An Approach of Adaptive Fuzzy Control System with Uncertainties

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Abstract— When using the Lyapunov synthesis approach to construct an adaptive fuzzy control system, one important way is to regard the fuzzy systems as approximators to approximate the unknown functions in the system to be controlled. Concerning the unknownness, generally there are two cases: a completely unknown case, and a partly unknown case. However, most of the schemes presented so far have only focused on the former. Clearly, if an unknown function belongs to the latter, the knowledge available about the function should be utilized as much as possible in the development of the control system. In this paper, our goal is to design an adaptive fuzzy controller for a class of nonlinear systems with uncertainty, which can correspond to the either case. Also, we propose a unique way to deal with the uncertainties, i.e., adopt a switching function with an alterable coefficient, which is tuned by adaptive law based on the tracking error.

I. Introduction

For the last decade and more, a large number of researches have been focused on the adaptive fuzzy control systems, and have achieved success in a sense. In such an adaptive fuzzy control system, the Lyapunov synthesis approach is used to construct a stable adaptive fuzzy controller. A key of element of these successes has been the merger of robust adaptive control theory with fuzzy approximation theory, where the unknown functions in the system are approximated by parameterized fuzzy approximators.

Obviously, before using a fuzzy approximator to approximate an unknown function, the extent of the unknownness should be examined. Generally, there are two cases: a completely unknown case, and a partly unknown case. Actually, most of the schemes presented so far have only focused on the former, and few studies pay attention to the latter. In a system to be controlled, if an unknown function belongs to the latter, the knowledge available about the function, clearly, should be utilized to the maximum in the development of the control system. Although some papers [1]-[2] focused such a problem, the proposed schemes did not involve the control gain, which is not a trivial problem indeed in a control system.

On the other hand, among the schemes of adaptive fuzzy control system proposed so far, the upper bounds of uncertainties, and the reconstruction errors between the optimal approximators and their corresponding functions to be approximated are assumed to be known. Actually, such an upper bound is not easy to be known in a practical control system.

In this paper, our goal is to design an adaptive fuzzy controller for a class of nonlinear systems with uncertainty of either of the previously mentioned types. Also, to deal with the uncertainty, we adopt a switching function with an alterable coefficient, which is tuned by an adaptive law based on the tracking error. The adaptive law to adjust all parameters will be developed based on the Lyapunov synthesis approach. It is shown that the proposed adaptive fuzzy controller guarantees tracking error, between the output of the considered system and the desired value, to be uniformly bounded, also the bound can be made arbitrarily small by choosing appropriately related parameters, while maintaining all signals in the system asymptotically stable.

II. Problem Statement

This paper focuses on the design of adaptive fuzzy control algorithms for a class of nonlinear systems whose equation of motion can be expressed in the canonical form:

$$x^{(n)}(t) + f_1(X(t)) = b_1(X(t))u(t) + d(t) \tag{1}$$

where $X^T(t) = \left[x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\right]$ is the state, $u(t)$ is the control input, f_1, and b_1 are linear or nonlinear functions, and d denotes the uncertainty of the system. In the above system, generally functions f_1, b_1 are not known as well as d. However, there is a case that they can be partly known prior to developing the control system. In this way, the knowledge about the functions, clearly, should be utilized as much as possible in the development of the control system to improve the control performance. Therefore, system (1) can be rewritten as,

$$\begin{aligned} x^{(n)}(t) &+ f_0(X(t)) + f(X(t)) \\ &= \left[b_0(X(t) + b(X(t))\right] u(t) + d(t) \end{aligned} \tag{2}$$

where f_0, and b_0 are the known parts in f_1, and b_1, respectively, which will be used in the controller structure directly, and f, and b are the unknown parts in f_1, and b_1, respectively. If f_1 or b_1 in (1) is completely unknown previously, f_0 or b_0 simply becomes 0. Clearly, form (2) can correspond to either case: f_1 or b_1 is completely unknown or partly unknown.

Let $x_d(t)$ be a desired trajectory and define the tracking error,

$$\tilde{x}(t) = x(t) - x_d(t) \tag{3}$$

The problem we consider in this paper is to design a controller $u(t)$ for (2) which ensures the tracking error is uniformly bounded, also the bound can be made arbitrarily small by choosing appropriately related parameters, while maintaining all signals in the system asymptotically stable.

The nonlinear functions f and b in (2) are unknown, so before developing our control algorithm we have to solve the problem of approximating f and b. In the following section, it will be shown that using fuzzy IF-THEN rules, the unknown functions f and b can be approximated by some parameterized fuzzy approximators.

To proceed with our development, we state our assumptions on the system.

Assumption 1: Uncertainty $d(t)$ is bounded by a constant d^*, i.e.,

$$|d(t)| \le d^* \tag{4}$$

Assumption 2: The control gains b_0, and b satisfy the following inequalities,

$$b_0 \ge 0 \tag{5}$$

$$b > 0 \tag{6}$$

Remark 1 Compared with other schemes, there is an important difference about the assumption postulated on the uncertainty. In this paper, we just suppose that the boundary d^* exists, but its real value does not need to be known.

Remark 2 As the regular assumptions postulated on the control gain b [3][4], a prior gradient boundary $|\frac{d}{dt}b|$ should be known as well as the boundary of $|b|$. Here we remove them to improve the reality of system.

III. Adaptive Fuzzy Control

A. Fuzzy approximator

We consider a subset $U \subset \mathcal{R}^n$ of the fuzzy system with singleton fuzzifier, product inference, and Gaussian membership function. Hence, such a fuzzy system can be written as

$$\mathcal{F}(X) = W^T(t) \cdot G(X) \tag{7}$$

where $X = [x_1, x_2, \ldots, x_n]^T \in U$, $W^T(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_N(t)]$ with $\omega_j(t)$ being the so-called connection weight; $G^T(X) = [g_1(X), g_2(X), \ldots, g_N(X)]$, and $g_j(X) = \frac{\prod_{i=1}^n \mu_{A_j^i}(x_i)}{\sum_{j=1}^N \prod_{i=1}^n \mu_{A_j^i}(x_i)}$ where $\mu_{A_j^i}(x_i)$ is a Gaussian membership function, defined by

$$\mu_{A_j^i}(x_i) = \exp\left[-\left(\frac{x_i - \xi_j^i}{\sigma_j^i}\right)^2\right] \tag{8}$$

where ξ_j^i indicates the position, and σ_j^i indicates the variance of the membership function. We now can show an important property of the above fuzzy system.

Theorem 1 For any given real continuous function f on the compact set $U \subset \mathcal{R}^n$ and arbitrary ε^*, there exists an optimal fuzzy system expansion $\mathcal{F}^*(X) = W^{*T} \cdot G(X)$ such that

$$\sup_{X \in U} |f(X) - \mathcal{F}^*(X)| < \varepsilon^* \tag{9}$$

The theorem above shows that the fuzzy system $\mathcal{F}$ can be viewed as an approximator to approximate a real continuous function f. In this paper such a approximator is referred to be as an optimal fuzzy approximator.

B. Design of Controller

In this paper, we adopt the variable structure theory to construct our adaptive fuzzy control system. The sliding mode hyperplane is firstly defined as

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}(t) \quad \text{with } \lambda > 0 \tag{10}$$

where λ defines the bandwidth of the error dynamics of the system. The equation defines a time-varying hyperplane in R^n on which the tracking error $\tilde{x}(t)$ decays exponentially to zero, so that perfect tracking can be asymptotically obtained by maintaining this condition. Similarly, if the magnitude of s can be shown to be bounded by a constant Φ, then the actual tracking errors can be shown [4, 5] to be asymptotically bounded by:

$$|\tilde{x}^{(i)}(t)| \le 2^i \lambda^{i-n+1} \Phi, \quad i = 1, 2, \ldots, n-1. \tag{11}$$

The time derivative of the error metric can be rewritten as

$$\begin{aligned} \dot{s}(t) &= (b_0 + b)\,u + d - f_0 - f \\ &\quad - x_d^{(n)} + \Lambda^T \tilde{X} \end{aligned} \tag{12}$$

where $\Lambda^T = \left[0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, (n-1)\lambda\right]$, $\tilde{X}^T = \left[\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}\right]$. Referring to system (2), it naturally suggests that when b, and f are known, a controller of form,

$$u(t) = (b_0 + b)^{-1}\left(-ks + f + a_r - d^* \cdot \mathrm{sgn}(s)\right) \tag{13}$$

leads to $\dot{s}s = -ks^2 - d^*|s| + ds \le -ks^2$, and hence, $\tilde{x}(t) \to 0$ as $t \to \infty$, where $k > 0$, $a_r = f_0 + x_d^{(n)} - \Lambda^T \tilde{X}$, and d^* is the upper boundary of d. However, as mentioned before, the functions f, and b in this paper are supposed to be unknown as well as uncertainty d. Therefore, the problem is how $u(t)$ can be determined when a system involves such a kind of unknown functions. Thus we have to approximate them to proceed with the development. Here the fuzzy approximator described in the previous subsection is used. Let us denote $b^*(X) = W_b^{*T} G_b(X), f^*(X) = W_f^{*T} G_f(X)$ to be the optimal fuzzy approximators of the unknown functions $b(X), f(X)$, respectively. According to Theorem 1, there are two small positive value $\varepsilon_b^*, \varepsilon_f^*$ such that, the errors,

$$\varepsilon_b = b - b^* \quad (14)$$

$$\varepsilon_f = f - f^* \quad (15)$$

which are referred to as reconstruction errors, satisfy the following inequalities,

$$|\varepsilon_b| < \varepsilon_b^* \quad (16)$$

$$|\varepsilon_f| < \varepsilon_f^* \quad (17)$$

We also should note that the boundaries of $\varepsilon_b^*, \varepsilon_f^*$ do not need to be known in this paper. In this way, in the approximations of b and f, the necessity to assume a prior knowledge of some bounds on the reconstruction errors can be removed. Apparently, the optimal vectors W_b^* and W_f^* in the optimal fuzzy approximators are unknown either, therefore, as usual, their estimates, denoted as $\hat{b}(X) = \hat{W}_b^T(t) G_b(X), \hat{f}(X) = \hat{W}_f^T(t) G_f(X)$ are adopted, and which will be tuned based on the error dynamics s.

Now, we are ready to develop our control system. Inspired by the above control structure in (13), our adaptive fuzzy controller is determined by,

$$u = (b_0 + \hat{b})^{-1} \left[-ks + \hat{f} + a_r - (\hat{\varepsilon}_b u + \hat{\varepsilon}_f + \hat{d}) \mathrm{sgn}(s) \right] \quad (18)$$

where $\hat{\varepsilon}_b$, and $\hat{\varepsilon}_f$ are the estimates of ε_b^*, and ε_f^*, respectively, and $\hat{d}$ is the estimate of d^*. The role of adopting $\hat{\varepsilon}_b, \hat{\varepsilon}_f$ is not only to avoid a prior knowledge about the reconstruction errors, but also to make compensation for their approximation errors. $\hat{\varepsilon}_b, \hat{\varepsilon}_f$, and $\hat{d}$ are estimated by,

$$\dot{\hat{\varepsilon}}_b = \mathcal{P}_\varepsilon \{ -\gamma_b \sigma \hat{\varepsilon}_b + \gamma_b |u||s| \} \quad (19)$$

$$\dot{\hat{\varepsilon}}_f = -\gamma_f \sigma \hat{\varepsilon}_f + \gamma_f |s| \quad (20)$$

$$\dot{\hat{d}} = -\gamma_d \sigma \hat{d} + \gamma_d |s| \quad (21)$$

where γ_b, γ_f, and γ_d are the adaptation rates, and $\sigma > 0$ is a leakage constant [6], which counteracts a draft of parameter values into regions of instability in the absence of persistent excitation, $\mathcal{P}_\varepsilon$ represents a projection operator [6] necessary to ensure that $\hat{\varepsilon}_b \in \mathcal{C}_\varepsilon \subset \mathcal{R}$ that is a subspace in which $\hat{\varepsilon}_b \leq 0$ is held.
The adaptive laws are synthesized by,

$$\dot{\hat{W}}_b = \mathcal{P}_w \{ -\Gamma_b \sigma \hat{W}_b + u \Gamma_b G_b(X) s \} \quad (22)$$

$$\dot{\hat{W}}_f = -\Gamma_f \sigma \hat{W}_f - \Gamma_f G_f(X) s \quad (23)$$

where $\hat{W}_b$, and $\hat{W}_f$ are the estimates of W_b^*, and W_f^*, respectively; Γ_b, and Γ_f are some appropriate symmetric positive definite matrices which determine the adaptation rates, and $\mathcal{P}_w$ also represents a projection operator necessary to ensure that $\hat{W}_b \in \mathcal{C}_b \subset \mathcal{R}^N$ that is a subspace in which $\hat{W}_b^T G_b(X) > 0$ is held.

Remark 3 Via the projection operator $\mathcal{P}_w$ in (22), the risk of a zero-denominator calculation in (18) is prevented. Also, the boundedness of $\hat{W}_b$ is assured, even it involves the control signal u. Besides, the projection operator $\mathcal{P}_\varepsilon$ in (19) is due to a need for the system stability which is shown later.

Remark 4 Let $\hat{\omega}$ be an updating scalar, and $\hat{\omega}_{np}$ denotes the parameter before projection, i.e., $\dot{\hat{\omega}} = \mathcal{P}\{\dot{\hat{\omega}}_{np}\}$. In general, the magnitudes of $|\dot{\hat{\omega}}_{np}|$, and $|\hat{\omega}_{np}|$ are greater then or equal to $|\dot{\hat{\omega}}|$, and $|\hat{\omega}|$, respectively. Namely,

$$|\dot{\hat{\omega}}_{np}| \geq |\dot{\hat{\omega}}| \quad (24)$$

$$|\hat{\omega}_{np}| \geq |\hat{\omega}| \quad (25)$$

Before the stability analysis of the control system showed above, we give a lemma which is needed in the stability analysis.

Lemma: The following inequality holds:

$$\tilde{\omega} \dot{\hat{\omega}} \leq \tilde{\omega} \dot{\hat{\omega}}_{np} \quad (26)$$

where $\tilde{\omega} = \hat{\omega} - \omega^*$, $\tilde{\omega}_{np} = \hat{\omega}_{np} - \omega^*$ with ω^* being a constant, and $\hat{\omega}, \hat{\omega}_{np}$ are same things as in Remark 4.

Proof:

$$\tilde{\omega} \dot{\hat{\omega}}_{np} - \tilde{\omega} \dot{\hat{\omega}} = \tilde{\omega} \left(\dot{\hat{\omega}}_{np} - \dot{\hat{\omega}} \right) \quad (27)$$

As to $\dot{\hat{\omega}}_{np}$, there are two cases: (a) $\dot{\hat{\omega}}_{np}$ is on an increasing direction; or (b) $\dot{\hat{\omega}}_{np}$ is on a decreasing direction. Now if we can show that (27) ≥ 0 in either case, then (26) holds.
(a) In case of $\dot{\hat{\omega}}_{np}$ increasing:
Because $\dot{\hat{\omega}}_{np}$ is increasing, the following relations are held:

$$\dot{\hat{\omega}}_{np} - \dot{\hat{\omega}} \begin{cases} = 0, & \text{if} \quad \hat{\omega}_{np} \in \mathcal{C} \\ > 0, & \text{if} \quad \hat{\omega}_{np} \notin \mathcal{C} \end{cases} \quad (28)$$

$$\tilde{\omega} > 0, \quad \text{if} \quad \hat{\omega}_{np} \notin \mathcal{C} \quad (29)$$

where $\mathcal{C} \subset \mathcal{R}$ is a subset in which $\dot{\hat{\omega}}_{np} = \dot{\hat{\omega}}$, and $\hat{\omega}_{np} = \hat{\omega}$ are held. Substituting (28-29) into (27), we can, apparently, get that (27) ≥ 0.
(b) In case of $\dot{\hat{\omega}}_{np}$ decreasing:
The proof of (27) ≥ 0 follows after straightforward getting the counterpart of case (a). □

We begin the stability analysis by defining a Lyapunov function as

$$V = \frac{1}{2} \Big(s^2 + \tilde{W}_b^T \Gamma_b^{-1} \tilde{W}_b + \tilde{W}_f^T \Gamma_f^{-1} \tilde{W}_f + \gamma_b^{-1} \tilde{\varepsilon}_b^2 + \gamma_f^{-1} \tilde{\varepsilon}_f^2 + \gamma_d^{-1} \tilde{d}^2 \Big) \quad (30)$$

where,

$$\tilde{W}_b = \hat{W}_b - W_b^* \quad (31)$$

$$\tilde{W}_f = \hat{W}_f - W_f^* \quad (32)$$

$$\tilde{\varepsilon}_b = \hat{\varepsilon}_b - \varepsilon_b^* \quad (33)$$

$$\tilde{\varepsilon}_f = \hat{\varepsilon}_f - \varepsilon_f^* \quad (34)$$

$$\tilde{d} = \hat{d} - d^* \quad (35)$$

Let $\hat{W}_{b,np}$ denote the parameter vector before projection, i.e., $\hat{W}_b = \mathcal{P}_w\{\hat{W}_{b,np}\}$. Thus,

$$\dot{\hat{W}}_{b,np} = -\Gamma_b \sigma \hat{W}_b + u\Gamma_b G_b(X)s \quad (36)$$

According to the lemma above, we have

$$\tilde{W}_b^T \Gamma_b^{-1} \dot{\hat{W}}_b \leq \tilde{W}_b^T \Gamma_b^{-1} \dot{\hat{W}}_{b,np} \quad (37)$$

Correspondingly,

$$\dot{\hat{\varepsilon}}_{b,np} = -\gamma_b \sigma \hat{\varepsilon}_b + \gamma_b |u||s| \quad (38)$$

$$\gamma_b^{-1} \tilde{\varepsilon}_b \dot{\hat{\varepsilon}}_b \leq \gamma_b^{-1} \tilde{\varepsilon}_b \dot{\hat{\varepsilon}}_{b,np} \quad (39)$$

Taking the time derivative of the Lyapunov function of (30), and substituting (37) and (39) in which yields,

$$\begin{aligned} \dot{V} \leq\ & s\dot{s} + \tilde{W}_b^T \Gamma_b^{-1} \dot{\hat{W}}_{b,np} + \tilde{W}_f^T \Gamma_f^{-1} \dot{\hat{W}}_f \\ & + \gamma_b^{-1} \tilde{\varepsilon}_b \dot{\hat{\varepsilon}}_{b,np} + \gamma_f^{-1} \tilde{\varepsilon}_f \dot{\hat{\varepsilon}}_f + \gamma_d^{-1} \tilde{d} \dot{\hat{d}} \end{aligned} \quad (40)$$

Substituting (12) into (40), we have,

$$\begin{aligned} \dot{V} \leq\ & [(b_0 + b)u - f + d - a_r]\, s \\ & + \tilde{W}_b^T \Gamma_b^{-1} \dot{\hat{W}}_{b,np} + \tilde{W}_f^T \Gamma_f^{-1} \dot{\hat{W}}_f \\ & + \gamma_b^{-1} \tilde{\varepsilon}_b \dot{\hat{\varepsilon}}_{b,np} + \gamma_f^{-1} \tilde{\varepsilon}_f \dot{\hat{\varepsilon}}_f + \gamma_d^{-1} \tilde{d} \dot{\hat{d}} \end{aligned} \quad (41)$$

Combining relations (14-15), and (31-34) with the first right-hand term in (41), one becomes,

$$\begin{aligned} & [(b_0 + b)u - f + d - a_r]\, s \\ =\ & [b_0 u + \varepsilon_b u + b^* u - \varepsilon_f - f^* + d - a_r]\, s \\ =\ & \Big[b_0 u + \varepsilon_b u + (\hat{W}_b^T G_b - \tilde{W}_b^T G_b) u \\ & - \varepsilon_f - (\hat{W}_f^T G_f - \tilde{W}_f^T G_f) + d - a_r \Big]\, s \end{aligned} \quad (42)$$

Substituting (42), (18-21), (23), (36), and (38) into (41) follows,

$$\begin{aligned} \dot{V} \leq\ & -ks^2 + \hat{\varepsilon}_b |s| \left(|u| - u \right) \\ & -\sigma \left(\tilde{W}_b^T \hat{W}_b + \tilde{W}_f^T \hat{W}_f + \tilde{\varepsilon}_b \hat{\varepsilon} + \varepsilon_f \hat{\varepsilon}_f + \tilde{d}\hat{d} \right) \end{aligned} \quad (43)$$

Let's pay attention to the second right-hand term in above expression. It leads to that $\hat{\varepsilon}_b |s| \left(|u| - u \right) \leq 0$, because $|u| - u \geq 0$ and $\hat{\varepsilon}_v \leq 0$ due to the projection algorithm in (19). Therefore, (43) becomes,

$$\begin{aligned} \dot{V} \leq\ & -ks^2 - \frac{\sigma}{2} \left(\tilde{W}_b^T \tilde{W}_b + \tilde{W}_f^T \tilde{W}_f + \tilde{\varepsilon}_b^2 + \tilde{\varepsilon}_f^2 + \tilde{d}^2 \right) \\ & + \frac{\sigma}{2} \left(W_b^{*T} W_b^* + W_f^{*T} W_f^* + \varepsilon_b^{*2} + \varepsilon_f^{*2} + d^{*2} \right) \\ \leq\ & -\alpha V + \epsilon \end{aligned} \quad (44)$$

where,

$$\epsilon = \frac{\sigma}{2} \left(W_b^{*T} W_b^* + W_f^{*T} W_f^* + \varepsilon_b^{*2} + \varepsilon_f^{*2} + d^{*2} \right) \quad (45)$$

$$\alpha = \min \left(2k, \lambda_{\min}(\Gamma_b)\sigma, \lambda_{\min}(\Gamma_f)\sigma, \gamma_b\sigma, \gamma_f\sigma, \gamma_d\sigma \right) \quad (46)$$

which implies,

$$\begin{aligned} V(t) \leq\ & e^{-\alpha(t-t_0)} V(t_0) + \int_{t_0}^{t} e^{-\alpha(t-\tau)} \epsilon d\tau \\ =\ & \left(V(t_0) + \frac{\epsilon}{\alpha} \right) e^{-\alpha(t-t_0)} + \frac{\epsilon}{\alpha} \end{aligned} \quad (47)$$

Therefore, all signals in (30), which also are signals involved in the system, are bounded. Besides, from (30) and (47), we can get that there exists $\mathcal{T}$ such that for $t \geq \mathcal{T}$, $s(t)$ satisfies

$$s(t) \leq \sqrt{\frac{2\epsilon}{\alpha}} \quad (48)$$

which implies $s(t)$ tends to a ball centered at the origin with radius $\sqrt{\frac{2\epsilon}{\alpha}}$. Also, from (11) and (48), we have,

$$|\tilde{x}(t)| \leq \frac{1}{\lambda^{n-1}} \sqrt{\frac{2\epsilon}{\alpha}} \quad (49)$$

which means the tracking error $\tilde{x}(t)$ is uniformly bounded. Further, from (49) and (45)-(46), we can see that the bunndary of $\tilde{x}(t)$ depends on the bandwidth of the error dynamics λ in (10), the coefficient for error dynamics k in (18), the leakage constant σ and the adaptation rates Γ $(\gamma_b, \gamma_f, \gamma_d, \Gamma_b, \Gamma_f)$ in (19)-(23), therefore, the magnitude of boundary of $\tilde{x}(t)$ can be made arbitrarily small by adjusting the parameters λ, k, σ, and Γ.

IV. Simulation

To clarify the proposed design procedure, we apply the adaptive fuzzy controller developed in previous section to control the following unstable nonlinear system:

$$\dot{x}(t) = 1 + f(x) + (1.1 + b(x))u(t) + d(t) \quad (50)$$

where $f(x) = 0.5 \times \frac{1-e^{-x(t)}}{1+e^{-x(t)}}$, and $b = \cos(2x)$ are two unknown functions, and $d(t) = 2.5\sin(7t)$ is an unknown uncertainty. Without the control, i.e., $u(t) = 0$, it can be easily seen that the system is unstable (Fig.1). The control objective is to force the system state $x(t)$ to the origin, i.e., $x_d(t) = 0$. To approximate the unknown functions $f(x)$ and $b(x)$, we use the fuzzy system as mentioned in subsection 3.1. For them, the simulation is conducted with the following fuzzy rules:

$$R_j : IF\ x\ is\ A_j\ THEN\ f\ is\ w_{fj}$$

or,

$$R_j : IF\ x\ is\ A_j\ THEN\ b\ is\ w_{bj}$$

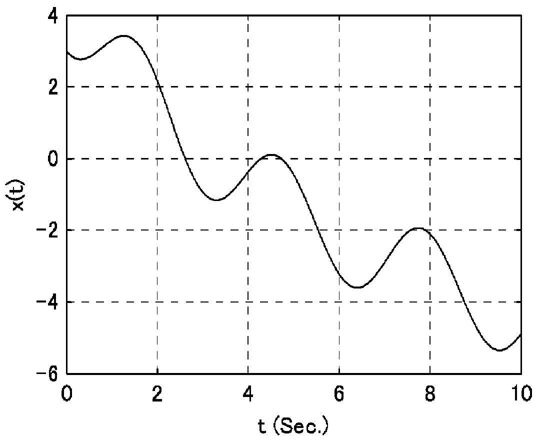

Fig. 1. Trajectory of state x(t) with u(t) = 0 and x(0) = 3

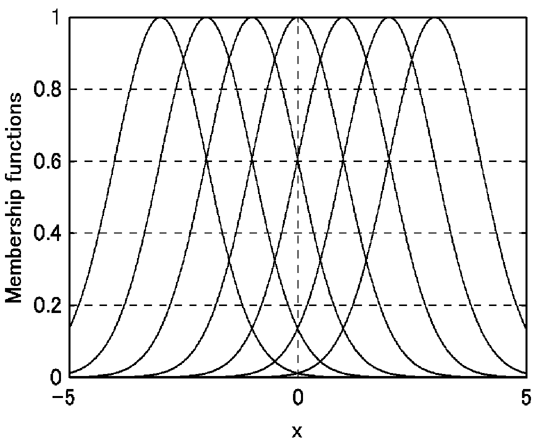

Fig. 2. Membership functions in precedent

where $j(= 1, 2, \ldots, 7)$ is rule's number; A_j is fuzzy set, which is shown as in Fig.2; and w_{fj}, and w_{bj} are some singleton values for which 1 is initially assigned.

Control law (18) was used where $k = 0.5$, with the adaptive laws (19)-(23) where $\sigma = 1, \gamma_b = 0.2, \gamma_f = 0.4, \gamma_d = 0.1$, and $\Gamma_b = 0.8I, \Gamma_f = 0.8I, \Gamma_B = 0.2I$ with I being some appropriate identity matrices. To perform the projectron algorithm in (19), we take $\hat{\varepsilon}_b = 0$, if $\hat{\varepsilon}_{b,np} > 0$ where $\hat{\varepsilon}_{b,np}$ is the estimate before projection. Correspondingly for $\hat{W}_b$ in (22), we take $\hat{W}_b = 0I$, if $\hat{W}_{b,np}^T G_b(X) < 0$. In addition, this simulation takes the values that $\lambda = 1$ and initial state $x(0) = 3$.

Simulation results are shown in Fig.3-4. Fig.4 shows the evolution of $x(t)$ where a good performance is observed. The amount of control effort required to achieve the above level of the performance is illustrated in Fig.4. Although the control effort is involved in the adaptive law (22), the boundedness can be confirmed in Fig.4. We also should note that, when the system state $x(t)$ enters around the sliding surface, sign function $\text{sgn}(s)$ begins working frequently so that such a control law leads to control chattering. Actually, by adopting a saturation function $\text{sat}(s/\phi)$ where ϕ is a little constant instead of $\text{sgn}(s)$, the control chattering can be surely prevented [7].

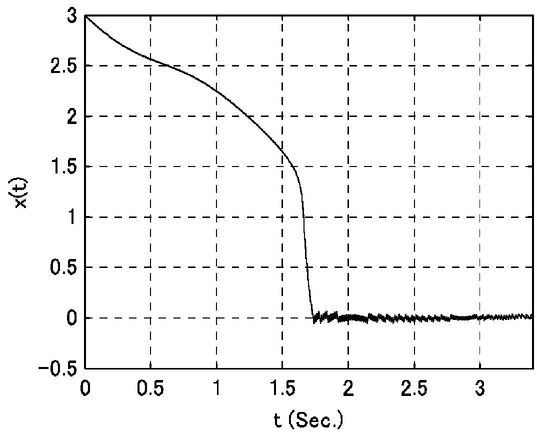

Fig. 3. Evolution of state x(t)

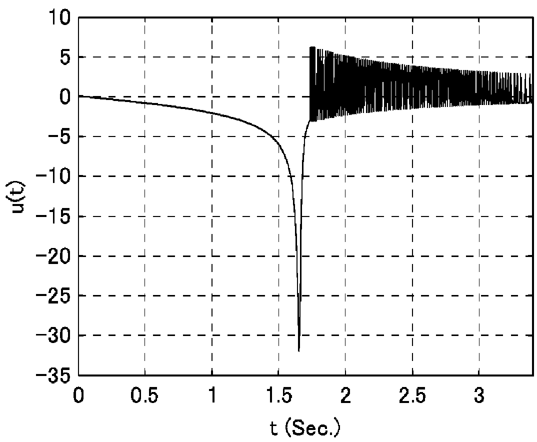

Fig. 4. Amount of control law u(t)

V. Conclusion

In this paper, we proposed an approach for a class of nonlinear systems with uncertainties. The results achieved in this paper can be summarized in a theorem as follows:

Theorem 2 If the plant (1) is controlled by (18) with the adaptive laws (19)-(23), then tracking errors will be uniformly bounded, also the bound can be made arbitrarily small by choosing appropriately control parameters, while maintaining all signals in the system asymptotically stable.

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