Causal Relations Dynamic Modeling for Control and Fault Diagnosis

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Abstract—**Simplified incremental type of dynamic models, named cause-effect relation (CER) models are proposed and investigated in the paper. They are able to identify a wide class of dynamic plants and systems by revealing the relationships between the current *change-of-output* and the past *changes-of-inputs* of the process. The Least-Squares estimation algorithm, including some specially proposed modifications are used for identifying the parameters: CER functions of these models. In most practical cases, the identification results lead to a meaningful as shape and a logically interpretable CER function that can be further used in different human-like decision- making or fault diagnosis problems.**

The second part of the paper deals with an implementation of the CER dynamic models for synthesis of a feed-forward controller unit in an open loop reference control scheme. The result of this synthesis is a CER-dynamic model that forms a serial connection with the plant and tries to match as close as possible the performance of the preliminary defined reference model.

The third part of the paper describes a special recursive computation scheme for calculation of the inverse dynamics of a single-input process. It is shown that this procedure can be successfully used for solving "backward tracking" and fault diagnosis problems in real dynamic systems.

Finally, extensive simulations have shown the merits of the proposed modeling and control approaches, as well as their possible practical applications.

I. Introduction

The smooth operation of complex industrial systems requires frequent or real time monitoring and analysis of their dynamic behavior. The results of such analysis are most often used for control purposes, but could also be useful in different procedures of decision making and fault diagnosis. Here the possible reason (origin) for the observed abnormal behavior of the system has to be discovered and analysed based on the preliminary identified dynamic system model.

In recent years the research area of simulation and identification of dynamic systems have been very popular and a lot of interesting results have been reported in the literature. Most of the approaches use neural networks and fuzzy models [1],[3] or auto regressive techniques [2],[4],[5] for representing the system behavior and its internal cause-effect relationships.

One problem in identification based on input-output data is, that the model accuracy and its interpretability and generalization ability are usually contradictory requirements. Standard identification procedures normally do not use human knowledge or hypothesis about the expected dynamics of the system in order to make it more interpretable. As a result accurate identified models with bad interpretability are often obtained as a result of a standard identification, based on pure numerical calculations.

In this paper a general cause-effect type of dynamic model is considered, that uses the cause-effect relations between the past input changes and the current process output. This model has an incremental structure and needs a special identification procedure, proposed in the paper. The so called Cause-Effect Relation (CER) function, obtained by the identification procedure, keeps in most cases clear physical understanding about the strengths of relationships in the dynamic process. Therefore such a model has a better interpretability even if it is not necessarily better in accuracy, compared to other types of dynamic models.

The CER dynamic model can be successfully used for identification of a wide class of dynamic systems and in fault diagnosis procedures. Further on, the proposed model is used in a special algorithmic scheme for synthesis of a reference model controller in open loop control structure. The obtained controller (the correction unit) converts the original input to the plant into appropriate control that tries to match as close as possible the output of a predefined reference model. The obtained controller has the same cause-effect structure as those of the reference model and the plant model, which makes the whole control structure more homogeneous. Finally, some results from various numerical simulations in this paper show the practical applicability and give ideas for further ideas and improvements.

II. The Cause-Effect Relations Dynamic Models

The dynamics of a process with control input $u(t)$ and a process output $y(t)$ can be represented in many different formats and structures [1]-[5]. When a discrete dynamics is taken into account, the general cause-effect relation scheme is a suitable way of modeling the dynamic behavior, as shown in Fig. 1.

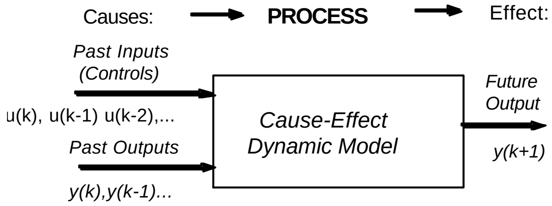

Fig.1. The Cause-Effect Relations in Dynamic Models.

Let k be the current time sampling. Then the basic idea is that in order to predict the future process output $y(k+1)$ we need to consider the previous inputs (controls) and previous outputs according to Fig. 1.

The autoregressive type models such as ARX or ARMA-models and their non-linear modifications: NARX and NARMA models are widely used for identification of dynamic systems [2],[4],[5]. They use preliminary assumed parameters m and n (a kind of *order* or *memory* of the process) that represent the number of the delayed inputs and outputs respectively, as follows:

$$y(k+1) = \sum_{i=1}^{m} g_i y(k+1-i) + + \sum_{i=0}^{n-1} f_i u(k-i) \quad (1)$$

However the above dynamic models do not have explicit physical interpretation for $m>1$. Therefore another *cause-effect* type model that has a better interpretability can be created if we assume $m=1$ (*one step* past time output only) and also if we take as inputs the *differences* of the past inputs of the model. Then the new model, called cause-effect relations (*CER*) model as in [4],[5] uses the following variables:

changes-of-the-past-inputs: $\Delta u(k)$, $\Delta u(k-1)$,... and the respective *change-of-the-current-output:* $\Delta y(k)$.

The CER dynamic model has a clear human-interpretable structure and could be regarded as a kind of "rule-based-model" since *differences*, but not absolute values are taken into account in it. Its computational structure is shown in Fig. 2.

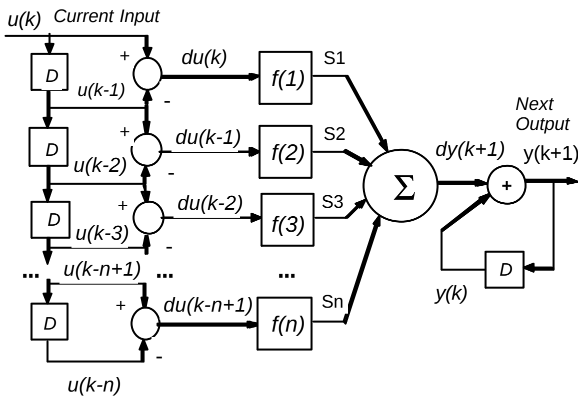

Fig. 2. Computational Structure of the One-Dimensional CER Dynamic Model.

Let us denote by n the *memory length* of the dynamic model. Then the incremental type of the CER dynamic model used for prediction of a system behavior is written according to Fig. 2. as:

$$y(k+1) = y(k) + \Delta y(k+1) \quad (2)$$

where:

$$\Delta y(k+1) = F[\Delta u(k),...,\Delta u(k-i),...,\Delta u(k-n+1)] \quad (3)$$

with

$$\Delta u(k-i) = u(k-i) - u(k-i-1) \quad (4)$$

Then the predicted increment of the output is :

$$\Delta y(k+1) = \sum_{i=1}^{n} \Delta u(k+1-i) f(i) = \sum_{i=1}^{n} S_i \quad (5)$$

In case of non-linear dynamic behavior, the strength levels $f(i)$, $i=1,2,...,n$ are rather functions than simple constants, i.e. the CER-function changes its shape within the operating area of the model. In this case, a *Fuzzy Inference System*, as shown in Fig. 3. could be used to compute the non-linear CER model.

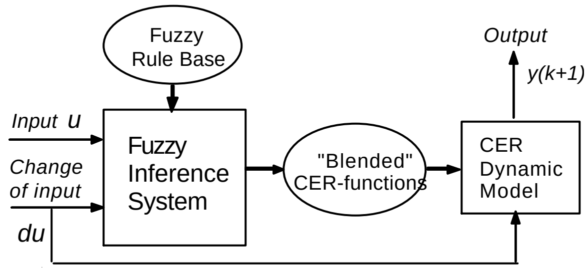

Fig. 3. Computation Scheme for Non-Linear CER Dynamic Model with Fuzzy Inference System.

In Fig. 4. an example of two different shapes of CER functions, from two different CER Dynamic Models: *M1* and *M2* are displayed. Their respective dynamic behavior is shown in Fig. 5.

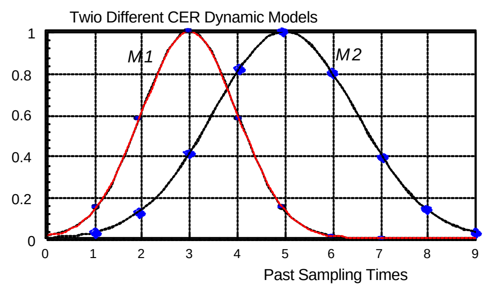

Fig. 4. Example of two Different CER Dynamic Models.

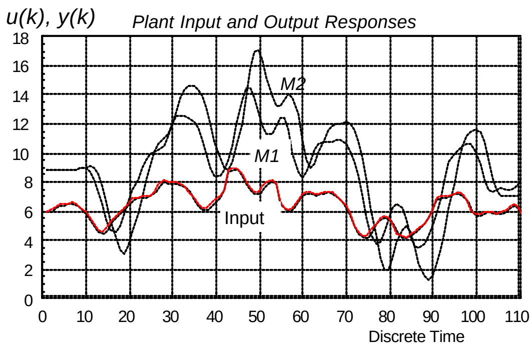

Fig. 5. Dynamic Responses of the CER Models from Fig. 4.

By increasing the length of the memory n more precise dynamic behavior can be expressed while by changing the shape of the CER-functions different dynamic behaviors can be expressed.

In most cases the CER-function is a physically interpretable due to its smooth shape (e.g. *increasing-decreasing*). However it has been found experimentally, that the CER dynamic models may lose easily their physical interpretability if noise data are used for identification. Quite often, even a slight noise may cause a significant distortion in the real (smooth) shape of the CER function, thus destroying its physical meaning. This demerit of the CER models could be avoided to some extent by introducing the concept of the Cellular Dynamic models (not presented in this paper), as discussed in [5]. It is proven that the Cellular Dynamic Models are more robust against noised input-output data thus producing more interpretable and meaningful CER-functions.

III. Identification of the Dynamic Model Based on the Least-Square Estimation Algorithm

This is an off-line identification problem where a preliminary defined number of n strength degrees $f(i),\ i=1,2,...,\ n$ of the CER-function have to be identified based on a collection of M input-output experimental data: $\{u(k), d(k)\},\ k=1,2,...,M$. During the Identification, the mean squared error (MSE) between the measured output $d(k)$ and the estimated output $y(k)$ should be minimized, as follows:

$$MSE = \frac{1}{m}\sum_{k=n+1}^{M}\{d(k)-y(k)\}^2 = \sum_{k=n+1}^{M}\{d(k)-[y(k-1)+\sum_{i=1}^{n}\Delta u(k-i)f(i)]\}^2 \tag{6}$$

The actual number of data for evaluation is: $m = M - n$. Then the direct implementation of the Least-Squares Estimation Algorithm, as shown in [4] leads to the following (rectangular in size) linear system of equations:

$$\mathbf{A}_{m\times n}\,\mathbf{f}_{n\times 1} = \mathbf{d}_{m\times 1}, \tag{7}$$

where $\mathbf{d}$ is a vector-column with the following structure:

$$\mathbf{d}_{m\times 1} = [\Delta d(n+1), \Delta d(n+2), ... , \Delta d(M)]^T \tag{8}$$

The unknown vector-column $\mathbf{f}$ with size n, that contains the strength degrees of the *CER-function*, has the format:

$$\mathbf{f}_{n\times 1} = [\ f(1)\quad f(2)\ \ ...\ \ f(i) ...\ f(n)]^T \tag{9}$$

The rectangular matrix $\mathbf{A}$ is constructed as follows:

$$\mathbf{A}_{m\times n} = \begin{bmatrix} \Delta u_{n+1}(k-1) & \Delta u_{n+1}(k-2) & ... & \Delta u_{n+1}(k-n) \\ \Delta u_{n+2}(k-1) & \Delta u_{n+2}(k-2) & ... & \Delta u_{n+2}(k-n) \\ ... & ... & ... & ... \\ \Delta u_{M}(k-1) & \Delta u_{M}(k-2) & ... & \Delta u_{M}(k-n) \end{bmatrix} \tag{10}$$

Then the Identification solution, based on the least-square estimation, is written as follows:

$$\mathbf{f} = (\mathbf{A}^{\mathbf{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{d} \tag{11}$$

IV. Identification Results with CER Dynamic Models

Further on we illustrate the above Identification scheme (11) on the well known and often used as a bench-mark example: the *Gas-Furnace* data set [3]. It consists of *296* pairs of input-output data, that represent the relationship in discrete time between the *Methane* gas (*CH4*) in the input and the concentration of *Carbon Dioxide* (*CO2*) in the exhaust gases of the furnace, as graphically depicted in Fig. 6.

We have identified 13 different CER dynamic models with different memory lengths from $n = 1$ to $n = 13$ by using the input-output data from Fig. 6.

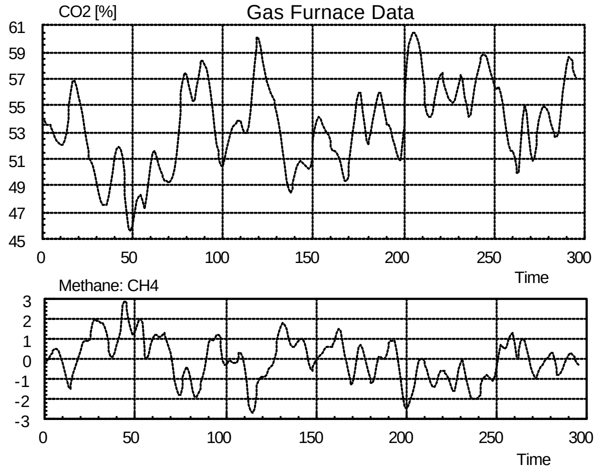

Fig. 6. Test Gas-Furnace Data Used in the Simulations.

The obtained CER functions are shown in Fig. 7. It is seen that the maximum strength of the relation between the past inputs and the current output is at the 5th past sampling time.

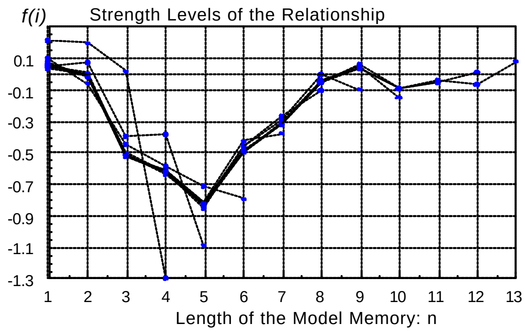

Fig. 7. Different CER Dynamic Models Identified from the Gas-Furnace Data in Fig. 6.

Even if not very smooth in shape, the most of the CER functions show the same tendency (and the same *extremum*). The deviation of the CER functions with shorter memory length ($n \le 7$) is due to the truncation of the model and the respective loss of useful information. At the same time, an excessively long memory ($n \ge 11$) is not needed and does not improve the model accuracy. It may even cause sometimes a "reverse effect", by "catching" nonexistent relationships from old past times. All these problems have to be considered during the preliminary phase of the "memory length selection" when creating the CER dynamic model.

Fig. 8. shows the accuracy of the different identified CER models that can be further used for selection of the most appropriate CER model.

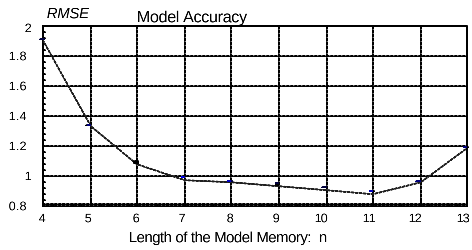

Fig. 8. Modeling Error of the Different CER Models from Fig. 7.

V. Feed-Forward Reference Model Control by use of CER Dynamic Models

The above described CER dynamic model in Section 2 can be successfully used in different control schemes for synthesis of a controller with the same CER structure, as the plant model. . One interesting control structure, that is widely used in robotics and many other engineering fields is the so called reference model control [2],[4]. Here the desired dynamic behavior of the plant is given by a predefined and stable *Reference Model*. The main goal of the controller is to force the process to follow the reference model output. In the simplest case, the controller could be of the type of *feed-forward controller*, as shown in Fig. 9.

From a control point of view, such a feed-forward controller can be regarded as a special "*correction unit*" that transforms the initial reference signal $r(t)$ into a modified control $u(t)$ which is further applied as a plant input.

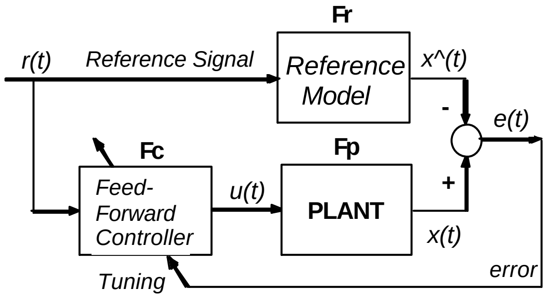

Fig. 9. Tuning the Feed-Forward Controller in a Reference Model Control Structure

In order to design such a controller (correction unit), an identification procedure based on experimental data is proposed in the sequel.

Let us assume that the dynamics of all units in Fig. 9. is represented by CER dynamic models with the following CER-functions and memory lengths:

F_r and n_r for the *Reference Model*;

F_p and n_p for the *Plant* and

F_c with n_c for the *Controller*.

Suppose that $k_{\max}$ sampling times from the reference signal $r(k)$ are also available for the identification.

Then the general statement of this identification problem is as follows:

Given F_r, n_r; F_p, n_p; $k_{\max}$ and n_c; determine all n_c points of the CER-function F_c of the *Controller* that minimize the following performance index:

$$Q = \frac{1}{2}\sum_{k=1}^{k_{\max}}[x(k) - \hat{x}(k)]^2 \tag{12}$$

The solution of this identification problem can be obtained in a non-iterative way by a proper modification of the LSM algorithm.

Let us first derive the output of the serial connection of the controller (correction unit) and the Plant, as shown in Fig. 10.

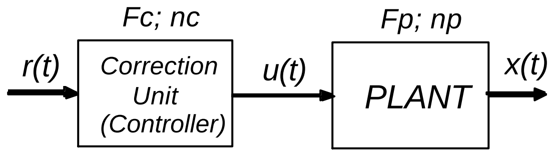

Fig.10. Serial Connection of two Dynamic Units.

$$x(k) = x(k-1) + \sum_{i=1}^{n_p}\Delta u(k-i) f_p(i) \tag{13}$$

$$\Delta u(k-i) = u(k-i) - u(k-i-1) = \sum_{j=1}^{n_c}\Delta r(k-i-j) f_c(j) \tag{14}$$

Finally the output of the plant is calculated as:

$$x(k) = x(k-1) + \sum_{i=1}^{n_p} f_p(i)\sum_{j=1}^{n_c}\Delta r(k-i-j) f_c(j) \tag{15}$$

Obviously, the total memory length n for the whole serial connection unit, which represents the number of all past times used for calculation of the current output $x(k)$ is: $p = n_p + n_c$.

According to Fig. 10., for the identification procedure we use the condition that the increment of the reference model and that one of the serial connection (controller plus plant) should be equal at each sampling time k, that is:

$$\Delta\hat{x}(k) = \Delta x(k) \tag{16}$$

which yields the following equations:

$$\Delta\hat{x}(k) = \sum_{i=1}^{n_r}\Delta r(k-i) f_r(i), \quad \text{and} \tag{17}$$

$$\Delta x(k) = \sum_{j=1}^{n_p}\Delta u(k-j) f_p(j) = \sum_{j=1}^{n_p} f_p(j)\sum_{l=1}^{n_c}\Delta r(k-j-l) f_c(l) \tag{18}$$

$$\sum_{i=1}^{n_r}\Delta r(k-i) f_r(i) = \sum_{j=1}^{n_p}\Delta u(k-j) f_p(j) = \sum_{j=1}^{n_p} f_p(j)\sum_{l=1}^{n_c}\Delta r(k-j-l) f_c(l) \tag{19}$$

Finally, the solution $f_c(l)$, $l = 1,2,\ldots,n_c$ for the correction unit is obtained by solving the following linear system of equations:

$$A_l(k).f_c(l) = B(k), \quad l = 1,2,\ldots,n_c;\ k = 1,2,\ldots,k_{\max};\ k_{\max} > n_c \tag{20}$$

where:

$$A_l(k) = \sum_{j=1}^{n_p} f_p(j)\Delta r(k-j-l) \tag{21}$$

$$B(k) = \sum_{i=1}^{n_r}\Delta r(k-i) f_r(i) \tag{22}$$

VI. Simulation Results for Reference Model Control

The above Identification scheme (20), (21) and (22) has been used for extensive simulations in synthesizing different CER-type feed-forward controllers, according to Fig. 9.

In Fig. 11. the predetermined CER-function for the *Reference Model* is shown, together with two different (supposed to have been preliminary identified) plant models, conditionally called: "*Quick Plant*" and "*Slow Plant*".

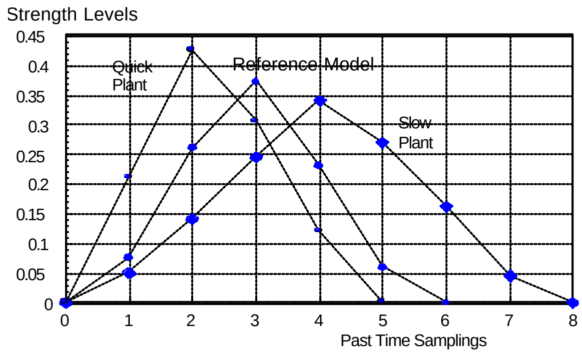

Fig. 11. CER functions of the Assumed Reference Model and two Different Plant Models, Used in the Simulations.

Then the same input, from the Gas-Furmace Example in Fig. 6. has been used for identification of two groups of feed-forward controllers with different memory lengths within the range $n_c \in [1, 7]$. Their respective identified CER-functions are shown in Fig. 12. for the "Q*uick Plant*" and in and Fig. 13. for the "*Slow Plant*".

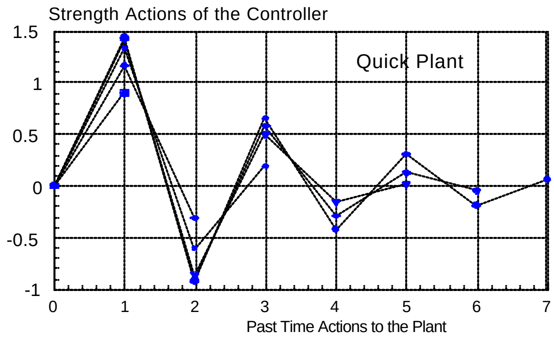

Fig. 12. Identified Feed-Forward Controllers with Different Memory Lengths: in the Case of "*Quick Plant*" Model

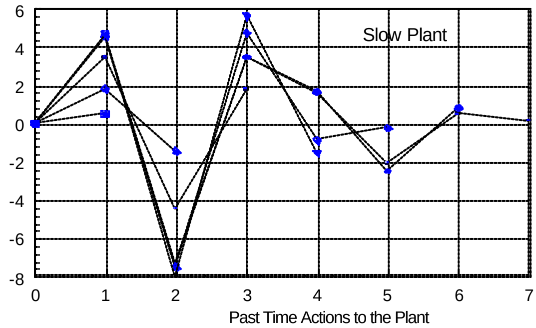

Fig. 13. Identified Feed-Forward Controllers with Different Memory Lengths in the Case of "*Slow Plant*" Model.

As seen, the CER-functions for the controllers have "broken-shape" lines compared to the relatively smooth CER-functions of the plant and the reference model in Fig. 11. Nevertheless these shapes have also a physical interpretability, namely that the controller "tries" to follow the reference model trajectory by oscillations in its control inputs to the plant. It is obvious, that in such case an exact (faultless) matching in the dynamics cannot be expected, as shown later on.

By comparing the strength levels of both groups of controllers, it is easy to notice that the controllers in Fig. 13. use much stronger actions (between *–8* and *+6*) in order to force the "Slow Plant" to follow the Reference model trajectory. For comparison, the "*Quick Plant*" can be controlled with milder strength actions: between *–1* and *+1.5*, as seen in Fig. 12. These results have also clear physical interpretability.

The following Fig. 14. shows the approximation error from the different identified feed-forward controllers. It is obvious that the case of "*Slow Plant*" is approximated with less accuracy compared to that of the "*Quick Plant*". Moreover, the controllers with longer memory buffer (up to certain extent) have in general a better performance accuracy.

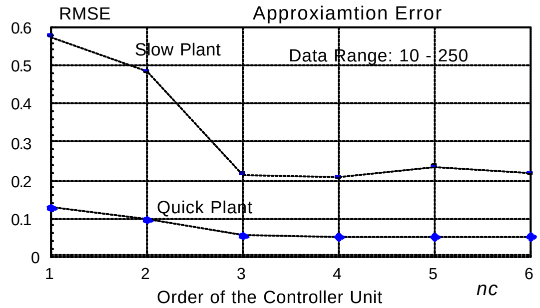

Fig. 14. Approximation Error for the Different Feed-Forward Controllers and Different Plant Models.

Finally, the satisfactory performance of the identified feed-forward controller with memory length $n_c = 4$ in the case of "Slow *Plant*" is shown in Fig. 15. A specially generated periodical reference input signal is presented as a new (unseen) for the controller to check its performance.

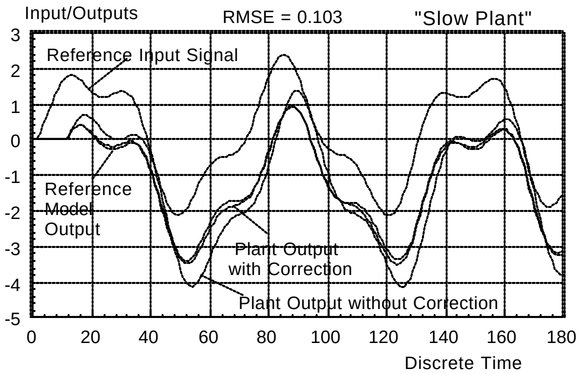

Fig. 15. Performance of the Reference Model Control by use of the Feed-Forward Controller with $n_c = 4$.

The accuracy of the performance depends on the length of the memory and some experimental adjustment is needed before the real application. In addition, the synchronization in sampling time is very important for the accuracy of simulations, because the serial connection: "*Controller-Plant*", from Fig. 9. needs a longer memory buffer to be loaded before the start of the real simulation.

VII. Inverse Dynamics Calculation Based on the CER Dynamic Models

In many practical cases such as fault diagnosis and *backward tracking* in dynamic systems, a possible input signal sequence to the system (believed to be a "cause of the failure") has to be found, that is able to "explain" the observed output behavior (i.e. the malfunction) of the system. Here the assumption of "known in advance" dynamic model of the system (in the form of CER dynamic model) is made.

In order to solve such an "*inverse dynamics*" problem we suppose that the following discrete time series for the output is known:

$$y(k), y(k+1), y(k+2), \ldots, y(k+m)\,. \qquad (23)$$

Then, the input sequence:

$$u(k-n), u(k-n+1), \ldots, u(k+m-n) \qquad (24)$$

which is able to produce the above measured output could be recovered by an appropriate recursive calculation procedure.

First, let us rewrite equation (5) in the following way:

$$\Delta y(k+1) = \Delta u(k) f(1) + \sum_{i=2}^{n} \Delta u(k+1-i) f(i) \qquad (25)$$

Taking into consideration that:

$$\Delta u(k) = u(k) - u(k-1) \qquad (26)$$

we obtain the following expression for calculating the latest (current) input $u(k)$ that contributes to the future output $y(k+1)$:

$$u(k) = \frac{1}{f(1)}\left[u(k-1) f(1) + \Delta y(k+1) - \sum_{i=2}^{n} \Delta u(k+1-i) f(i)\right] \qquad (27)$$

It is clear that at the beginning, all the initial $n-1$ inputs: $u(k-1), u(k-2), \ldots, u(k-n+1)$ have to be known in advance in order to start the recursive procedure (27) that would calculate the next inputs: $u(k),\ u(k+1),\ u(k+2), \ldots$ until the last measured output $y(k+m)$ is used for the calculations. Fig. 16. gives a graphical illustration of this recursive procedure for finding the inverse dynamics.

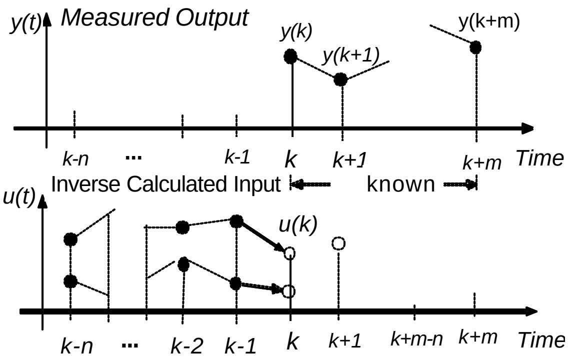

Fig. 16. Calculation Scheme for the Inverse Dynamic.

Here different optimization strategies with plausible constraints over the generated input sequence can be used for a practical implementation of the recursive procedure (27).

It is important to notice that the above proposed calculation scheme leads to a deviation (*bias)* in the computed input signal compared to the real input. It means that the real input remains still unknown, but its true *incremental behavior* is reproduced. In many fault diagnosis problems such a solution is sufficient and meaningful.

One example of an input signal recovery is presented in Fig. 17. for the case of a known CER dynamic model and a pure (noiseless) measured output data set.

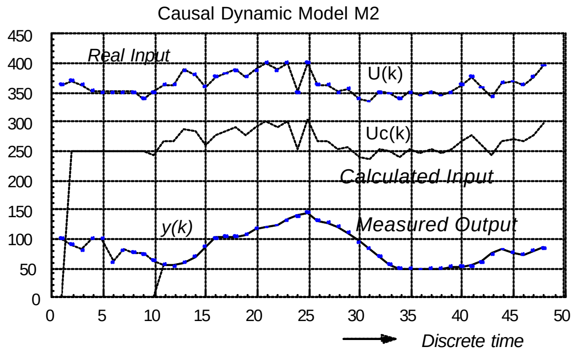

Fig. 17. Results from the Inverse Dynamics Calculation of the Causal Dynamic *Model 2* from Fig. 4.

VIII. Discussions and Future Work

Simplified incremental type of dynamic models that reveal the cause-effect relations in dynamic systems are proposed and identified in this paper. An important property of such models is their good interpretability of the system behavior. Two algorithms, namely for a Reference Model Control as well as for a Backward Tracking are derived as possible applications of such dynamic models for control and fault diagnosis.

Some open problems for further research could be the proper (or optimal in some way) selection of the memory length of the models as well as the method of improving their robustness and interpretability in the presence of highly noised data.

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