A New Stereo Matching Algorithm Based on Synergetics for Occlusion and Reversal Position

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Abstract—We propose a new stereo matching algorithm based on a pattern recognition algorithm using Synergetics. A matching process between left and right images is represented by the differential equations with the two parameters of order parameter and attention parameter. We can have the freehand designing the construction of those parameters according to a recognized world. This leads the flexibility of the method. This paper presents how to design those parameters and then shows the experiment results that the proposal algorithm has high matching precision in comparison with the conventional methods of template matching and DP-matching.

I. INTRODUCTION

In computer vision, 3-D reconstruction from stereo images is one of the most important issues. The 3-D reconstruction can be applied to many fields of robot vision, virtual reality and so on. A main issue in processing stereo images is to decide correspondence points between multiple images. It is called a stereo matching problem. Many studies [1] have been presented for this issue. However, stereo matching on the images including occlusion (that correspondence points partially hide) and reversal position (that an order of corresponding points interchanges among multiple images) is still a remained difficult issue. While a multi-eye stereo method employing nine cameras [2] is proposed to solve this problem, we employ orthodox two cameras and propose a new stereo matching algorithm in this paper.

Template matching and dynamic programming matching (DP-matching) are well known as one of conventional stereo matching methods. On the template matching, a sequential similarity algorithm and a block correlation matching [3] are representative. Those have a quite simple procedure but low matching precision. DP-matching is a good method [4] for simple stereo images from the viewpoint of simplicity of algorithm and matching precision, but cannot well process stereo images including occlusion and reversal position.

Ohta and Kanade [5] proposed a three-dimensional search method using DP-matching. This method was not enough for solving a reversal position problem. Fujii and Matsuyama [6] proposed two-stage DP-matching method in order to solve a reversal position problem. This method, which employed pixel point matching, classified all the pixels into four categories (correct, left occlusion, right occlusion, unknown) in the first stage DP-matching and then applied DP-matching again to unknown category in the second stage. In fact, this might be effective for several examples. However, evaluation such as quantitative measurement for matching precision was not presented.

Recently, Synergetics Computer was proposed by H. Haken [7], which is a pattern recognition method based on his Synergetics theory. Synergetics is a mathematical model

representing self-organization dynamics in a complex system. The Synergetics describes self-organization with differential equations, called order parameter equations, including two parameters of order parameter and attention parameter. The order parameters are determined by inner product of an input pattern vector and adjoint vector of prototype pattern vectors. Those express the degree of relationship between an input pattern and prototype patterns, and an attention parameter can control the dynamics of order parameter. We can have a freehand designing those parameters in accordance with an object. Here, we applied this pattern recognition method based on Synergetics to a stereo matching problem.

In this paper, we propose 1) a new stereo matching algorithm based on Synergetics, 2) a new stereo matching algorithm employing two-stage DP-matching, and show 3) the experimental results for the measurement of matching precision. For reference, the result by template matching is also presented. The construction of this paper is as follows:

Section II describes the outline of a pattern recognition based on Synergetics. Section III is a main part of this paper. It presents how to deign order parameter and attention parameter, and then gives a new stereo matching algorithm based on Synergetics. Section IV describes the outline of conventional DP-matching and a new stereo matching algorithm using two-stage DP-matching. Section V shows the experimental results using building blocks. Section VI is conclusive remarks.

II. PATTERN RECOGNITION BASED ON SYNERGETICS

A. Outline

A certain state in a system autonomously changes to another ordered state by external controls or fluctuation forces. This process is called self-organization. Synergetics explains self-organization as follows: When a system receives an external environmental change, the system prepares stable modes and unstable modes for the reconstruction of its state. The stable modes are dominated by the unstable modes, and then vanished. This mechanism is called a slaving principle of Synergetics. The unstable modes are the candidates for a future ordered state in the system. A specific unstable mode wins the growing competition among the modes, and then forms a new ordered state in the system. The equations representing this competition are called order parameters equations of Synergetics.

Let us consider an autonomous system equation (1) with an external control parameter α and nonfluctuation force and a state vector be q(x, t),

$$\dot{q} = N(q(x,t),\alpha) . \tag{1}$$

After this, q(x, t) is written in q for simplicity. Furthermore, let us suppose equation (1) has a stable solution q_0 for a

certain value of α and then $q = q_0 + w(x, t)$, where x is a position vector, and w represents a small change. Here, expanding N in a power series around q_0 ,

$$\dot{\boldsymbol{q}} = N(\boldsymbol{q}_0) + \boldsymbol{L}\boldsymbol{w} + \hat{N}(\boldsymbol{w}) . \tag{2}$$

Here, the third term of right-hand side contains the second and / or higher powers of w. Furthermore, disregarding terms of more than second orders and adjusting the scale $N(q_0)=0$, (2) is transformed into

$$\dot{\boldsymbol{w}} = \boldsymbol{L}\boldsymbol{w}$$
, $\boldsymbol{L} = (L_{ij}) = (\partial N_i / \partial q_j | \boldsymbol{q} = \boldsymbol{q}_0)$. (3)

The solutions of (3) can be written in the general form

$$\boldsymbol{w}(\boldsymbol{x},t) = e^{\lambda t} \boldsymbol{v}(\boldsymbol{x}) . \tag{4}$$

In the case of nondegenerate eigenvalues, let us suppose the state vector q is presented as

$$q = q_0 + \sum_j \xi_j(t) v_j(x), \ \xi_j(t) = A_j e^{\lambda_j t}.$$
 (5)

Here, λ_j and v_j are the *j*-th eigenvalue and its eigenvector. Now, a set of adjoint vector v_k^+ , which satisfies an orthogonal condition, is introduced. The symbol < > means inner product.

$$\langle \mathbf{v}_{k}^{+} \mathbf{v}_{i} \rangle = \delta_{ki}$$
, $\delta_{ki} = \begin{cases} 1 & \text{for } k = i, \\ 0 & \text{otherwise} \end{cases}$ (6)

Then, inserting (5) into (2) and multiplying (2) by v_k^+ ,

$$\mathbf{v}_{k}^{+} \sum_{j} \dot{\xi}_{j}(t) \mathbf{v}_{j} = \mathbf{v}_{k}^{+} \sum_{j} \xi_{j}(t) L \mathbf{v}_{j} + \mathbf{v}_{k}^{+} \hat{N} \left(\sum_{j} \xi_{j}(t) \mathbf{v}_{j} \right). \quad (7)$$

From (6), the following is obtained.

$$\dot{\xi}_{k} = \lambda_{k}\xi_{k} + \tilde{N}_{k}(\xi_{j}) . \tag{8}$$

Equations (8) are divided into two set of unstable modes (the real part of eigenvalue is non-negative) and stable modes (the real part of eigenvalue is negative). Applying the slaving principle here, the stable modes are represented by the unstable modes and eliminated from (8). Consequently, the following order parameter equations of Synergetics

$$\hat{K}_{k} = k - B \sum_{k' \neq k} \hat{L}_{k'-k} - C \left(\sum_{k'} \hat{L}_{k'} \right) = k, \ k = 1, \dots, N$$
 (9)

are led, where *B* and *C* are positive constants, and ξ_k and λ_k are called order parameter and attention parameter, respectively. One of ξ_k 's survives, namely, grows to a positive value, in (9) and forms a new ordered state in the system.

In the application of equation (9) to pattern recognition, q and v_k correspond to an input pattern vector and the *k*-th prototype pattern vector, respectively. Thus, ξ_k represents a kind of similarity between an input pattern vector and the *k*-th prototype vector. λ_k plays a roll to control the increase of ξ_k . In equation (9), a surviving order parameter converges to a positive value and the others converge to 0. This means, from equation (5), the prototype pattern corresponding to the index of the surviving order parameter is recognized.

B. Order parameter equations

Let us consider a simple case of order parameter equations with two dimensions in order to explain those meaning. We put N=2, B=1, and C=1 in equation (9). That is

$$\dot{\xi}_1 = -\xi_1(\xi_1^2 + 2\xi_2^2 - \lambda_1) = F_1(\xi_1, \xi_2), \qquad (10)$$

$$\dot{\xi}_2 = -\xi_2(\xi_2^2 + 2\xi_1^2 - \lambda_2) = F_2(\xi_1, \xi_2).$$
(11)

In calculating the singular points of equations (10) and (11), the following four cases are obtained.

$$(i)\xi_1 = \xi_2 = 0. (12)$$

(ii)
$$\xi_1 = 0$$
 , $\xi_2 = \pm \sqrt{\lambda_2}$ (13)

(iii)
$$\xi_1 = \pm \sqrt{\lambda_1}$$
 , $\xi_2 = 0$. (14)

$$(iv) \xi_1 = \pm \sqrt{(2\lambda_2 - \lambda_1)/3} , \xi_2 = \pm \sqrt{(2\lambda_1 - \lambda_2)/3} , 2\lambda_2 - \lambda_1 > 0, 2\lambda_1 - \lambda_2 > 0.$$
 (15)

In satisfying the condition (15), singular points are shown in Fig. 1. Here (i), (ii) and (iii), and (iv) represent unstable focus, stable focus, and saddle point, respectively. In the following, we discuss only the first quadrant because of symmetry.



Fig. 1. Configuration of singular points point (2D)

Let us introduce a potential function *V* as

$$\dot{\xi}_{k} = -\frac{\partial V}{\partial \xi_{k}} \,. \tag{16}$$

Using (16) and (9), we find

$$V = -\frac{1}{2}(\lambda_1\xi_1^2 + \lambda_2\xi_2^2) + \frac{1}{2}\xi_1^2\xi_2^2 + \frac{1}{4}(\xi_1^2 + \xi_2^2)^2.$$
(17)

The contour map and landscape of potential (17) are shown in Fig. 2(a) and Fig. 2(b), where $\lambda_1 = 1.0$, $\lambda_2 = 0.8$, and a saddle point, stable focuses, unstable focuses are shown as symbols , . The order parameter equations (10) and (11) represent the motion of a particle on potential surface. If the initial value of the k-th order parameter is given as the location \mathbf{X} in Fig. 2(a), the particle, which corresponds to the value of order parameter, roles down on the potential surface and settles in one stable focus (ξ_1 =1.0). Since this stable focus is corresponding to a prototype pattern, the corresponding pattern is recognized. At this time the ridge of potential, which is represented by the line through to in Fig. 2(a), is the watershed to determine which stable focus is chosen. And Fig. 3 shows dynamics of the order parameter, where $\xi_1(0)=0.5, \xi_2(0)=0.6, \lambda_1=1.0, \text{ and } \lambda_2=0.8$. This case shows that ξ_1 grew to 1, namely pattern v_1 was recognized.



Fig. 2. Potential function (λ_1 =1.0, λ_2 =0.8)



Fig. 3. Order parameter ($\xi_1(0)=0.5, \xi_2(0)=0.6, \lambda_1=1.0, \lambda_2=0.8$)

III. NEW STEREO MATCHING ALGORITHM BASED ON SYNERGETICS

A. Stereo matching problem

This paper considers the following stereo vision system:

(1) Camera geometry is parallel as shown in Fig.'s 4(a) and (d), so that epipolar lines are parallel with *X*-axis in a camera coordinate system (Fig.'s 4(b), (c), (e), and (f)).

(2) The object of stereo vision is building blocks.

(3) Feature points are the edges of the building blocks and obtained by Smooth filter of 3×3 pixels window, Laplacian operator of a 3×3 pixels window, and the Thinning operator of a 3×3 pixels window.

(4) Matching primitives are edge points as shown in Fig.'s 4(b), (c), (e), and (f), where occlusion (a point C) appears in the right image (Fig. 4(c)), and reversal position (points G, H, and I) appears in the both image (Fig.'s 4(e) and (f)).

Hence, stereo images generally include occlusion and reversal position. The stereo images with occlusion have feature points not to be match. And the stereo image with reversal position cannot obtain correct correspondence by the method based on Markov condition such as DP-matching. Then, we take a two-stage matching approach. In the first stage, the feature points to be obviously corresponded are found. Here, it is expected the occluded feature points are not corresponded. The remaining points are matched in the second stage.



Fig. 4. Stereo matching

In a stereo matching problem, we assign an input pattern vector on an edge point in one image to q and do the *k*-th prototype pattern vector on edge points in the other image to v_k . Further, the initial value of the *k*-th order parameter $\xi_k(0)$ is obtained by inner products of v_k^+ and q (equation (18)).

$$\langle \mathbf{v}_k^+ | \mathbf{q} \rangle = \xi_k(0) \,. \tag{18}$$

B. Construction of the parameters in the first stage

Let us consider the construction of pattern vectors v_k , and attention parameter round a current edge. We set up the parameters in the first stage so as to find the edge points to be obviously corresponded. For pattern vectors, we mainly use the brightness information of the vicinity of the edge, and further do the phase information of edges for attention parameter.

1) Construction of pattern vectors \mathbf{v}_k

In the first stage, we construct pattern vector v_k with the following elements (see Fig. 5):

(1) Brightness values of pixels of four area windows (small 7 \times 7, large 7 \times 5 of 3 \times 5 unit, perpendicular 1 \times 48, and both ends area of edge a couple of 11 \times 11).

(2) Length and angle of an edge sequence, which extends in direction of *Y*-axis from the current edge.

Consequently v_k has the pixel elements of (1) and (2).

2) Construction of attention parameter λ_k

The following elements are adopted as the determination of an attention parameter (see Fig. 6):

(1) Brightness values of pixes of two area windows (horizontal 106×1 : *hori* and perpendicular 1×240 : *perp*).

(2) Average brightness of pixels of small area window small.

(3) Information about position of the feature point on the image: *posi*.

(4) Information about distance to the neighborhood edge of the feature point: *edge-dist*.

(5) Length and angle of an edge sequence, which extends in the direction of *Y*-axis from the current edge: *edge*.

Let us define the similarity between two vectors X and Y. Then, x_j and y_j represent the *j*-th element of the respective vector. If X and Y are scalar, N is equal to 1.

$$r(X,Y) = 1.0 - \frac{1}{N} \sum_{j=1}^{N} \frac{|x_j - y_j|}{\max(x_j, y_j)},$$
(19)

$$r(X,Y) = 1.0 - \frac{1}{2} \sum_{j=1}^{N} |x_j - y_j|, \qquad \sum_{j=1}^{N} x_j = \sum_{j=1}^{N} y_j = 1$$
(20)

$$r(X,Y) = \frac{1}{\sigma_x \sigma_y} \frac{1}{N} \sum_{j=1}^{N} (x_j - \bar{x})(y_j - \bar{y}) .$$
(21)

Equation (19) is used for *small*, *posi*, *edge-dist*, (20) is used for *perp*, and (21) is used for *hori*. Let the attention parameter of the *k*-th edge for an input pattern be λ_k . Then, λ_k is defined as the following liner combination of 6 similarities.

$$\lambda_{k} = \sum_{j} w_{j} r_{j}, \ j \in \{hori, perp, small, posi, edge-dist, edge\} (22)$$

C. Construction of the parameters in the second stage

In the second stage, pattern vectors are set up to a fixed scalar value (=1.0) because the brightness information for the pattern vectors is not effective for correct correspondence in the first stage. Thus, only the attention parameter is used in

the second stage. The attention parameter is determined by the same information of *hori*, *small*, *edge-dist*, *edge* as the first stage, and the following in addition:

(5) Information about position from the camera using the matching result of the first stage: *depth* ((19) is used). Then, λ_k is obtained by the following:

$$\lambda_{k} = \sum_{j} w_{j} r_{j}, \quad j \in \{hori, small, edge-dist, edge, depth\}$$
(23)

D. Whole process of stereo matching

Let the number of edges on an epipolar line in a left image and a right image be N and M, respectively. Let the *i*-th edge of the left image and the *k*-th edge of the right image be an input pattern vector \boldsymbol{q}_i and a prototype pattern vector \boldsymbol{v}_k . The algorithm of stereo matching is given as the following steps: *First stage*

Step 1: Calculation of an initial value $\xi_k(0)$ by equation (18)

- *Step 2*: Construction of λ_k by equation (22)
- Step 3: Calculation of order parameter equations (9)
- *Step 4*: Determination of a matching edge to q_i
- Step 5: Repeat the steps from 1 to 4 for q_i (*i*=1 ~ N)
- Step 6: Replace the right image with input and the left image with prototype (cross-reference), and then repeat the steps form 1 to 5
- Step 7:Define the edges that are the same result for crossreference as correspondence points

The remained edge points without the correspondence edges in the first stage are processed in the second stage. The procedure is the same as the first stage except for using equation (23) instead of (22) in *Step 2*.



Fig. 5. The information elements for pattern vectors





Fig. 6. The information elements for attention parameter

IV. STEREO MATCHING ALGORITHM BASED ON DP-MATCHING

A. Conventional method based on DP-matching

A stereo matching algorithm based on DP-matching constructs a two-dimensional search plane on the left and right epipolar lines as shown in Fig. 7. Let the number of edges on epipolar lines in a left image and a right image be M_L and M_R , respectively and we introduce the following notations:

- (1) $m = (m_R, m_L)$ is a determination node constructed by edge m_R of the right and edge m_L of the left.
- (2) $g(\boldsymbol{m}, \boldsymbol{k})$ is a cost function from \boldsymbol{k} to \boldsymbol{m} .
- (3) f(m, k) is a cost of optimal path from k to m, and expressed with the following formula,

 $f(\mathbf{m}, \mathbf{k}) = \max\{f(\mathbf{m}-i, \mathbf{k}) + g(\mathbf{m}, \mathbf{m}-i)\}, f(\mathbf{k}, \mathbf{k}) = 0.$ (24) Here, $\mathbf{k} = (k, l), i = (i, j), 0 \quad i \quad m_R - k, 0 \quad j \quad m_L - l, \text{ and } i + j \quad 0.$

Using conventional DP-matching, the next path from m is chosen so as to maximize or minimize a cost function (24). Hence, the DP-matching cannot get correct correspondence for the stereo image with reversal position because of Markov condition that an order of edge points do not interchange between the left and the right. Then, we take the same approach of two-stage matching as the case employing Synergetics.



Fig. 7. 2-D search plane of DP-matching

B. New stereo matching algorithm based on two-stage DPmatching

We try a stereo matching algorithm based on the two-stage DP-matching. Here, a cost function for DP-matching is employed with the same elements as the method based on Synergetics. Namely, the cost function of the first stage is equation (22), and that of the second stage is equation (23). This is because the information for stereo matching needs to be unified between the two methods for fair comparison. The correspondence edges are determined by the same cross-reference as the case of the method based on Synergetics.

V. EXPERIMENT

A. Experimental environment

The environment of experiment is as follows:

- (1) Camera geometry is parallel, and camera parameter is known.
- (2) A stereo image has 320×240 pixels with 256 gray scales.
- (3) 30 stereo images of building blocks including occlusion and reversal position are taken.

(4) Computer environment is as follows:

CPU: Pentium II (400MHz),

Memory: 128MB,

OS: Windows98, and

Programming language: Microsoft Visual C++ 6.0

Table I shows the classification of the 30 images. Some typical examples are shown in Fig. 8. Here, the edges of objects are emphasized. Fig.'s 8(a), 8(b), and 8(c) show the case of occlusion, the case of reversal position, and the case of occlusion and reversal position.

		The number of building blocks				
		1	2	3	4	5
Feature of stereo image	(a)	2	1	0	0	0
	(b)	0	6	4	0	0
	(c)	0	0	3	2	0
	(d)	0	3	1	0	0
	(e)	0	1	5	1	1

(a) None (b) Occlusion (single image) (c) Occlusion (both images) (d) Reversal position (e) Reversal position and occlusion





Fig. 8. Typical examples of stereo image

We evaluate matching precision and computation time for the method based on Synergetics, the method based on twostage DP-matching and template matching. Here, the matching precision is defined as

The number of correct correspondence points

The number of possible correspondence points where the dominator is counted up by a man.

B. Experimental results

1) An example of stereo matching

First, we show the 3-D reconstruction from the result of stereo matching. Fig. 9 shows the sample stereo image that includes occlusion and reversal position. Fig. 10 shows the 3-D reconstruction of Fig. 9 from the matching result. Here, Fig.'s 10(a), 10(b), and 10(c) show the results of Synergetics, two-stage DP-matching, and correlation (7×7 pixels window), respectively. It is confirmed that the method of Synergetics realizes correct matching for occlusion and reversal position, but two-stage DP-matching and correlation show incorrect matching. Secondly, we measured the computation time and the matching precision of each method. The results are summarized in Table II. It is confirmed that the stereo matching algorithm of Synergetics has a quite high matching precision but requires much computation time.



Fig. 9. A sample stereo image of experiment



TABLE II COMPUTATION TIME AND MATCHING PRECISION

Method	Computation time (sec)	Matching precision (%)
Synergetics	19.6	99.09 (325/328)
Two-stage DP	6.18	57.01 (187/328)
Correlation	5.16	53.96 (177/328)

2) Experimental results of all the images

First, the measurement of matching precision for all the stereo images is shown in Fig. 11. Here, symbols , , and

show the Synergetics, the two-stage DP-matching, and the correlation. In Fig. 11, it is confirmed that high precision matching is realized by Synergetics. On the other hand, the method of two-stage DP-matching and the correlation are obviously inferior to Synergetics. Especially, the matching precision of images with the complicated correspondence relation was quite low (see image No. 29). The followings can be considered as the reason:

(1) Although the evaluation function of DP-matching is a linear combination of evaluation elements, the order parameter equations of Synergetics are nonlinear functions with order parameter and attention parameter.

(2) In the DP-matching, all the feature points do not become correspondence candidates because of Markov condition. However, in the method based on Synergetics, all the feature points simultaneously become correspondence candidates.

Secondly, the results of computation time that obtained from all the experiment images are shown in Fig. 12. It is confirmed that the computation time of Synergetics method increases exponentially with the increase of the number of feature points. It is due to increase the number of order parameter equations to be solved numerically.

VI. CONCLUSIONS

We proposed a new stereo matching algorithm based on Synergetics. The method had quite high matching precision. In this sense, our proposed method was quite effective. It, however, has a shortcoming with much computation time. For the improvement of computation time, we are going to develop a high-speed algorithm that does not numerically solve order parameter equations. Furthermore, there is still a room of improvement of the construction of the parameters.

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Fig. 11. Matching precision of each method



Fig. 12. Computation time of each method