Portfolio Selection Model Based on Block of Shares [1]

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Abstract– **In a real investment, a stock should be dealt according to a block of shares. In the formulation of a conventional portfolio selection model, it is not discussed about a block of shares for each of stocks. However, in a real investment, a decision maker should buy each stock according to a block of shares. Therefore, for example, in the case where the decision maker allocates the funds according to the solution of the portfolio selection model, if the amount of funds for investment is short, he might not buy the stock only one block of shares.**

In this paper, we propose a portfolio selection model in consideration of the amount of funds and block of shares.

Keywords: Portfolio Selection, Amount of funds for investing, a block of shares

1.Introduction

H. Markowitz proposed a method to allocate of an amount of funds for investment to many stocks, which was named a portfolio selection problem in 1952 [1]. Based on time-series data of return rate, it theoretically decides the best investing rate to each of stocks, which minimizes the risk or the variance of the profits in keeping the least rate or the expected return rate that a decision maker expects. The characteristic of this model is to reduce the risk to allocate the amount of funds for investing to many stocks. This is a excellently concise model for real problems, researches have been done about various aspects of this model, such as increase in efficiency of calculation [2, 4, 5, 6].

In a real investment, stocks are dealt with based on the block of shares [3]. For example, in Japanese market, the stocks are traded in blocks of 1000 in the case where the face value is 50 yen, in a block of 100 in the case where the face value is 500 yen and in a block of 1 in the case where the face value is 50,000 yen.

By the way, the portfolio selection problem proposed by H. Markowitz is formulated based on the premise that the amount of funds can be assumed to be infinity without limit. However, if a decision maker invests the amount of funds according to investing ratio, it is clear that there are surplus funds. Moreover, the solution is the same result which is not concerned with the size of the funds which a decision maker manages. However, even if it is the same investment ratio, naturally ral allocations to each stock should differ in the cases where the funds are 1 billion yen and 500 million yen. Furthermore, it is possible that a decision maker may unable to buy only one stock because of the investing ratio for an objective stocks is much smaller.

In consideration of the above problem, in this paper, we propose the portfolio selection model in consideration of the amount of investing funds and a block of shares. A proposed model is formulated based on Markowitz model. In this model, we express the investing ratio using a stock unit price, a block of shares, a number of blocks and the amount of funds for investing which a decision maker can manage. Therefore, our model can obtain a solution not with an investment ratio but with the number of blocks of shares. It is possible to obtain a solution more realistically than the conventional model by this formulation.

In this paper, as a numerical example, we calculate in an example the amount of funds 1 billion yen, 500 million yen and 250 million yen, and the difference among results will be compared depending on each of the amount of funds. Moreover, we will campare between Markowitz model using the investing ratio and the amount of funds and the result of our model.

2.Portfolio Selection Model Based on a Block of Shares

In this section we explain the portfolio model in consideration of the amount funds and a block of shares. First, we explain the portfolio selection ploblem which is proposed by H. Markowitz. Next, we explain the formulation of our model.

A. Markowitz Model

The mean-variance approach to the portfolio selection was originally proposed by H. Markowitz [1]. Generally, a decision maker is satisfied of that a risk is much smaller and that the expected return rate is much larger. According to this point of view, the portfolio selection

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ploblem is formulated as the following two objective quadratic programming problem.

FORMULATION 1.

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j, \tag{1}$$

$$\text{maximize} \quad \sum_{i=1}^{n}\mu_i x_i, \tag{2}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_i = 1, \tag{3}$$

$$x_i \geq 0 \quad (i = 1, 2, \ldots, n), \tag{4}$$

where σ_{ij} denotes a covariance between stocks i and j, μ_i an expected return rate of stock i. x_i is an investing rate to stock i, respectively.

However, it is difficult to obtain the optimal solution of FORMULATION 1 efficiently. Therefore, H. Markowitz assumed almost all the decision maker to be a risk aversion person. According to this assumption, H. Markowitz has formulated a portfolio selection problem as the following quadratic programming problem under the restriction that the expected return rate is made sure to be more than some amount.

FORMULATION 2.

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \tag{5}$$

$$\text{subject to} \quad \sum_{i=1}^{n}\mu_i x_i \geq R \tag{6}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{7}$$

$$x_i \geq 0 \quad (i = 1, 2, \ldots, n) \tag{8}$$

where R denotes an acceptable least rate of the expected return, σ_{ij} a covariance between stocks i and j , μ_i an expected return rate of stock i, and x_i an investing rate to stock i, respectively.

The characteristic of this model is to reduce the risk to allocate the amount of funds for investing to many stocks.

In the real investment problem, there is a limitation in the amount of funds which a decision maker can manage. Furthermore, since dealings of stocks are conducted based on the block of shares of each stock, it is impossible for using up the fund prepared in fact 100% because of an odd dealing problem. However, the portfolio selection problem proposed by H. Markowitz is formulated based on the premise that the amount of funds can be infinity without limit.

Therefore, in this paper, we propose the portfolio selection model in consideration of the amount of funds for investment and a block of shares.

Table 1: Price of each stock

	Unit price of the each stock(yen)	The stock price per block(yen)
Toray Industries	520	520,000
Obayashi Corp	375	375,000
Snow Brand Milk	446	446,000
Teijin	494	494,000
Kao	1,010	1,010,000
Asahi Glass	935	935,000
Daikin Industries	638	638,000
Hitachi	770	770,000
Honda Motor	1,190	1,190,000
Dai Nippon Printing	1,400	1,400,000
Taisho Marine & Fire	540	540,000
Tobu Rail	429	429,000
All Nippon Airways	775	775,000
Osaka Gas	250	250,000
Shochiku	1,150	1,150,000

B. Fourmulation

Our model is fundamentally formulated based on the Markowitz model, which is illustrated in Formulation 2. In our model, we express the investment ratio of a Markowitz model using a stock unit price, a block of shares and the amount of funds for investing which a decision maker can manage. Our model can be rewritten as follows:

FORMULATION 3.

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\frac{n_i p_i q_i}{B}\frac{n_j p_j q_j}{B} \tag{9}$$

$$\text{subject to} \quad \sum_{i=1}^{n}\mu_i\frac{n_i p_i q_i}{B} \geq R \tag{10}$$

$$\sum_{i=1}^{n} n_i p_i q_i \leq B \tag{11}$$

$$\frac{n_i p_i q_i}{B} \geq 0 \quad (i = 1, 2, \ldots, n) \tag{12}$$

where, n_i denotes the number of dealing blocks of shares of the ith stock, p_i a unit dealing price of the ith stock, q_i the equity volume per block of shares of the ith stock and B an amount of funds for investment which a decision maker can manage, respectively.

The characteristic of our model can obtain a solution not with an investment ratio but with the number of a block of shares. Therefore, This formulation enables us to obtain more realistic a solution than the conventional model.

3.Numerical Example

In this section, we exemplify the effectiveness of our model employing a numerical example.

The nominated stocks for investing are 15 Japanese stocks including Toray Industries, Obayashi Corp, Snow Brand Milk, Teijin, Kao, Asahi Glass, Daikin Industries, Hitachi, Honda Motor, Dai Nippon Printing, Taishou Marine & Fire, Tobu Rail, All Nippon Airways (ANA), Osaka Gas and Shouchiku. The price of the each stock is illustrated in Table 1. We treat a block of shares as 1000.

In this simulation, an acceptable least rate of the expected return R is 0.015. Moreover, the amount of funds for investing which a decision maker can manage is as 1 billion yen, 500 million yen, and 250 million yen.

The results of our model is shown in Table 2.

In Table 2, in the case where the amount of funds is 1 billion yen, the stock of Toray Industries and Tobu is purchased with a small number of blocks. However, these stocks are not purchased in the case where the amount of funds are smaller than 1 billion yen. This means that it is more efficient to purchase other stocks rather than purchasing these stocks a little number of blocks.

Comparing a risk and the expected return rate, there is not large difference from each value by the difference of the amount of funds.

Next we calculate the Markowitz model in order to compare with the result of our model. Where an acceptable least rate of the expected return R is 0.015. The amount of funds for investing which a decision maker can manage is 1 billion yen, 500 million yen, and 250 million yen, respectively.

The results are illustrated in Table 3.

In Table 3, it is shown that the number of blocks are not integer. For example, In the case of Toray Industries, the number of blocks is 38.60, so the decision maker can buy this stock only for 38 blocks. The funds for 0.60 blocks are to surplus fund. Therefore, the funds allocation rate of solution of Markowitz model is not 100% in the real investment.

So we re-calculated the number of blocks and allocated funds of Markowitz model. The results are shown in Table 4.

Comparing between Table 2 and Table 4, the total of allocated funds of our model are larger than the each of Markowitz model. Especially, in the case where the amount of funds is smaller, a differece of total value is larger. We calcurate the allocation efficiency of the funds, which is value showing the ability to be allocated efficiently, employing a following equation.

$$\frac{1}{B}\sum_{i=1}^{n} n_i p_i q_i \times 100 \tag{13}$$

The each result is shown in Table 5.

Table 5 shows that the allocation efficiency of our model is better than the efficiency of Markowitz model. When the amount of funds is 1 billion yen, the allocation efficiency is simmilar between our model and Markowitz model. However, as the amount of funds decrease, the allocation efficiency of Markowitz model is worse remarkably. On the other hand, the efficiency of our model is better value.

Furthermore, comparing the risk and expected return rate, our model can obtain the satisfiable value to be compared with the value of Markowitz model.

As the results of a numerical example, we could show that the proposed model can obtain the portfolio which realizes the efficient allocation of fund according to the amount of funds which a decision maker can manage.

4.Concluding Remarks

In a real investment, a stock is dealt according to a block of shares. in the formulation of a conventional portfolio selection model, it is not discussed about a block of shares for each of stocks. However, in a real investment, a decision maker should buy each stock according to a block of shares. Therefore, for example, in the case where the amount of funds for investment is short, if the decision maker allocates the amount of funds according to solution of the portfolio selection model, He might not buy the stock only one block of shares.

In this paper, we proposed the portfolio selection model in consideration of the amount of funds for investment and a block of shares. A proposed model is formulated based on Markowitz model. In this model, we expressed the investing ratio using a stock unit price, a block of shares, a number of blocks and the amount of funds for investing which a decision maker can manage.

The following points should be emphasized from the result of numerical examples :

- We can obtain the portfolio which realizes the efficient allocation of fund according to the amount of funds which a decision maker can manage employing our model. Our model can obatin a solution not with an investment ratio but with the number of blocks of shares. So the solution of our model is more realistic than Markowitz model.
- As compared with the Markowitz model, we can allocate the amount of funds more efficiently.
- Comparing the risk and expected return rate, our model can obtain the satisfied value to be compared with the value of Markowitz model.

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Table 2: Results of our model

	Amount of investing funds					
	1 billion yen		500 million yen		250 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	10	5,200,000	0	-	0	-
Obayashi Corp	35	13,125,000	0	-	0	-
Snow Brand Milk	719	320,674,000	366	163,236,000	181	80,726,000
Teijin	0	-	0	-	0	-
Kao	86	86,860,000	65	65,650,000	21	21,210,000
Asahi Glass	0	-	0	-	0	-
Daikin Industries	0	-	0	-	0	-
Hitachi	44	33,880,000	0	-	3	2,310,000
Honda Motor	37	44,030,000	21	24,990,000	6	7,140,000
Dai Nippon Printing	138	193,200,000	75	105,000,000	47	65,800,000
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	22	9,438,000	0	-	0	-
All Nippon Airways	114	88,350,000	48	37,200,000	26	20,150,000
Osaka Gas	394	98,500,000	253	63,250,000	114	28,500,000
Shochiku	91	104,650,000	35	40,250,000	21	24,150,000
Total		997,907,000		499,576,000		249,986,000
Expected Return Rate	0.01634		0.01647		0.01643	
Risk	0.000706		0.000744		0.000751	

Table 3: Results of Markowitz Model

	Amount of investing funds					
	1 billion yen		500 million yen		250 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	38.60	20,069,590.0	19.30	10,034,795.0	9.65	5,017,397.5
Obayashi Corp	0.00	0.0	0.00	0.0	0.00	0.0
Snow Brand Milk	695.23	310,074,300.0	347.62	155,037,150.0	173.81	77,518,575.0
Teijin	0.00	-	0.00	-	0.00	-
Kao	80.21	81,016,520.0	40.11	40,508,260.0	20.05	20,254,130.0
Asahi Glass	20.65	19,305,840.0	10.32	9,652,920.0	5.16	4,826,460.0
Daikin Industries	0.00	-	0.00	-	0.00	-
Hitachi	87.83	67,630,660.0	43.92	33,815,330.0	21.96	16,907,665.0
Honda Motor	40.99	48,781,180.0	20.50	24,390,590.0	10.25	12,195,295.0
Dai Nippon Printing	64.15	89,809,370.0	32.07	44,904,685.0	16.04	22,452,342.5
Taisho Marine & Fire	0.00	-	0.00	-	0.00	-
Tobu Rail	0.00	-	0.00	-	0.00	-
All Nippon Airways	143.80	111,443,300.0	71.90	55,721,650.0	35.95	27,860,825.0
Osaka Gas	563.47	140,868,600.0	281.74	70,434,300.0	140.87	35,217,150.0
Shochiku	96.52	111,000,600.0	48.26	55,500,300.0	24.13	27,750,150.0
Total		1,000,000,000		500,000,000		250,000,000
Expected Return Rate	0.01611		0.01611		0.01611	
Risk	0.000666		0.000666		0.000666	

Table 4: Results of Markowitz Model(Re-calculation)

	Amount of investing funds					
	1 billion yen		500 million yen		250 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	38	19,760,000	19	9,880,000	9	4,680,000
Obayashi Corp	0	-	0	-	0	-
Snow Brand Milk	695	309,970,000	347	154,762,000	173	77,158,000
Teijin	0	-	0	-	0	-
Kao	80	80,800,000	40	40,400,000	20	20,200,000
Asahi Glass	20	18,700,000	10	9,350,000	5	4,675,000
Daikin Industries	0	-	0	-	0	-
Hitachi	87	66,990,000	43	33,110,000	21	16,170,000
Honda Motor	40	47,600,000	20	23,800,000	10	11,900,000
Dai Nippon Printing	64	89,600,000	32	44,800,000	16	22,400,000
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	0	-	0	-	0	-
All Nippon Airways	143	110,825,000	71	55,025,000	35	27,125,000
Osaka Gas	563	140,750,000	281	70,250,000	140	35,000,000
Shochiku	96	110,400,000	48	55,200,000	24	27,600,000
Total		995,395,000		496,577,000		246,908,000
Expected Return Rate	0.01603		0.01600		0.01591	
Risk	0.00066		0.000657		0.00065	

Table 5: Allocation efficiency of the amount of funds

	Amount of investing funds		
	1 billion yen	500 million yen	250 million yen
Our model	99.79%	99.92%	99.99%
Markowitz model	99.54%	99.32%	98.76%

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