in various processes of management such as forecasting the trend of market, decision making of a new product, development of a product and so on.

The concept of a project plays an important role in strategic management.

In the project, the project members are an employee or a group of people in a company, a subordinate company in a conglomerate.

In this paper a project member is named a member. We have proposed a project portfolio management [8] in order to manage a project smoothly and quickly.

The project portfolio management is based on a mean-variance analysis which is proposed by H. Markowitz [9]. The mean-variance analysis is a mathematical programming model to determine an allocation of funds to many stocks. In our method, a project portfolio analysis is employed on the basis of the achievement evaluation of each project members to decide participation of members, and to decide the reallocation of funds in the next term of the allocation of funds in preceding term in minimizing the difference from the previous allocation of project funds [14].

The future result of a project (named expected evaluation in the following) is to be maximized in executing a strategy under the condition that the project commits a risk in the minimum.

It can be formulated as a two-objective non-linear programming problem. Furthermore, we introduce a concept of fuzzy sets to illustrate a decision maker's vague aspiration level for each of expected evaluation and risk [6, 10]. The decision maker can obtain a solution that satisfies his aspiration level required. In the mean-variance analysis, we will not only evaluate the expected evaluation and its risk, but also minimize the difference between the preceding and next allocations of project funds. This is called a fuzzy re-

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allocation portfolio selection model. If we decide the fund distribution to each project member after a project was executed one term, the past evaluation of project members should be taken into consideration and the difference of the fund distribution of continuity selected members between the previous and next terms should be minimized in order to guarantee the continuously of their plan as possible.

2. Project Portfolio Management Considering Change of an Investment

In this paper, we illustrate a project portfolio analysis based on the mean-variance analysis [4, 5, 6, 7]. The mean-variance analysis proposed by H. Markowitz is mathematically to decide the investing ratio over securities or funds which realizes, using time-series data of return rates in the past, the minimum risk (i.e. minimum variance) for the certain expected return rate previously given by a decision-maker. We rewrite the terms of the mean-variance model into the terminology of achievement evaluation method as we discuss about the achievement evaluation method. The expected return should be an achievement evaluation value in the measuring scale given by the decision-maker.

Formulation 1.

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} G_i G_j \\ & \text{maximize} && \sum_{i=1}^n \mu_i G_i \\ & \text{subject to} && \sum_{i=1}^n G_i = 1 \end{aligned}$$

where σ_{ij} is a covariance between evaluation values of project members i and j , μ_i expected evaluation value of project member i , and G_i allocation ratio of funds to project member i , respectively.

In this paper, we formulated the mean-variance analysis as a two objective model which maximizes the expected evaluation and minimizes its risk. The project portfolio management is pursued as the procedure in Figure 2. We will explain its procedure mentioned below.

2.1. Stage 1. The decision of the new project team

In Stage 1, we explain the methods to select more adaptable project members for the new project according to their past achievement evaluation and organize the better project team based on

[PROCEDURE]

- Stage 1. The decision of the new project team
- Stage 2. The decision of fund allocation and the selection of project members
- Stage 3. The selection of the new participating member
- Stage 4. The fund allocation of the continuing member
- Stage 5. The allocation of project funds to new team members

Figure 1: Procedure of project portfolio management

a project portfolio method. In this paper, the states of each section and a project should be accounted with an unified evaluation value. That process is shown in the following. First, we select adaptable members for a new project among the feasible candidate nominated. We employ a project portfolio model to the selection of candidates. The model enables us to select the appropriate project members and decide the allocation of the project funds under the consideration of the past evaluation of the members. The project portfolio analysis proposed in this paper can be employed to evaluate each of project members. Analysis procedure is shown in Figure 2.

[PROCEDURE]

- Step 1: Select the objective direction in project evaluation
- Step 2: Select the explanatory attributes in evaluation
- Step 3: Select the necessity (V_L, E_L) and sufficiency (V_U, E_U) levels of an expected evaluation value and its risk
- Step 4: Determine membership parameters for ratios α_V, α_E

Figure 2: Procedure the achievement evaluation of the project member in mean-variance analysis

Step 1 in Figure 2 is to weigh the direction of the project mission experientially obtained by the decision-maker. These parameters are employed in the fuzzy mean-variance analysis.

Next, evaluation value for the project is established. In this paper, we analyze a project activity using time-series of evaluation values about the past achievement of the project.

Let us denote the number of members in the project m , the number of attributes which evaluate a member n and the number of the times T . Time-series evaluations of n attributes for member i is denoted \mathbf{X}_{it} as:

$$\mathbf{X}^i = \begin{bmatrix} x_{11}^i & x_{21}^i & \cdots & x_{n1}^i \\ x_{12}^i & x_{22}^i & \cdots & x_{n2}^i \\ \vdots & \vdots & \ddots & \vdots \\ x_{1T}^i & x_{2T}^i & \cdots & x_{nT}^i \end{bmatrix} \quad (1)$$

$(i = 1, 2, \dots, n; t = 1, 2, \dots, T)$

The direction of the project is expressed using a direction vector where each value of an attribute denotes an importance degree in the project. Therefore, the evaluation of a member should be pursued in the project direction. The project vector, which shows the direction of the objective evaluation given by a decision-maker, is denoted as:

$$\mathbf{a}^T = [a_1, a_2, \dots, a_n] \quad (2)$$

This project vector shows a direction to weigh the attribute toward which the decision-maker intends to move the project as a performance evaluation index. The size of a project vector is denoted as:

$$A = \sqrt{(\mathbf{a}, \mathbf{a})} = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2} \quad (3)$$

A unit vector toward a project vector is written as follows:

$$\mathbf{u}^T = \left[\frac{a_1}{A}, \frac{a_2}{A}, \dots, \frac{a_n}{A} \right], \quad (\mathbf{u}, \mathbf{u}) = 1 \quad (4)$$

The inner product between time-series evaluation vector \mathbf{x}_{it} and standardized project vector \mathbf{u} provides project evolution based on the measuring scale of the project mission, that is, in the direction of the project vector.

$$B_i = \mathbf{X}^i \mathbf{u} \quad (5)$$

In this paper, we employ the mean-variance analysis to evaluate investment for each project on the basis not only of the maximization of the total project result, but also of the minimization of its risk of the project.

In the conventional mean-variance, this problem is formulated as a quadratic programming which minimizes risk or variance under the realization of the expected return rate of which a decision maker is satisfied, where the expected return and risk hold the trade-off relation. Therefore,

we should better formulate two-objective mean-variance model, which can obtain the best solution under the consideration of aspiration levels that a decision maker might have for both indices of expected return rate and its risk.

The decision-maker defines membership functions for each of expected return rate and risk using necessity and sufficiency levels. The objective of this paper is to obtain a solution that satisfies an aspiration level required.

In this paper, we should employ a non-linear membership function such as a goal one defined by H. Leberling [11], which has asymptotic lines $\lambda = 1$ and $\lambda = 0$. We employ the logistic function for a non-linear membership function as follows :

$$f(\mathbf{x}) = \frac{1}{(1 + \exp(-\alpha(\mathbf{x})))} \quad (6)$$

The logistic function has a similar shape as the hyperbolic function employed by H. Leberling, but it is more easily handled than the hyperbola. And also a trapezoidal membership function is an approximation of the logistic function. Therefore, the logistic function is much more appropriate to denote a vague goal level.

The goal rate for an expected return can be described using the logistic membership function in the following:

$$\mu_E(E(\mathbf{G})) = \frac{1}{1 + \exp(-\alpha_E(E(\mathbf{G}) - E_M))} \quad (7)$$

where E_M is the mid point where membership grade λ is 0.5.

The goal for risk can be described using the logistic membership function in the following:

$$\mu_V(V(\mathbf{G})) = \frac{1}{1 + \exp(\alpha_V(V(\mathbf{G}) - V_M))} \quad (8)$$

where V_M is the mid point where membership value λ is 0.5.

Values α_E and α_V influence respectively on the shapes of membership functions μ_E and μ_V , where $\alpha_E > 0$ and $\alpha_V > 0$. The larger parameters α_E and α_V get, the less their vagueness becomes.

Formulation 2.

$$\begin{aligned} & \text{maximize} && \lambda_t \\ & \text{subject to} && \alpha_V V(\mathbf{G}) + \lambda_t \leq \alpha_V V_M \\ & && \alpha_E E(\mathbf{G}) - \lambda_t \geq \alpha_E E_M \\ & && \sum_{i=1}^n G_i = 1 \\ & && \lambda_t, G_i \geq 0 \quad (i = 1, 2, \dots, n) \end{aligned}$$

where λ_t is a value to which extent the decision maker will be satisfied by the following solution

Table 1: The fund allocation method signary

sign	meaning after the change
σ_{ij}	A covariance between member i and j
μ_i	Expected evaluation value of member i
$G_i \rightarrow F_i$	The allocation rate of project funds to member i

obtained from **Formulation 2.**:

$$V(\mathbf{G}) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} G_i G_j ,$$

$$E(\mathbf{G}) = \sum_{i=1}^n \mu_i G_i ,$$

where σ_{ij} denote a covariance between evaluation, B_i , of member i and evaluation B_j of member j , μ_i expected evaluation value of member i , and G_i allocation ratio of funds to member i , respectively.

2.2.Stage 2. The decision of fund allocation and the selection of project members

In Stage 2, according to this evaluation value, we decide allocation of amount of funds for each of project members, and select the continuous project member organized project is continuously. That process is shown in the following.

First, we determine the fund allocation rate employing a fuzzy mean-variance analysis. That is, even if a certain member obtains some good result, this member will not be evaluated so much when its result is not stable. In this evaluation, we can obtain the best result in the direction of the project mission and also make it stable.

In analysis a member is evaluated using a gross profit data of the last ten terms and the contribution of the member to the project is quantized. The allocation of project funds will be decided based on the result of the project portfolio management. When a member shows a high contribution to the project, the member will be selected as a team member to the next project but a member with less contribution to the project will not be selected to the next project and changed to a new member.

In this stage, we rewrite **Formulation 2.** as we discuss about the fund allocation method. It is written as shown in Table 1 from **Formulation 2.**.

$V(\mathbf{G})$ and $E(\mathbf{G})$ in **Formulation 2.** are rewrit-

ten in the following:

$$V(\mathbf{F}) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} F_i F_j ,$$

$$E(\mathbf{F}) = \sum_{i=1}^n \mu_i F_i ,$$

After we obtain the allocation rate vector \mathbf{F} , we can decide the allocation of the project funds to member i by multiplication between the project funds and allocation rate F_i to member i .

2.3.Stage 3. The selection of the new participating member

In Stage 3, we explain how to select the project members which could not contribute the project and the new project members participating the project.

That process is shown in the following.

In this stage, we should select new project members based on **Formulation 2.**. The selected members can enter the project instead of members which could not contribute to the project in the last term.

2.4.Stage 4. The fund allocation of the Continuing member

In Stage 4, we explain the method to take the preceding allocation of funds into consideration in deciding the next reallocation. So, we employ a fuzzy reallocation model of mean-variance analysis considering the allocation of funds in preceding term. That process is shown in the following.

In this stage, we should explain the formulation of the fuzzy reallocation model of the project portfolio management. We intend to make the plan of a project member assured as possible as we can. In other words, if the allocation of the project funds is changed drastically, the plan of the member cannot be continuously executed over a fiscal year. We distribute the project funds taking the last allocation rate into consideration as long as the performance of the project can be obtained.

In the formulation the linear function of a risk and a difference between the preceding and the next allocation of project funds to a project member is employed as the objective function [15]. However it is easy to define an objective function for the difference between the preceding and the present project member in various ways. In this paper, we define a fuzzy reallocation model of a fuzzy mean-variance analysis based on the following three objective functions.

Formulation 4.

$$\text{minimize} \quad \sum_{i=1}^n \sum_{j=1}^n \sigma_{t,(ij)} F_{t,i} F_{t,j} \quad (9)$$

$$\text{maximize} \quad \sum_{i=1}^n \mu_{t,i} F_{t,i} \quad (10)$$

$$\text{minimize} \quad \frac{1}{n} \sum_{i=1}^n (F_{t,i} - F_{(t-1),i})^2 \quad (11)$$

$$\text{subject to} \quad \sum_{i=1}^n F_{t,i} = 1 \quad (12)$$

$$F_{t,i} \geq 0 \quad (i = 1, 2, \dots, n) \quad (13)$$

where $\sigma_{t,(ij)}$ denotes a covariance between stocks i and j in the t th term and $\mu_{t,i}$ an expected return rate of stock i in the t th term. $F_{t,i}$ and $F_{(t-1),i}$ are allocation rates of funds to project member i in the t th and the $(t-1)$ th terms, respectively.

In this formulation, the third objective function shows the distance between the preceding allocation of funds to project member and the next allocation funds to project member. In a real problem of investing, stocks should not be hugely traded, because of the influence on a price of stocks and the amount of a dealing fee. Therefore, the third objective function should be minimized.

We transform this three objective portfolio model into a fuzzy model based on a fuzzy mean-variance analysis which is proposed by J. Watada et al [13]. We first describe equations as follows:

$$\begin{aligned} V(F) &= \sum_{i=1}^n \sum_{j=1}^n \sigma_{t,(ij)} F_{t,i} F_{t,j} \\ E(F) &= \sum_{i=1}^n \mu_{t,i} F_{t,i} \\ D(F) &= \frac{1}{n} \sum_{i=1}^n (F_{t,i} - F_{(t-1),i})^2 \end{aligned}$$

Employing these functions, we transform Formulation 4 into a fuzzy model employing sigmoid membership functions. The following model is obtained as Formulation 5.

Formulation 5.

$$\text{maximize} \quad \lambda \quad (14)$$

$$\text{subject to} \quad -\alpha_V (V(F) - V_M) \geq \log \frac{\lambda}{1-\lambda} \quad (15)$$

$$\alpha_E (E(F) - E_M) \geq \log \frac{\lambda}{1-\lambda} \quad (16)$$

$$-\alpha_D (D(F) - D_M) \geq \log \frac{\lambda}{1-\lambda} \quad (17)$$

$$\sum_{i=1}^n F_{t,i} = 1 \quad (18)$$

$$F_{t,i} \geq 0 \quad (i = 1, 2, \dots, n) \quad (19)$$

where λ denotes a membership grade, α_V , α_E and α_D determine respectively the shapes of member-

ship functions for risk, expected return and distance. V_M , E_M and D_M denote the mid points at which the membership functions of the risk, the expected evaluation value and the difference between the preceding and next terms have 0.5.

Substituting $\lambda_t = \log\{\lambda/(1-\lambda)\}$, we have

$$\begin{aligned} \log \frac{\lambda}{1-\lambda} &= \lambda_t \\ \lambda &= \frac{1}{1 + \exp(-\lambda_t)} \end{aligned} \quad (20)$$

Since a logistic function is monotonously increase, maximizing λ makes λ_t maximize, that is, maximizes $\log\{\lambda/(1-\lambda)\}$. Accordingly, Formulation 5 is equivalent to Formulation 6 as follows:

Formulation 6.

$$\text{maximize} \quad \lambda_t \quad (21)$$

$$\text{subject to} \quad \alpha_V V(F) + \lambda_t \leq \alpha_V V_M \quad (22)$$

$$\alpha_E E(F) - \lambda_t \geq \alpha_E E_M \quad (23)$$

$$\alpha_D D(F) + \lambda_t \leq \alpha_D D_M \quad (24)$$

$$\sum_{i=1}^n F_{t,i} = 1 \quad (25)$$

$$F_{t,i} \geq 0 \quad (i = 1, 2, \dots, n) \quad (26)$$

In this paper, we employ necessity and sufficiency levels to decide the value of V_M , E_M and D_M . V_L and V_U denote necessity and sufficiency levels for risk, E_L and E_U necessity and sufficiency levels for expected evaluation value, and D_L and D_U necessity and sufficiency levels for distance, respectively. Values V_M , E_M and D_M are decided as $(V_L + V_U)/2$, $(E_L + E_U)/2$ and $(D_L + D_U)/2$, respectively.

Fund allocation can be decided with the following calculation.

$$P_{t-1} = \sum_{i=1}^n O_i C_i \quad (27)$$

$$O_i \in \{0, 1\} \quad (28)$$

$$C_i = F_{i,t-1} \cdot A \quad (i = 1, 2, \dots, n) \quad (29)$$

where P_{t-1} the first half fund of allocation sum total to member, m , O_i is shown whether it was participation or it did not participate, C_i the first half fund of allocation of member, A the total fund of a first half project.

2.5.Stage 5. The allocation of project funds to new team members

In Stage 5, we explain how to distribute the fund among the new project member. That process is shown in the following.

The fund which the fund distributed with Stage 4 in the fund of allocation to the project was

deducted from is distributed in the new project members. Formulation 3 is used for the analysis.

$$(A - P)F_{t,i} = P_t \quad (i = 1, 2, \dots, n) \quad (30)$$

where P_t a new project member fund total of allocation .

3.Concluding Remakes

In this paper, we proposed a project portfolio management in order to progress a project quickly. In our method, we employed a mean-variance analysis based on the achievement evaluation of the project members to select participating project members, and employed a mean-variance analysis considering the reallocation of funds in the preceding term to decide the allocation of funds in the next term. Achievement evaluation was done using the past actual results of the project member, and proper project adaptability could be obtained. The model obtained fund allocation to consider the preceding about the fund allocation problem again was explained. The method can be pursued so as the decision maker can obtain a solution that satisfies an required aspiration level based on their own behavior.

References

- [1] A.Andersen, Group Management, Seisansei Publishing, 1999.
- [2] A.Andersen, Management of Achievement Evaluation, Seisansei Publishing, 2000.
- [3] S.L.Goldman, R.N.Nagel, K.Preiss, Agile Competitors and Virtual Organizations Strategies for Enriching the Customer, Van Nostrand Reinhold, 1995.
- [4] T.Kawaura, J.Watada, "Mean - Variance Analysis of Agricultural Management based on a Boltzmann Machine", Proceedings of 1999 IEEE International Conference on Fuzzy Systems, , Seoul, Korea, pp.1196-1201, 1999.
- [5] T.Kawaura, J.Watada, T.Watanabe, "Neural Network Approach to Sales Portfolio Management", The International Symposium on Medical Informatics and Fuzzy Technology Proceedings, Hanoi, Vietnam, pp.120-123, 1999.
- [6] T.Kawaura, J.Watada, "Fuzzy Mean - Variance Analysis for Production Management", The Fourth Asian Fuzzy Systems Symposium, Tsukuba, Japan, Vol.2, pp.1027-1032, 2000.
- [7] T.Kawaura, J.Watada, "Mean - Variance Analysis of Medical Investment and Medical Products", Biomedical Soft Computing and Human Science, Vol.5, No.2, pp.91-96, 2000.
- [8] Takayuki Kawaura, Junzo Watada and Teruyuki Watanabe, "Analysis of Group Management based on Fuzzy Project Portfolio Model", The 2nd Int. Symposium of Advanced Intelligent Systems, Korea, August 24-25, pp.124-128, 2001
- [9] H. Markowitz, Mean-Variance Analysis in Portfolio Choice and Capital Markets, Blackell, 1987.
- [10] H.Mizunuma, H.Matsuda, J.Watada, "Decision Making in Management Based on Fuzzy-Mean Variance Analysis" , Journal of Japan Society for Fuzzy Theory and Systems , Vol.8, No.5, pp.854-860, 1996 (in Japanese).
- [11] H. Leberling, "On Finding Compromise Solutions in Multicriteria Problems Using the Fuzzy Min-Operator, Fuzzy Sets and Systems, Vol.6, pp.105-118, 1981.
- [12] H. Tanaka, P. Guo, Possibilistic Data Analysis for Operations Research, Heidelberg, New York, Physica-Verlag, 1999.
- [13] J.Watada,"Fuzzy portfolio selection and its applications to decision making", Tatra Moutains Math. Publ. 13, pp.219-248, 1997.
- [14] T. Watanabe and J. Watada, "A Formulation of Fuzzy Rebalance Portfolio", The 2nd Int. Symposium of Advanced Intelligent Systems, Korea, August 24-25, pp.114-118, 2001
- [15] T. Watanabe, J. Watada,"Portfolio Selection Problem with Minimum Fluctuations of Portfolio Pattern" , Proc. The 4th Asian Fuzzy System Symposium 2000 at Tsukuba, Japan, Vol.2 , pp1021-1026 , 2000