Chaotic Short-term Forecasting based on Wavelet Transform

Yoshiyuki Matsumoto
Tohwa University
Multimedia Department
Phone: +81-92-541-1511
FAX: +81-92-541-2707
e-mail: matchan@tohwa-u.ac.jp

Junzo Watada
Osaka Institute of Technology
School of Industrial Management
Phone: +81-6-6954-4327
FAX: +81-6-6952-6197
e-mail: junzow@osb.att.ne.jp

Abstract:--- Recently, the chaotic method is employed to forecast a near future using uncertain data. This method makes it possible by restructuring the attractor of given time-series data in the multi-dimensional space through Takens' embedding theory. However, many economical time-series data do not have a chaotic characteristic. In other words, it is hard to forecast the future trend of such economical data on the basis of chaotic theory. In this paper, time-series data are divided into wave components using wavelet transform. It is shown that the divided components of time-series data is much more chaotic based on correlation dimension than the original time-series data. The highly chaotic characteristic of the divided component of the time-series data enables us to forecast the value or the movement of time-series data in near future. The up and down movement of TOPICS value is shown so highly predicted by this method as 70%.
Keywords: Chaos theory, Short-term forecasting, Wavelet transformation.

I Introduction

The chaotic short-term forecasting method[1-3] based on time-series data enables us to know a value, which we could not predict before. Nevertheless, it is still difficult to forecast a value in near future because many kinds of data have little chaotic characteristics.[4] Even though such data are very low chaotic, it is possible to find the chaotic characteristics in the partial portion of the data[4].

In this research, wavelet transformation[5] is employed to divide the original time-series data into chaotic and non-chaotic portions and we can find the highly chaotic component out of the original data by measuring these correlated dimension. If we can successfully find the highly chaotic portion out of the original data, it is possible to improve the forecasting precision by the wavelet transformation.

The correlation dimension[6][7] will be measured lower than the one of the original data, if the divided components are more highly chaotic than the original data.

II. Chaotic Approach and Forecasting

A. Chaos Theory

The understanding of chaos in common is that it means greatly disturbed state different from an ordered state. Although the chaos in science means a disturbed state, it doesn't mean so large a disturbed state, but does a medium-sized disturbed phenomenon, which changes irregularly along a time in a sense. In other words, it means an irregularity of a changing phenomenon that is controlled by relatively simple rules or a simple structure. One typical example of a chaos system is a logistic mapping. The logistic mapping can be defined by a very simple relation. But the resulted state seems to show very random movement on a graph.

Especially, one characteristic of the chaos system is sensitivity to an initial state [8]. When the initial state is changed a little, the mapping by a chaotic mechanism shows almost the same trajectory in the initial state, but shows very different trajectory beyond a short term. This phenomenon is called "a sensitivity to an initial state" Because of "the sensitivity to an initial state", it is not appropriate to employ the chaos method in forecasting a value in long-term future using the chaos method. That is, it is possible to predict the state in near future, which is sufficiently influenced, by the present state.

B. Forecasting by Chaotic Method

The general objective to employ the chaos method in forecasting is 1) to find a deterministic structure in given time-series data and 2) to predict a value in such a near future from a certain point using this structure, that the present state can sufficiently influence.

This chaotic method enables us to forecast with high precision its values in near future using time-series data which show very unpredictable and unpatriotically changes.

This forecasting bases on the Takens' embedding theory [9] which tells us that it is possible to restructure the trajectory of a dynamic system in a high dimensional space by using only the information (that is, time-series data) of one component dimension (variable).

Using time-series data *x(t)*, let us define vector *Z(t)* as follows:

$$Z(t)=\left(x(t),x(t-t),x(t-2t),\cdots,x(t-(n-1)t)\right) \qquad (1)$$

where τ denotes an arbitrary constant time interval. The vector *z(t)* shows one point in *n* dimensional space (Data Space). Therefore, changing *t* generates a trajectory in the *n* dimensional data space. When *n* is sufficiently large, this trajectory shows a smoothly changed one of the high dimensional dynamic system. That is, if the dynamic system has some attractor, the attractor obtained from the original one should come out on the data space. In other words, the original attractor of the dynamic system can be embedded in the *n* dimensional topological space. Number *n* is named an embedded dimension. Denoting the dimension of the original dynamic system by *m*, it can be proved that this *n* dimension is sufficiently large if *n* holds the following:

$$n = 2m + 1 \qquad (2)$$

Equation (2) is a sufficient condition on the embedded dimension. It is required to employ data with more than *3m+1* to *4m+1* samples in time length in short-term forecasting.

Next, let us describe the deterministic structure using a restructured trajectory. There are several methods. Figure 1 illustrates short-term forecasting using the chaotic method that is embedding discrete time-series data with equal time interval $\tau = 15$ in embedded dimension $n=3$.

Observed discrete time-series samples can be mapped into a topological space of embedded 3 dimensions as shown in Figure 1. As a result, the mapped vector is denoted as follows:

$$Z(t)=\left(x(t),x(t-t),x(t-2t)\right) \qquad (3)$$

Let *z(i)* denote a vector of 3 dimensions that observed data including the most recent time are mapped on a topological space.

Figure 2 illustrates the relation of data which are mapped around the neighborhood of *z(i)* in 3 dimensional space can be shown as Figure 2.

These data in the neighborhood of *z(i)* is the data observed in past.

The trajectory of *z(i+1)* at one step future has been observed as shown in Figure 1.

These relations enable us to forecast behavior *z(i+1)* in near future.

The future trajectory *x(i+1)* of the given time-series data *(x(t), x(t-1),⋯)* can be calculated

1) by deciding the nearest point *z(j)* included in the neighborhood with diameter ε from *z(i)*,

2) by calculating the distance $I_{\{t+1\}}$ between *z(i+1)* and *z(j+1)* using the Jacobi matrix A_j of the nearest point z(j) and the distance *It* between *z(i)* and *z(j)* and

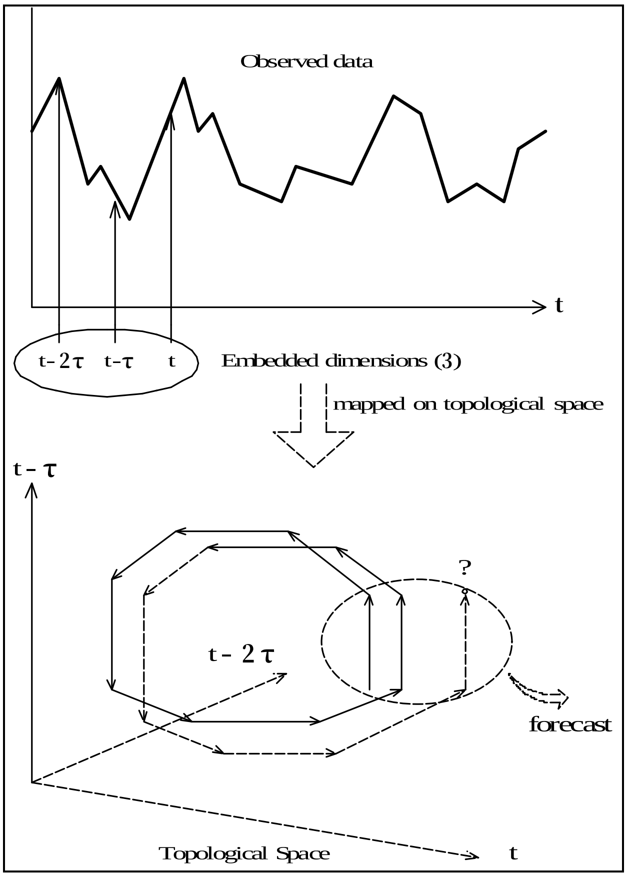

Figure 1. ***z(t)*** **mapped into n dimensional topological space**

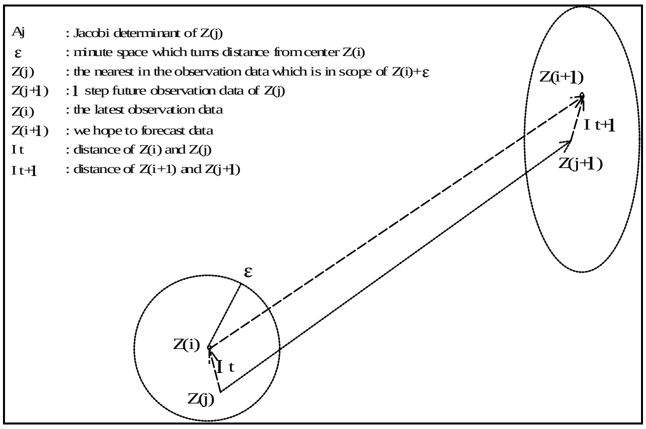

Figure 2. Illustration of Chaotic Short-term Forecasting

3) by deciding the trajectory *x(i+1)* in one step future of the original time-series data.

Ⅲ. Correlation Dimension

Generally measurement of correlation dimension is employed in a method to evaluate whether the time-series data are chaotic or not. The method of correlation dimension is pursued, by checking whether the time-series data distribute in the less dimension space than m dimension, if the data is embedded in *m* dimension space.

At first, let us embed the time-series data into *m* dimension space. Then, the procedure is written as follows:

1)draw the circle with radius *r* at the center of the points which each embedded vector has.
2)count how many points are included within the drawn circle and measure its number *C*.

When the radius is large, then the large number of points should be included in the circle with radius *r*. Therefore, as *C* is an increasing function of *r*, let us denote it as *C(r)*. If plotted points are distributed evenly in the m dimensional space, the number of points included within the circle should increase proportionally to the area of the circle, as the r increases.

$$C(r) = ar^{m} \tag{4}$$

On the other hand, if the structure has any regularity, C(r) should increase proportionally to the less value than *m* powered value

$$C(r) = br^{(m-x)} \tag{5}$$

The value *(m-x)* is named correlation dimension. In the case of random data, the regularity could not be found in the space even if the embedded dimension is increased. Therefore, the correlation dimension should increase as the embedded dimension does. When the time-series data has the deterministic structure in the embedded space, the correlation dimension can not increase and should be matured at the same value, even if the embedded dimension increases.

IV. Wavelet Transformation

FFT is a famous method to transform signal into the portions of each frequencies. A sin function is employed as a base function. The sin function is a function, which is an infinitive smooth function. Therefore, the information obtained by the FFT transformation does not include the local information such as the place and the frequency where the original signals have which frequency.

On the other hand the wavelet transformation employs a compact portion of a wave as a base function. Therefore, it is a time and frequencies analysis such as it is possible to determine the signal using time and frequency

The mother wavelet transformation $\psi(x)$ of a function *f(x)* can be defined as follows:

$$(W_j f)(b,a) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{|a|}} \overline{y\left(\frac{x-b}{a}\right)} f(x)dx \tag{6}$$

Where *a* is a scale of the wavelet, *b* is a translate. $\overline{y(x)}$ is a conjunction of a complex number.

It is also possible to recover the original signal *f(x)* using wavelet transformation. That is, we can realize the inverse wavelet transformation as follows:

$$f(x) = \frac{1}{C_y} \iint_R (W_y f)(b,a) \frac{1}{\sqrt{|a|}} y\left(\frac{x-b}{a}\right) \frac{dadb}{a^2} \tag{7}$$

The wavelet transformation is a useful method to know the characteristics of the signal but not an efficient one. It is because the signal has a minimum unit and the wavelet method expresses many-duplicated information's. This point can be resolved by discrediting a dimensional axis. Let us denote a dimension as $(b,1/a)=(2^{-j}k, 2^{j})$, then the discrete wavelet transform can be written as

$$d_k^{(j)} = 2^j \int_{-\infty}^{\infty} \overline{y(2^j x - k)} f(x)dx \tag{8}$$

Inverse wavelet transform is

$$f(x) \sim \sum_j \sum_k d_k^{(j)} y(2^j x - k). \tag{9}$$

Let us denote the summation $\sum_k d_k^{(j)} y(2^j x - k)$ of the right term as

$$g_j(x) = \sum_k d_k^{(j)} y(2^j x - k) \tag{10}$$

Then let us define $f_j(x)$ as

$$f_j(x) = g_{j-1}(x) + g_{j-2}(x) + \cdots \tag{11}$$

where an integer *j* is named a level. If we can denote *f(x)* as $f_0(x)$, then

$$f_0(x) = g_{-1}(x) + g_{-2}(x) + \cdots \tag{12}$$

This equation illustrates that the function $f_0(x)$ is transformed as wavelet components $g_{-1}(x)$, $g_{-2}(x)$, …, . It is required that the left side should be transformed uniquely into the right side and also the left side should be realized by composition from the right side components. They can be realized by using a mother wavelet y as a base function.

The function $f_j(x)$ can be rewritten using a recursive forms

$$f_j(x) = g_{j-1}(x) + f_{j-1}(x) \tag{13}$$

This equation means that the original signal $f_j(x)$ can be transformed into wavelet components $g_{j-1}(x)$ and $f_{j1}(x)$. This equation enables us to denote the original into the wavelet components step by step. This method is named multi-resolution signal decomposition.

V. Measurement of correlation dimension transformed wavelet components

Here we transformed the time-series data into frequent components by Wavelet multi-resolve analysis. Spline4 is employed as a mother wavelet function and the transformation was done until level 4. The time-series data analyzed is Tokyo stock average index TOPIX. The data are 2048 from January 1991. Figure 4 shows the result obtained by the multi-resolve analysis. The first figure in Figure 4 is the original one. The smaller value j is the lower frequency component it shows.

The result of Wavelet transformation shows TOPIX has each frequency component smoothly.

We measured the correlation dimension of each component divided by Wavelet transformation. The results are shown in Figure 5. The original TOPIX data shows that correlation dimension is matured at around 7. On the other hand, the wavelet component j=-1 is matured at around 6, the correlation dimension of the Wavelet component whose j is less than or equal to -2 is matured around 4 to 5.
Therefore, the results illustrates that the transformed components has more chaotic than the original TOPIX time-series data.

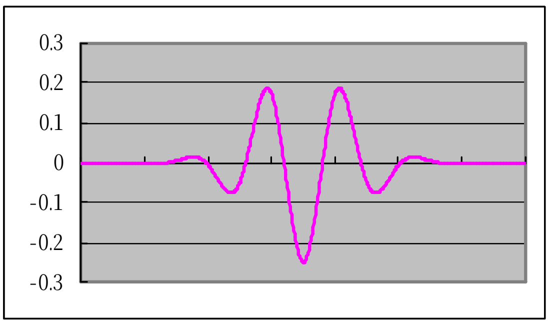

Figure 3 Spline4: Mother wavelet

The measurement result of correlation dimensions shows that the component time-series data are more chaotic than the original time-series data. Let us forecast the short-term future using the original data and divided wavelet transformed component data and compare the preciseness between both results. The data is the same TOPIX data employed above. In this discussion the data were normalized into mean 0 and variance 1. The embedded dimensions are examined from dimensions 3 to 9. We measured the forecast errors about them.

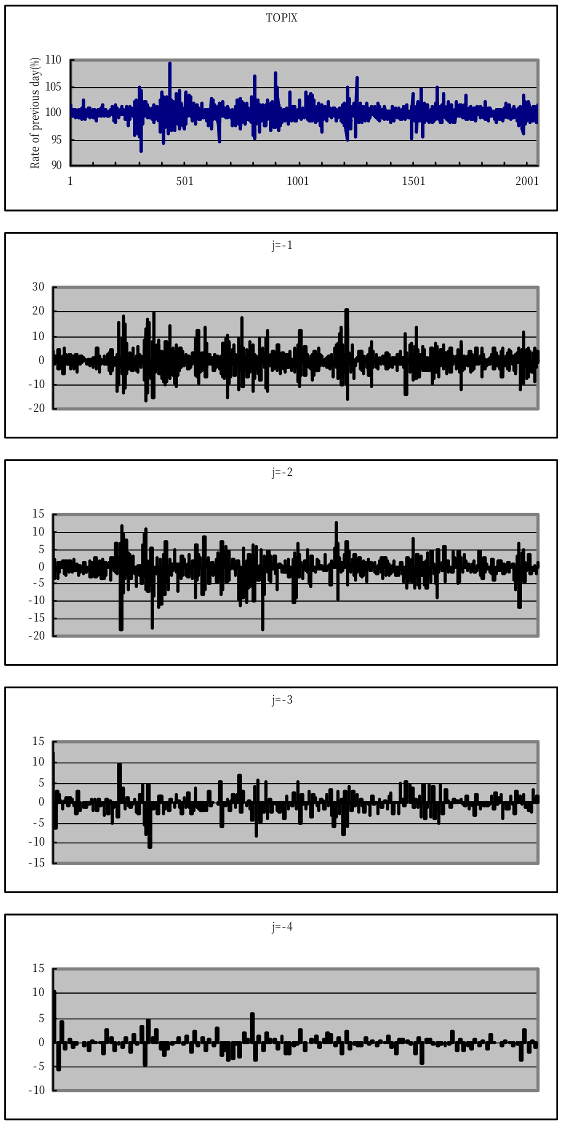

Figure 4. Divided Time-series Data

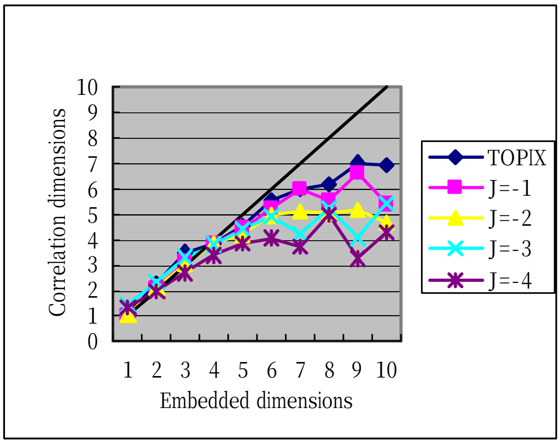

Figure 5. Measurement of Correlation dimension about transformed component data and the original data

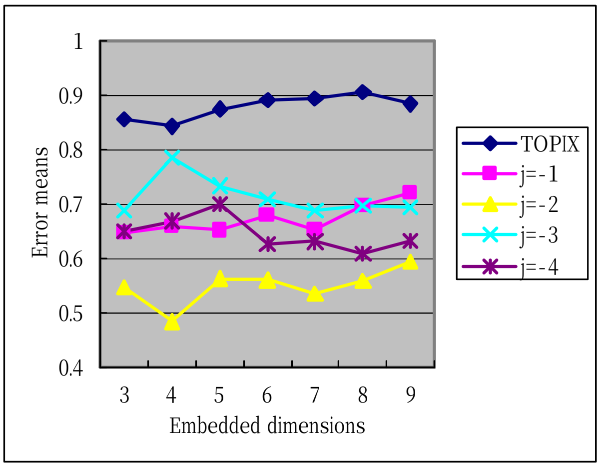

Figure 6. The prediction error of transformed component time-series data

In the forecast, we employed remaining 100 data out of 2024 in the recent portion for the original examination and the transformed component examination.

Figure 6 shows the forecasted results employing wavelet transformed component time-series data. Vertical axis denotes error means and horizontal axis shows embedded dimension.

The transformed component data shows lower forecasting error than the original data. This shows that the component data is much more chaotic than the original TOPIX.

We could show that the wavelet transformed component time-series data are more chaotic than the original one both from the measurement of correlation dimension and from the forecasting error.

Let us examine the forecasting of the up and down movements of the price. This is to forecast the movement to which direction the today's price of a stock goes from the yesterday's price. This forecasting is only done the movement instead of on the value.

Let us check the prediction of up and down movements of the stock price based on the forecasted results. The up and down prediction was done for the movement of their following day's price using the TOPICS index data. The prediction is correct, if the direction of the movement is the same between the real and predicted movements. The percentage of the correct predictions is shown for the 100 trials.

As shown in Figure 7, the vertical axis is correct prediction rate and the horizontal axis is embedded dimension.

The component data by Wavelet transform shows better than the original time-series data. The transformed component of the original data is much easier to predict the chaotic movement. In the case j=-4 the prediction is better than 70%. This is very high prediction.

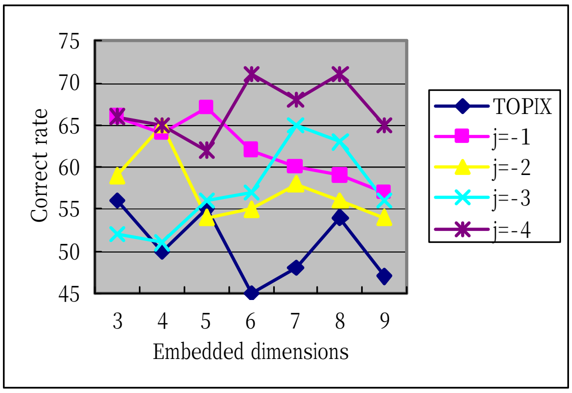

Figure 7 Correct rates on up and down prediction

VI. Concluding Remarks

The objective of this paper is to realize the short-term forecasting based on Wavelet transform. Original time-series data are divided into components through Wavelet transform. The chaotic short-term forecasting method is applied to the transformed component abstract by Wavelet transform.

It is noted that even if the given time-series data have lower chaotic characteristics, we can derive the chaotic structure in the components obtained by Wavelet transform. This means that even if the given data is lower chaotic, we can derive partially chaotic component from the original data through the Wavelet transform.

We discussed about the measurement of the correlation dimension, forecasting error and the correct prediction rate by the comparison between the original data and the component of the Wavelet transform using TOPIX data which is a Japanese index of the stock market. The correlation dimension of the component is lower than one of the original data. TOPIX data obtained lower correlation dimension than individual stock-price data. The lower correlation dimension means higher chaotic characteristics. TOPIX showed that the divided component data obtained by Wavelet transform obtained higher prediction rate than the original data. This shows the wavelet transform can abstract the chaotic component well from the original data.

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