Image Reconstruction from Projection Using Unsupervised Neural Network

Robert Cierniak

Technical University of Czestochowa, Poland
42-200 Czestochowa, Armii Krajowej Str. 36
email: cierniak@kik.pcz.czest.pl

***Abstract*—This paper reports the most imported results of the author's investigations on the key problem in the computer tomography: image reconstruction from projections. To solve this problem an energy expression is proposed like Hopfield net. During the minimizing of the energy function could be done the reconstruction process. To improve the convergence of the algorithm the entropy term in energy expression is introduced. The dedicated to realize the minimizing task an unsupervised neural network structure consists of two layer. Some interesting results of the performed computer simulations are shown.**

***Index Terms*—Image Reconstruction, Tomography, Neural Networks.**

I. INTRODUCTION

Computer tomography, especially its algorithmic aspect, is still a very rewarding field of investigations. Since Cormack's publication [4], one of the key tasks in the field has been to integrate into the studies many new algorithms and apply them to the task of reconstructing an image from projections. The most important reconstruction methods are those using convolution and back propagation [11][17][23], Fourier inversion [17] or an algebraic reconstruction technique (ART) [1][8][12]. Considering the increasing amount of soft computing algorithms applicable to different science disciplines, it is possible that in the foreseeable future these algorithms will occupy an important place in computer tomography. This paper represents an attempt to approach image reconstruction from projections by using neural networks — the very popular and important tool of artificial intelligence systems to solve different image processing problems [3]. The idea of a neural network in application to image reconstruction from projection is presented in [13-16][20]. The investigated in this paper neural network structure is based on the formulated energy expression including entropy term. Maximum entropy criterion is met and has been endorsed by scientists representing different disciplines, were studied in papers [7][9]. In this paper a new approach to construct neural network solving deconvolution problem [18] is applied and investigated. The advantages of presented definition of neural network have been demonstrated during computer simulations designed to prove the assumption that an appropriately constructed neural network is able to reconstruct an image using projections.

II. PRELIMINARIES

A. Projections

Computer tomography, which has its beginnings in the work of Allan M. Cormack [4], who in 1979 was awarded the Nobel Prize for medicine and physiology, has became a diagnostic method in medicine and in many other fields of science. It is regarded as one of the most important inventions of the twentieth century. One of the most remarkable features of computer tomography is the possibility of using it to examine of the inside of an object, for example the human body. A three-dimensional image of the object is given by applying an appropriate method of projection and an image reconstruction algorithm. Currently there are several methods of projection (for example PET, NMR etc.), but the most popular and the most widespread one is the X-ray method. In diagnostics, knowledge of the distribution of the attenuation coefficient in the investigated object gives extremely useful information about both the tissue layers and any pathological changes. The depth of the shadow cast by the object onto a screen positioned opposite the radiation source gives us information about the two coefficients that have an influence on the attenuation of radiation. The function below gives the distribution of radiation attenuation reaching the screen when a projection is made at a specific angle. This is called the Radon's transformation [22]:

$$p(s,\alpha)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(s\cos\alpha-u\sin\alpha, s\sin\alpha+u\cos\alpha)\mathrm{d}s\mathrm{d}u\,, \tag{1}$$

where: α — is the angle of projection, x, y — the co-ordinates of the examined object, $s = x\cos\alpha + y\sin\alpha$ — the distance from the centre of rotation to the axis of the ray falling on the projection screen, $u = -x\sin\alpha + y\cos\alpha$ — the distance between a given point on the investigated object with co-ordinates (x,y) and the centre of rotation, measured along the axis of the falling ray.

B. Back-projections

Projections performed in this way may be subjected to the next phase of the signal processing: the values of the projection function $p(s,\alpha)$ passing through a fixed point (x,y) are accumulated to generate information about the attenuation coefficient $\mu(x,y)$ at this point. The accumulation given by the following expression is called the back-projection or Radon's back-projection operator:

$$\tilde{\mu}(x,y)=\int_{0}^{\pi} p(x\cos\alpha+y\sin\alpha,\alpha)\mathrm{d}\alpha \tag{2}$$

where: $\tilde{\mu}(x,y)$ — the image of the distribution of the X-ray attenuation in a section of the investigated body formed on the basis of the performed projections.

It is easy to show that the function $\tilde{\mu}(x,y)$ gives an image of the original distribution of the attenuation $\mu(x,y)$ in the investigated cross-section of the object. The relationship between the function and the attenuation coefficient is given by [11]:

$$\begin{aligned}\tilde{\mu}(x,y)&=\mu(x,y)*\left(x^2+y^2\right)^{-\frac{1}{2}}=\\&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x',y')\cdot\frac{1}{\sqrt{(x-x')^2+(y-y')^2}}\mathrm{d}x'\mathrm{d}y'\end{aligned} \tag{3}$$

The above equation is true only when projections are performed continuously and the object has an unlimited size. In practice, the physical methods that are applied give only a finite number of projections, there are only a limited number of measurement points, and the investigation takes place within a circle, which is limited by the object's rotation around the X-ray tube. These constraints and their consequences have been widely discussed in existing literature, see for example [11][17].

Given the available technical and computational tools, the only sensible solution in tomography seems to be the use of a digital image, which from now on we will employ exclusively. In this case, the equivalent of the linear functions $\mu(x,y)$ and $\tilde{\mu}(x,y)$ will be $\mu(i\Delta x, j\Delta y)$ and $\tilde{\mu}(i\Delta x, j\Delta y)$ respectively, where i, j are integers and the functions have discrete values.

Linear equation (3) has its discrete equivalent in the form:

$$\tilde{\mu}(i,j)=\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{L}} h_{ijkl}\,\mu(k,l)\,, \tag{4}$$

where: h_{ijkl} — gives the discrete impulse response of the signal, accountable for the geometric distortion of the original image .

Because, for any given projection, the X-ray sensors are placed on the screen with raster Δ_s, it is possible that no ray passes through a given point of the image. To take this into account we will apply linear interpolation. This will ensure that the points which are outside of the rays falling directly on the sensors on the screen will be given a projection value according to the equation:

$$\tilde{p}(s,\alpha)\cong p(m,\alpha)+(s-m)(p(m+1,\alpha)-p(m,\alpha)), \tag{5}$$

where: $m=\mathrm{Trunc}(s)$ — is the integer part of the variable s and: $s=i\Delta_s\cos\alpha+j\Delta_s\sin\alpha$, $-\mathrm{M}\le m\le \mathrm{M}-1$, where: M — is the radius of the circle defined by the tube around the investigated object.

In this way we can obtain the value of a given projection at every point of the discreet image. The results of the projections at each point are then collected to obtain values of the function $\tilde{\mu}(i\Delta_s, j\Delta_s)$. Aggregation is applied to all the projections passing through a given point, satisfying following expression:

$$\tilde{\mu}(i\Delta_s, j\Delta_s)\cong\Delta_\alpha\sum_{n=0}^{\mathrm{N}-1}\tilde{p}(i\Delta_s\cos n\Delta_\alpha + j\Delta_s\sin n\Delta_\alpha, n\Delta_\alpha), \tag{6}$$

where: Δ_α — the angle, given in radians, by which the pair tube-screen is rotated after each projection, $\mathrm{N}=\mathrm{Trunc}(\pi/\Delta_\alpha)$ — the number of projections (integer value).

As one can see from equation (6), the distribution of the attenuation coefficient in a given cross-section of the object, obtained in the way described above, is equal to the amalgamation of the original distribution function of the attenuation coefficient and the geometrical distortion element. In more complex presentations of this effect, the image of the cross-section would have stripes resembling a shadow cast by the radiation-absorbing element. The shadow can be at the front and back of the radiation-absorbing element.

Existing methods of image reconstruction from projection apply different ways of filtering projections $p(s,\alpha)$ to avoid the effects of the geometrical distortion term, shown in equation (3). Almost all existing commercial solutions are based on reconstruction using convolution and back-projection, but there are many other techniques of image reconstruction, which can sometimes be more efficient. Scientists keep looking for faster algorithms to shorten the time required to perform a complete reconstruction and have an image ready for diagnostics, they are also trying to improve the quality of the reconstructed image. Therefore reconstruction methods using parallel processing could be very promising. Most of them are based on the well-known algebraic reconstruction technique called ART, which, unfortunately, has some major defects limiting its potential.

In the above context, the usage of neural networks could give a new impulse to the investigations of image reconstruction from projection. When we take into account the fact that neural networks are one of the pillars of the newly fashionable discipline of science called soft computing, their usage seems to be even more tempting. This is especially so given all the potential benefits that the application of artificial intelligence in this field of medical engineering could bring.

III. THE NEURAL ALGORITHM OF THE IMAGE RECONSTRUCTION FROM PROJECTIONS

Along with such criteria as the least square error and the steepest descent, the entropy criterion is a very popular method to determine the direction and the rate of change in algorithms. This criterion also has some interesting application in the image processing. Many scientists see the entropy criterion as a very appealing tool for the whole domain of image reconstruction. In publications [2][5][6][18][21][24] they present methods for eliminating distortion from images, reconstructing of the incomplete images using an approach which maximises the entropy of the distorted images. Additionally, in paper [7][9] proposals for the removal of artefacts from reconstructed image with the help of the maximum entropy criterion were described. The most important advantage of using neural networks in such application is parallel computation.

Neural network structure using the maximum entropy criterion presented below was proposed at the first time in [17]. In application to image reconstruction from projections this structure is a new concept in this area of image processing.

Optimisation problem taking advantage of the maximum entropy criterion in order to reconstruct discreet image can be formulated as the constraints:

$$\begin{cases} \max\limits_{\mathbf{F}} Ent(\mathbf{F}) \\ \tilde{\mathbf{F}} = \mathbf{HF} \end{cases} \tag{7}$$

where: $\mathbf{F} = [\mu(i,j)]$ — the matrix with elements from the original image of the given object; $\tilde{\mathbf{F}} = [\tilde{\mu}(i,j)]$ — the matrix with elements from the distorted image of the given object; $\mathbf{H} = [h_{ijkl}]$ — the matrix of the impulse responses; $Ent(\mathbf{F})$ — the image entropy, which can take the form of the Shannon's entropy:

$$Ent(\mathbf{F}) = -\sum_{m=1}^{\mathrm{M}} \sum_{n=1}^{\mathrm{N}} \chi(m,n) \ln(\chi(m,n)) \tag{8}$$

or another function [17], where:

$$\chi(m,n) = \frac{\mu(m,n)}{\sum\limits_{m=1}^{\mathrm{M}} \sum\limits_{n=1}^{\mathrm{N}} \mu(m,n)} \ . \tag{9}$$

The problem expressed in equation (7) can be reformulated using a penalty method and take the form:

$$\min_{\mathbf{F}} \left(-Ent(\mathbf{F}) + \mathrm{wsp} \sum_{i=1}^{\mathrm{I}} \sum_{j=1}^{\mathrm{J}} f(e_{ij}(\mathbf{F})) \right), \tag{10}$$

where: $e_{ij}(\mathbf{F}) = \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{L}} h_{ijkl} \mu(k,l) - \tilde{\mu}(i,j)$ — i-th row of matrix, $\mathbf{HF} - \tilde{\mathbf{F}}$, wsp — suitable large positive coefficient, $f(\bullet)$ — penalty function.

Selecting a suitable form for the penalty function could have great importance for the learning progress of the neural network. Research has shown that the following function yields the best results:

$$f(e) = \lambda^2 \ln\cosh\left(\frac{e}{\lambda}\right), \text{ where } \lambda > 0 , \tag{11}$$

which derivation has the quite practical form:

$$f'(e) = \frac{\partial f(e)}{\partial e} = p\mathrm{tgh}\left(\frac{e}{\lambda}\right), \tag{12}$$

where: λ — slope coefficient.

It is worth underlining the importance of the coefficient represented by variable *wsp*. It has been proved that if the value of this coefficient is infinity, then the solution to equation (12) is identical to the solution to equation (9). If a value of this coefficient tends to infinity or in other words is suitably large, then the solution to equation (10) tends to the solution of the equation (7).

After conditioning in the expression (10) we can start to construct a neural network to realise the deconvolution task of equation (4). To this end, an energy expression is formulated by minimizing the value of this function using a neural network. The energy expression could take the form:

$$E^t = -Ent\left(\mathbf{F}^t\right) + \mathrm{wsp}\sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{\mathrm{J}} f\left(e_{ij}\left(\mathbf{F}^t\right)\right) \tag{13}$$

In order to find a maximum of this function is defined its derivation with the following form:

$$\begin{aligned}\frac{\mathrm{d}E^t}{\mathrm{d}t} = &-\sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{\mathrm{J}} \frac{\partial Ent\left(\mathbf{F}^t\right)}{\partial \mu^t(i,j)} \frac{\partial \mu^t(i,j)}{\partial t} + \\ &+\mathrm{wsp}\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{L}}\sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{\mathrm{J}} \frac{\partial f\left(e_{kl}\left(\mathbf{F}^t\right)\right)}{\partial e^t{}_{kl}} \frac{\partial e^t{}_{kl}}{\partial \mu^t(i,j)} \frac{\partial \mu^t(i,j)}{\partial t}\end{aligned}, \tag{14}$$

or in other words

$$\begin{aligned}\frac{\mathrm{d}E^t}{\mathrm{d}t} = &-\sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{\mathrm{J}} \frac{\mathrm{d}\mu^t(i,j)}{\mathrm{d}t} \cdot \\ &\left(\frac{\partial Ent\left(\mathbf{F}^t\right)}{\partial \mu^t(i,j)} + \mathrm{wsp}\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{L}} \frac{\partial f\left(e_{kl}\left(\mathbf{F}^t\right)\right)}{\partial e^t{}_{kl}} \frac{\partial e^t{}_{kl}}{\partial \mu^t(i,j)}\right)\end{aligned}, \tag{15}$$

where index *t* means the addiction of the variable denoted by this symbol to time in the learning progress of the neural network:

If we provide:

$$\begin{aligned}\frac{\mathrm{d}\mu^t(i,j)}{\mathrm{d}t} = &\frac{\partial Ent\left(\mathbf{F}^t\right)}{\partial \mu^t(i,j)} + \\ &-\mathrm{wsp}\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{L}} \frac{\partial f\left(e_{kl}\left(\mathbf{F}^t\right)\right)}{\partial e^t{}_{kl}} \frac{\partial e^t{}_{kl}}{\partial \mu^t(i,j)}\end{aligned} \tag{16}$$

to aim the looping of learning progress, then equation (18) takes the form:

$$\frac{\mathrm{d}E^t}{\mathrm{d}t} = -\sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{\mathrm{J}} \left(\frac{\mathrm{d}\mu^t(i,j)}{\mathrm{d}t}\right)^2 \tag{17}$$

One can see that the values of the equation (17) are always less or equal to zero, that is $\frac{\mathrm{d}E^t}{\mathrm{d}t} \le 0$, because $\mathrm{wsp} >> 0$, and functions $Ent(\bullet) < 0$ and $f(\bullet) >= 0$. Therefore, if $\frac{\mathrm{d}E^t}{\mathrm{d}t} = 0$, then it results from (17) that $\frac{\mathrm{d}\mu^t(i,j)}{\mathrm{d}t} = 0$, which has the sense of the global maximum achievement.

The structure of the neural network performing such formulated task is depicted in Fig. 2.

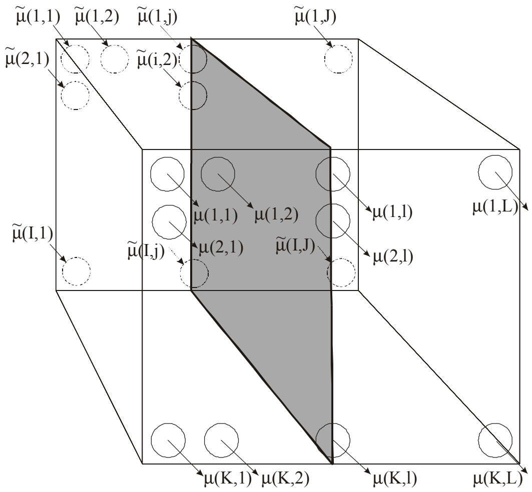


Fig. 2. Structure of the neural network.

The neural network presented above consists of two layers, with the same topology of neurons. For convenience of the used symbols, it is marked and next depicted one vertical cross-section through the neural network structure. So carried out section is presented in Fig. 3 together with all symbols used in relation to the quantities from the neural network.

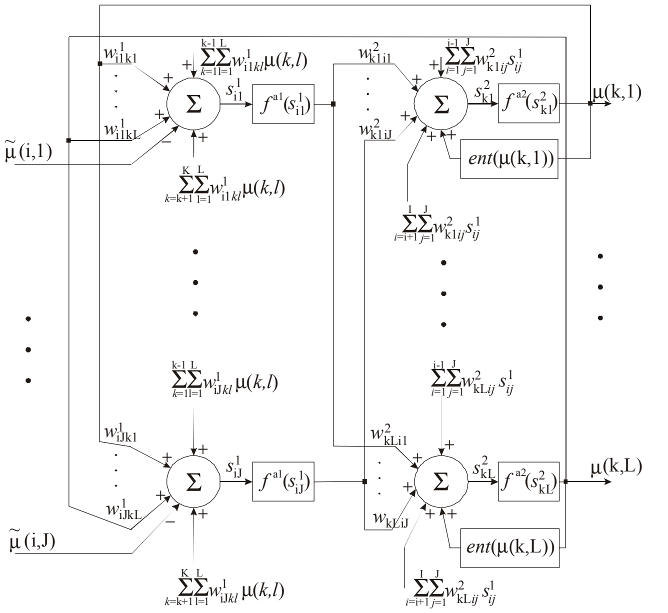


Fig. 3. Vertical cross-section of the neural network structure.

All of the symbols which appear in the neural network structure are listed in order from the input to the output of the network.

I. layer

Weights of the connections:

$$w_{ij}^{1} = h_{klij}\,, \tag{18}$$

Weighting sum:

$$s_{ij}^{1} = e_{ij}^{t}(\mathbf{F}) = \sum_{k=1}^{K}\sum_{l=1}^{L} w_{ij}(1)\mu^{t}(k,l) - \tilde{\mu}(i,j)\,, \tag{19}$$

Activation function:

$$f^{a1}\left(s_{ij}^{1}\right) = \frac{\partial f\left(e_{ij}\left(\mathbf{F}^{t}\right)\right)}{\partial e^{t}{}_{ij}}\,. \tag{20}$$

II. layer:

weights of the connections:

$$w_{ij}^{2} = -\mathrm{w}sp \cdot \frac{\partial e^{t}{}_{kl}}{\partial \mu^{t}{}_{ij}} = -\mathrm{w}sp \cdot h_{klij}\,, \tag{24}$$

weighting sum:

$$s_{ij}^{2} = \frac{\partial Ent\left(\mathbf{F}^{t}\right)}{\partial \mu^{t}(i,j)} + \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}^{2} f^{a1}\left(s_{ij}^{1}\right), \tag{21}$$

and: $s_{ij}^{2} = \dfrac{\mathrm{d}\mu^{t}(i,j)}{\mathrm{d}t}$,

activation function:

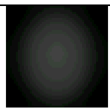
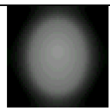
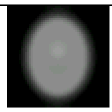
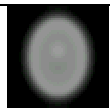
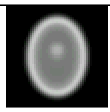
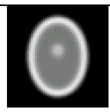
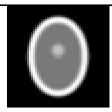
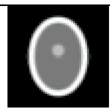
$$f^{a2}\left(s_{ij}^{2}\right) = \mu^{t}(i,j) = \int_{0} s_{ij}^{2}\,\mathrm{d}t\,, \tag{22}$$

additionally the following notation is used in the figure above:

$$ent\left(\mu^{t}(k,l)\right) = \frac{\partial Ent\left(\mathbf{F}^{t}\right)}{\partial \mu^{t}(k,l)}\,. \tag{23}$$

Neural network with the above structure was investigated using sequential simulations. The results of these experiments are presented in the next section.

TABLE I
PROCESS OF THE IMAGE RECONSTRUCTION FROM PROJECTION USING THE MATHEMATICAL MODEL OF THE CROSS-SECTION OF THE SKULL.

Number of iterations	3	12	25	50
View of the reconstructed image				
Number of iterations	200	400	1000	2000
View of the reconstructed image				

IV. EXPERIMENTAL RESULTS

Owing to computing complexity of image reconstruction from projections problem (the number of elements in matrix **H** increases by the power of four in proportion to extension in the size of the reconstructed image) the size of the processed image was fixed at 50×50 pixels.

Owing to the lack of physical projection data from the tomographic investigation, it is necessary to construct a mathematical model of the projected object — a so-called phantom. This was first proposed in papers [10][11].

Three ellipses with constant attenuation coefficients were applied to model the cross-section of a skull.

The proposed form of interpolation presented in equation (7) could be applied to obtain the values of projections assigned to every discreet point of the image.

In this way obtained image was next subjected to an process of reconstructions using neural network, which structure was explained in previous chapter. The progress in this process is presented in Tab. I.

One can see above that after about one thousand iterations the results of the reconstruction process are stabilized at the satisfactory level. Therefore, one can say that at this point the image is reconstructed and the process can be stopped.

V. CONCLUSIONS

The performed simulations demonstrated a convergence of the algorithm of image reconstruction from projections based on the neural network described in this work. The image of the cross-section of the investigated mathematical model, obtained after sufficient number of iterations is similar to the original image. Therefore, this method is acceptable from a diagnostic point of view.

Sizes of the processed image are not acceptable for a model of the more complicated mathematical object. The next step of investigations on the image reconstruction from projections using neural network must be an extension of the image sizes to apply in the simulations the standard mathematical model presented in literature.

The time consuming (very important factor in the practical computer tomography) of this algorithm is inadmissibly long. However, the simulations ware done using the sequence realization of this algorithm contrary to the natural parallel calculations in the neural networks. The hardware implementation of this neural network structure could give incomparable with another methods of the image reconstruction from projections concerning the time of reconstruction and the quality of reconstruction.

Very interesting problem is also the resistance of described algorithm from distortions arising in the physical projections which nature is explained for example in work [19].

References

[1] Censor Y., Finite series-expansion reconstruction methods, Proceeding of the IEEE, vol. 71, No. 3, pp. 409-419, 1983.

[2] Cichocki A., Unbehauen R., Lendl M., Weinzierl K., Neural networks for linear inverse problems with incomplete data especially in application to signal and Image Reconstruction, Neurocomputing, vol. 8, pp. 7-41, 1995.

[3] Cierniak R., Rutkowski L., *On image compression by competitive neural networks and optimal linear predictors*, Signal Processing: Image Communication - a Eurosip Journal, Elsevier Science B. V., vol. 15, No 6, Amsterdam, February 2000, pp. 559-565.

[4] Cormack A. M., Representation of a function by its line integrals with some radiological application, J. Appl. Phys., vol. 34, pp. 2722-2727, 1963.

[5] Elfing T., On some methods for entropy maximization and matrix scaling, Linear Alg. Its Appl., vol. 35, pp. 321-338, 1980.

[6] Erlander S. Entropy in linear programs, Math. Program., vol. 21, pp. 137-151, 1981.

[7] Frieden B. R., Zoltani C. R., Maximum bounded entropy: application to tomographic reconstruction, Appl. Optics, vol. 24, pp. 201-207, 1985.

[8] Gordon R., Bender R., Herman G. T., Algebraic reconstruction techniques (ART) for three-dimensional electron microscopy and X-ray photography, Journal of Theoretical Biology, vol. 29, pp. 471-481, 1970.

[9] Ingman D., Merlis Y., Maximum entropy signal reconstruction with neural networks, IEEE Trans. on Neural Networks, vol. 3, pp. 195-201, 1992.

[10] Jaene B., Digital Image Processing – Concepts, Algoritms and Scientific Applications, Springer-Verlag Berlin, Heidelberg, 1991.

[11] Jain A. K., Fundamentals of Digital Image Processing, Prentice Hall, New Jersey, 1989.

[12] Kaczmarz S., Angeneaherte Aufloesung von Systemen Linearer Gleichungen, Bull. Acad. Polon. Sci. Lett. A., vol. 35, pp. 355-357, 1937.

[13] Kerr J. P., Barlett E. B., A statistically tailored neural network approach to tomographic image reconstruction, Med. Phys., vol. 22, pp. 601-610, 1995.

[14] Kerr J. P., Barlett E. B., Medical image processing utilizing neural networks trained on a massively parallel computer, Comput. Biol. Med., vol. 25, pp. 393-403, 1995.

[15] Kerr J. P., Barlett E. B., Neural network reconstruction of single-photon emission computed tomography images, Journal of Digital Imaging, vol. 8, pp. 116-126, 1995.

[16] Knoll P., Mirzaei S., Muller A., Leitha T., Koriska K., Kohn H., Neumann M., An artificial neural net and error backpropagation to reconstruct single photon emission computerized tomography data, Med. Phys, vol. 26, pp. 244-248, 1999.

[17] Lewitt, R. M., Reconstruction algorithms: Transform methods, Proceeding of the IEEE, vol. 71, No. 3, pp. 390-408, 1983.

[18] Luo Fa-Long, Unbehauen R., Applied Neural Networks for Signal Processing, Cambridge University Press, 1998.

[19] Macovski A., Physical problems of computerized tomography, Proceeding of the IEEE, vol. 71, No. 3, pp. 373-378, 1983.

[20] Munllay M. T., Floyd C. E., Bowsher J. E., Coleman R. E., An artificial neural network approach to quantitative single photon emission computed tomographic reconstruction with collimator, attenuation, and scatter compensation, Med. Phys, vol. 21, pp. 1889-1899, 1994.

[21] Papoulis A., Maximum entropy and spectral estimation: A rewiew, IEEE Trans. on Acoustic, Speech and Signal Processing, vol. 29, pp. 1176-1186, 1981.

[22] Radon J., Ueber die Bestimmung von Functionen durch ihre Integralwerte Tangs gewisser Mannigfaltigkeiten, Berichte Saechsiche Akad. Wissenschaften, Math. Phys. Klass, vol. 69, pp. 262-277, Leipzig, 1917.

[23] Ramachandran G. N., Lakshminarayanan A. V., Three-dimensional reconstruction from radiographs and electron micrographs: II. Application of convolutions instead of Fourier transforms, Proc. Nat. Acad. Sci., vol 68, pp. 2236-2240, 1971.

[24] Skilling J., The axiom of maximum entropy, Maximum Entropy and Bayesian Methods in Science and Engineering, vol. 1, pp. 173-188, Kluwer, Dortrecht, 1988.